Introduction

- Theoretical exploration of link between growth process and income distribution in the closed and open economies
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- Many other mechanisms are absent; e.g.,
  - Differences in savings propensity between rich and poor (Kaldor)
  - Poor households face credit constraints (Galor and Zeira)
  - Greater inequality generates more redistribution via political process (Alesina and Rodrik; Persson and Tabellini)
Demand and Supply of Consumption Goods

- Mass $N$ of heterogeneous individuals, indexed by $a$
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$$X = \left[ \int_{\omega \in \Omega} X(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$
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• Choice of numeraire: $q_t = 1$ for all $t$. 
Production of Intermediates

- Production of intermediates

\[ x_\omega = \int_{a \in L_\omega} \psi(\varphi_\omega, a) \ell_\omega(a) \, da \]
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Demand for intermediate $\omega$

$x(\omega) = X p(\omega)^{-\sigma}$
Optimal pricing in monopolistic competition (after change of variable):

\[ p(\varphi) = \left( \frac{\sigma}{\sigma - 1} \right) \frac{w[m(\varphi)]}{\psi[\varphi, m(\varphi)]} \]
Pricing and Profits

- Optimal pricing in monopolistic competition (after change of variable):
  \[ p(\phi) = \left( \frac{\sigma}{\sigma - 1} \right) \frac{w[m(\phi)]}{\psi[\phi, m(\phi)]} \]

- Profits:
  \[ \pi(\phi) = \sigma^{-\sigma} (\sigma - 1)^{-(\sigma-1)} X \left\{ \frac{w[m(\phi)]}{\psi[\phi, m(\phi)]} \right\}^{1-\sigma} \]
Inventing New Varieties

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  - Stock of knowledge: $\theta_K M$
  - Worker of type $a$ has productivity $T(a)$ in research sector

Growth of varieties

Each invention generates a “Melitz draw” of $\phi$ from $G(\phi)$

Allow free entry into innovation:

$$\rho + g_M = w(a) T(a) \theta_K M$$ for all $a^2 L R$. 

Inventing New Varieties

- **Invention of new varieties à la Romer**
  - Stock of knowledge: $\theta_K M$
  - Worker of type $a$ has productivity $T(a)$ in research sector
  - $\ell_R(a)$ workers of type $a$ invent $dM = \theta_K MT(a) \ell_R(a) dt$ new varieties per time interval $dt$ (strong scale effects)
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$$g_M = \theta K N \int_{a \in L_R} T(a) dH(a)$$
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g_M = \theta_K N \int_{a \in L_R} T(a) dH(a)
  \]

- Each invention generates a “Melitz draw” of $\phi$ from $G(\phi)$

- **Allow free entry into innovation:**
  \[
  \frac{\int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \pi(\phi) dG(\phi)}{\rho + g_M} = \frac{w(a)}{T(a) \theta_K M} \quad \text{for all } a \in L_R.
  \]
Assumption  $T(a)/\psi(\varphi, a)$ is increasing in $a$ for all $(\varphi, a)$
Assumption \( T(a) / \psi(\varphi, a) \) is increasing in \( a \) for all \( (\varphi, a) \)

**Lemma 1** For any closed interval \([a', a''] \in L_M\),

\[
\frac{w'(a)}{w(a)} = \frac{\psi_a [m^{-1}(a), a]}{\psi [m^{-1}(a), a]} \quad \text{for all } a \in (a', a'')
\]
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**Sorting:** Assumption implies \( \exists \ a_R \) ("cutoff") such that \( a < a_R \Rightarrow a \in L_M \) and \( a > a_R \Rightarrow a \in L_R \) (like "occupational choice" in Lucas 78)
Labor-Market Equilibrium

- **Labor market clearing**: Supply of workers of type $m(\varphi)$ equals demand for workers by firms of type $\varphi$

\[
m'(\varphi) H'[m(\varphi)] = \frac{MX}{N} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \frac{w[m(\varphi)]^{-\sigma}}{\psi[\varphi, m(\varphi)]^{1-\sigma}} G'(\varphi)
\]
Labor-Market Equilibrium

