(1) **Introduction to groups**: group, subgroup and order, examples, fields, the dihedral and symmetric groups $D_n$ and $S_n$, matrix groups and quaternions, direct products, cosets and Lagrange’s theorem, cyclic groups, $\mathbb{F}^\times$ is cyclic, commutator, centralizer, normalizer, normal subgroups and quotient groups.

(2) **The isomorphism theorems**: homomorphisms, behavior of subgroups under homomorphisms, the three isomorphism theorems.

(3) **Group actions on sets**: basic definitions and properties, Cayley’s theorem, the coset representation, a formula for the number of orbits, applications, the game of 16 squares, Rubik’s cube.

(4) **The symmetric group**: conjugacy classes, simplicity of $A_n$.

(5) **Sylow’s theorems**: $p$-groups, groups of order $p$, $p^2$, $p^3$, Cauchy’s theorem, $S_3$, $S_4$, groups of order $pq$ and $p^2q$, $GL_n(\mathbb{F})$.

(6) **Composition series, the Jordan-Holder theorem**, solvable groups, nilpotent groups.

(7) **Free groups**: properties, reduced words, generators and relations, famous problems.

(8) **Finitely generated abelian groups, semi-direct products, groups of order 12**: the structure theorem for f.g. abelian groups, application of semi direct products.

(9) **Rings**: the integers and the integers modulo $n$, matrices over a ring, polynomial and power series rings, the ring of quotient, ring homomorphisms and isomorphism theorems, the Chinese remainder theorem.

(10) **Euclidean, principal ideal and unique factorization domains**: division and gcd”s. Euclidean algorithm, irreducible and prime elements, UFD, arithmetic in UFD, Gauss’ lemma, the ring of polynomials over a UFD.

(11) **Introduction to the module theory including tensor products and basic homological algebra**.