Defense of Dissertation

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Location: Math Thesis Room

Title: Turaev Surfaces and Toroidally Alternating Knots

Abstract: In this thesis, we study knots and links via their alternating diagrams on closed orientable surfaces. Every knot or link has such a diagram by a construction of Turaev, which is called the Turaev surface of the link. Links that have an alternating diagram on a torus are called toroidally alternating. For a toroidally alternating link, the minimal genus of its Turaev surface may be greater than one. Hence, these surfaces provide different topological measures of how far a link is from being alternating.

First, we classify link diagrams with Turaev genus one and two in terms of an alternating tangle structure of the link diagram. The proof involves surgery along simple loops on the Turaev surface, called cutting loops, which have corresponding cutting arcs that are visible on the planar link diagram. These also provide new obstructions for a link diagram on a surface to come from the Turaev surface algorithm. We also show that inadequate Turaev genus one links are almost-alternating.

We also extend Howie's characterization of alternating knots to give a topological characterization of toroidally alternating knots, which were defined by Adams. We provide necessary and sufficient conditions for a knot to be toroidally alternating. We also give a topological characterization of almost-alternating knots which is different from Ito's recent characterization.