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- Differentiate and substitute wage equation:

$$\frac{m''(\varphi)}{m'(\varphi)} = (\sigma - 1) \frac{\psi_\varphi [\varphi, m(\varphi)]}{\psi [\varphi, m(\varphi)]} - \frac{\psi_a [\varphi, m(\varphi)] m'(\varphi)}{\psi [\varphi, m(\varphi)]} + \frac{G''(\varphi)}{G'(\varphi)} - \frac{H''[m(\varphi)] m'(\varphi)}{H'[m(\varphi)]}$$
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$$+ \frac{G''(\phi)}{G'(\phi)} - \frac{H''[m(\phi)] m'(\phi)}{H'[m(\phi)]}$$

- Boundary conditions

$$m(\phi_{\text{min}}) = a_{\text{min}}, \quad m(\phi_{\text{max}}) = a_R$$
Equilibrium Matching Function

- Differential equations for $w$ and $m$ have unique solution for given $a_R$
Equilibrium Matching Function

- Differential equations for \( w \) and \( m \) have unique solution for given \( a_R \).
- If boundary points change and no term in the second-order differential equation changes, new and old matching functions can intersect at most once.
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So, $a_R \uparrow \Rightarrow$ (inverse)-matching function shifts down
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So, $a_R \uparrow \Rightarrow$ (inverse)-matching function shifts down
- every worker matches with lower productivity firm
- due to log supermodularity of $\psi(\cdot)$, log wage profile on $[a_{\min}, a_R]$ must flatten (steepen) when $a_R$ increases (decreases)
Balanced-Growth Path

- **Sorting** of workers implies:

\[
g_M = \theta_K N \int_{a_R}^{a_{\text{max}}} T(a) \, dH(a) \quad \text{(RR)}
\]
Balanced-Growth Path

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(RR)

- Combining labor-market clearing and free-entry condition:

\[
\rho + g_M = \theta_K N \Lambda(a_R)
\]

(AA)
Balanced-Growth Path

- **Sorting** of workers implies:

\[ g_M = \theta_K N \int_{a_R}^{a_{\max}} T(a) \, dH(a) \]  \hspace{1cm} (RR)

- Combining labor-market clearing and free-entry condition:

\[ \rho + g_M = \theta_K N \Lambda(a_R) \]  \hspace{1cm} (AA)

- These two conditions yield a solution for \((a_R, g_M)\):

![Diagram showing the relationship between \(a_R\) and \(g_M\) with points labeled \(R\), \(E\), and \(A\).]
Two Types of Results

Autarky
- How do cross-country differences generate differences in autarky (steady-state) growth rates and wage inequality?

Integration
- How does trade integration affect countries’ growth rates and inequality?
- How do growth and inequality compare across countries in a trade equilibrium?
Hicks-neutral technology differences: $\psi_c (\varphi, a) = \theta \psi_c \psi (\varphi, a)$
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Matching function $m(\varphi; a_R)$ is common to both countries.
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- Same AA and RR curves $\Rightarrow$ same cutoff $a_R$
Cross-country Comparisons in Autarky
Manufacturing Productivity

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  - Relative wages same in both countries if same cutoff $a_R$
  - Same $AA$ and $RR$ curves $\Rightarrow$ same cutoff $a_R$
  - Same long-run growth and inequality

- **Hicks-neutral technology differences generate income level differences**, but do not affect growth and inequality
Capacity to innovate described by three parameters:

- Size of labor force: $N_c$
- Efficiency of knowledge accumulation: $\theta_k$
- Productivity of inventors: $\theta_T$

In $RR$ and $AA$ curves, these parameters enter as product:

$$N_c \theta_k \theta_T$$

If $N_i \theta_k \theta_T > N_j \theta_k \theta_T$, then $a_R < a_R$ and $g_M > g_M$.
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If $N_i \theta_K \theta_T > N_j \theta_K \theta_T$, then $a_R(i) < a_R(j)$ and $g_M(i) > g_M(j)$.

