Computations with Matrices and Polynomials

SYLLABUS

CSc 87100/CRN 64061 **SPRING 2020**

3 credits Fridays 2-4 pm Room TBA

Victor Pan, Distinguished Professor,

Fellow of the American Math. Society for his "Contributions to the Mathematical Theory of Computation"

Lecture Notes are available upon the request to

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Optional: no credit seminar on Fridays, 4.15-6.15 pm., Room TBA

OVERVIEW: The course covers topics from Polynomial, Linear and Multilinear Algebras, and can be of interest for students in Mathematics and Computational Mathematics. One can reach research frontiers and research challenges in these areas much more easily than in classical Mathematics.

Rationale

Algorithms for matrix and polynomial computations are the backbone of modern computations in Sciences, Engineering, and Signal and Image Processing. They are routinely invoked when one turns on computer, TV or radio. The subjects have been extensively studied in both fields of Computer Science and Computational Mathematics and are the source of exciting research challenges.

The course will introduce students to some most fundamental methods and techniques of the design, analysis and implementation of modern algorithms.

It can lead students to research in Computer Science and Mathematics, currently supported by Instructor's current NSF Grants ($1,056,291) and PSC CUNY Award ($11,998), to refereed publications in Journals and Proceedings of Conferences, and eventually to the defense of PhD Theses. So far the Instructor has advised and mentored 12 students in each of the CUNY Programs in Computer Science and Mathematics (24 students overall) towards successful PhD defenses, but *the students would also obtain 3 credits just for successful learning the materials of this course.*

Description

The course will cover some fundamental topics of matrix and polynomial computations. The instructor has published on these subjects four books (over 1500 pages) and has about 300 hundred other refereed research and survey publications. He will *ADJUST THE LIST OF TOPICS TO STUDENTS' INTERESTS* and will facilitate the study by supplying reading materials.

In his previous teaching of such a course in the Graduate Center even students having no previous knowledge and experience in these areas have reached the research frontiers in Computer Science and Computational Mathematics. Students will be offered participation (under support from instructors' Grants and Award) in the advanced study of recent and new algorithms, in their formal mathematical analysis and computer implementation.

Topic List

- Fast Fourier transform
- Basic operations of computer algebra (polynomials and rational multiplication, division, multipoint evaluation, interpolation, computing GCDs and LCMs)
- Structured matrices such as Toeplitz, Hankel, circulant, Vandermonde, Cauchy, and Hierarchical Semiseparable (HSS) matrices
- Efficient algorithms for structured matrices. Their link to computations with polynomials and rational functions
- Data compression by using matrix structures
- Low rank approximation of matrices and tensors
- General matrices, their factorizations, norms and other basic concepts and techniques of matrix computations
- Techniques for error control and estimation in numerical computing with rounding
- Randomization methods

Leaning Goals

Students are expected to

- Understand the basic principles, concepts and techniques of symbolic and numerical computing
- Learn some fundamentals of algorithm design and analysis
- Learn efficient algorithms for operations with polynomials, rational functions and general and structured matrices
- Learn the basic techniques of data compression for structured matrices and matrices admitting their low rank approximation
- Learn and possibly practice the basics of the implementation of symbolic and numerical algorithms
- Get a chance to advance in research, publications and preparation to PhD defense

Assessment

- Class participations and discussions will be used to evaluate students' understanding of concepts of algebraic and numerical computations. The attendance and participation account for 10% of the final grade
- Homework assignments (40% of the final grade) will be designed to provide the

opportunities for students to verify their understanding of the current subjects of the study and their ability to employ the relevant techniques and algorithms

- Final and possibly midterm tests will represent 50% of the final grade. They will give students chances to show their overall understanding of the course subjects
- The students' advances in research and implementation of Matrix and Polynomial algorithms can demonstrate their knowledge and understanding of the course materials and will be counted as partial substitution for homework and exams towards final grade.