Income inequality:

- More unequal wages in manufacturing in $i$ than in $j$ due to better technology matches
- Larger size of research sector, which pays higher reward to ability
- More inequality overall
Cross-country Comparisons in Autarky
Capacity to Innovate

- **Capacity to innovate** described by three parameters

  - Size of labor force: \(N_c\)
  - Efficiency of knowledge accumulation: \(\theta_{KC}\)

In RR and AA curves, these parameters enter as product:

\[N_c \theta_{KC}\]

If \(N_i \theta_{Ki} \theta_{Ti} > N_j \theta_{Kj} \theta_{Tj}\) then \(a_Ri < a_Rj\) and \(g_Mi > g_Mj\)

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Grossman and Helpman (2015) *Growth, Trade, and Inequality*
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- If $N_i \theta_K^i \theta_T^i > N_j \theta_K^j \theta_T^j \Rightarrow a_{Ri} < a_{Rj}$ and $g_{Mi} > g_{Mj}$
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- If $N_i\theta_{Ki}\theta_{Ti} > N_j\theta_{Kj}\theta_{Tj}$ $\Rightarrow$ $a_{Ri} < a_{Rj}$ and $g_{Mi} > g_{Mj}$

- **Income inequality**:
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- **Income inequality:**
  - More unequal wages in manufacturing in \( i \) than in \( j \) due to better technology matches
  - Larger size of research sector, which pays higher reward to ability

  \( \Rightarrow \) **More inequality overall**
Modified AA curve: \((1 - s_c) (\rho + g_{Mc}) = \theta_K \Lambda (a_{Rc})\)
- Modified AA curve: $(1 - s_c) (\rho + g_{Mc}) = \theta_K N \Lambda (a_{Rc})$
- $s_i > s_j \Rightarrow$ AA curve lies above and to left in $i$
R&D Subsidies

- Modified AA curve: \((1 - s_c)(\rho + g_{Mc}) = \theta_K N\Lambda(a_{Rc})\)
- \(s_i > s_j \Rightarrow AA\) curve lies above and to left in \(i\)
- \(\Rightarrow\) Faster growth and more wage inequality in \(i\)
Suppose $i$ and $j$ differ in their sets of manufacturing technologies.

- Matching in manufacturing: Better technologies in $i$ imply better matches in $i$ for given $a$. But $a_{Ri} > a_{Rj}$ (larger manuf sector) means worse matches in $i$. First effect dominates: matches better for worker type $a$ in $i$ than $j$.

- Growth faster in $j$ than $i$ (comparative advantage in research).

- Income inequality: More unequal in $i$ at the bottom end of the distribution. $j$ has a larger research sector and R&D pays a greater return to ability. There exists a middle range of abilities such that for $a$ in this range, relative wage is higher in $i$ than in $j$ compared to $a_{\min}$ and compared to $a_{\max}$.
Suppose $i$ and $j$ differ in their sets of manufacturing technologies.

Let $G_c$ be truncated Pareto with common shape parameter $k$, common lower bound $\varphi_{\text{min}}$, and upper bounds $\bar{\varphi}_i > \bar{\varphi}_j$. 

Better technologies in $i$) better matches in $i$ for given $a_R$.

Matching in manufacturing:

But $a_{Ri} > a_{Rj}$ (larger manuf sector) \quad \Rightarrow \text{worse matches in } i$.

First effect dominates: matches better for worker type $a$ in $i$ than $j$.

Growth faster in $j$ than $i$ (comparative advantage in research).

Income inequality: More unequal in $i$ at bottom end of distribution.

There exists middle range of abilities such that for $a$ in this range, relative wage is higher in $i$ than in $j$ compared to $a_{\text{min}}$ and compared to $a_{\text{max}}$.
Suppose \( i \) and \( j \) differ in their sets of manufacturing technologies.

Let \( G_c \) be truncated Pareto with common shape parameter \( k \), common lower bound \( \phi_{\text{min}} \), and upper bounds \( \bar{\phi}_i > \bar{\phi}_j \).

Matching in manufacturing:

- Better technologies in \( i \) yield better matches in \( i \)
- But if \( a_{Ri} > a_{Rj} \) (larger manufacturing sector): worse matches in \( i \)
- First effect dominates: matches better for worker type \( a \) in \( i \) than \( j \)
- Growth faster in \( j \) than \( i \) (comparative advantage in research)
- Income inequality:
  - More unequal in \( i \) at bottom end of distribution
  - \( j \) has larger research sector and R&D pays greater return to ability
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International Integration: Trade and Knowledge Spillovers

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Find that *market access* plays role of demand level $X$:

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Effects of Trade on Growth and Inequality

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International Asymmetries

1 Differences in Manufacturing Productivity and Trade Barriers

Convergence in growth rates and wage inequality

An increase in $\theta K_{jc}$ raises growth and inequality in all countries

Greater inequality in $i$ than in $j$ at bottom of distribution, but at least as great inequality in $j$ at top

Grossman and Helpman (2015)
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