Credit Market Imperfections, Self-fulfilling Expectations, and Dynamics of Income Inequality *

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Abstract

This paper argues that complementarity between wage and aggregate employment is possible if there are imperfections in the credit market and the demand for labor is inelastic, at least locally. Complementarity between wage and aggregate employment implies multiplicity of equilibria. When equilibrium wage is sufficiently low then, compared to depositors, entrepreneurs supply more labor, save more, and rely less on external funding in order to guarantee participation in the credit market. This leads to endogenous income inequality among ex-ante identical agents. When equilibrium wage is sufficiently high then income inequality disappears because entrepreneurs are no longer credit constrained and, like depositors, supply the same amount of labor and earn the same amount of income. That is, self-fulfilling wage fluctuations lead to self-fulfilling fluctuations of income inequality. The model exhibits a wide range of dynamic phenomena, such as (a) indeterminacy of long run income inequality, (b) persistence of poverty and income inequality, (c) occurrence of poverty and income inequality due to self-fulfilling prophecy, and (d) endogenous fluctuation of income inequality due to fluctuations of self-fulfilling expectations. The model is capable of explaining the excessive volatility of income inequality and a sudden change in income inequality without any policy change or without shocks to fundamentals.

JEL Classification: E21, E32, E44, O11, O16

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1 Introduction

Empirical evidence reveals striking and persistent differences in evolution of income inequality among different nations. Countries sometimes experience a sudden change in income inequality. This change may either persist over time or reverse without any apparent policy change or without any shocks to fundamentals. For example, Figure 2(a) displays the time evolution of the Gini index of income inequality in Ukraine. As the figure indicates, Gini index fluctuated around a constant mean value of about 0.21 – 0.27 with no trend during the period 1968 – 92. In year 1993, Gini index increased more than 70% and reached 0.41. Since then income inequality declined gradually and reached 0.27 in 2003. Gini index stabilized to that level thereafter.\(^1\) As argued by Tedstrom (1995), Kuzio (1998), and Hare et al. (1998), such a rapid increase and then decrease of income inequality remains largely unaccounted because economic reforms in Ukraine during the period 1993 – 96 proceeded very slowly and occupational composition of the economy remained stable.

Gini index of income inequality exhibits a similar pattern (rapid change and then rapid return to the initial position) in many other countries, such as Argentina, Brazil, India, and Indonesia. For example, Figure 2(b) displays the time evolution of Gini index of income inequality in Indonesia. As the figure indicates, income inequality in Indonesia was about 0.32 during the period 1987 – 93. Gini index more than doubled during the period 1994 – 2000 and reached 0.75 in year 2000. After that, income inequality declined sharply and reached 0.44 in 2004.\(^2\) What explains such an enormous increase and then so rapid decline of income inequality?

The central hypothesis developed in this paper is that the imperfections in the credit market can lead to self-fulfilling fluctuations of income inequality. For this purpose we analyze a neoclassical overlapping generations model with capital accumulation, elastic labor supply, and imperfections in the credit market. Young agents decide how much labor to supply and whether to work for a firm or to become a self-employed worker. Old agents decide whether to become a depositor or an entrepreneur. Depositors lend their entire saving while entrepreneurs borrow and set up a firm by investing a minimum one unit of final commodity. Firms hire labor and pay wage according to the marginal product rule. Limited pledgeability of firms’ profit with inelastic demand for labor implies multiplicity of equilibria and leads to equilibrium complementarity between wage and aggregate employment. When the equilibrium wage is below the critical level then entrepreneurs work harder, save more, and rely less on external funding in order to guarantee participation in the credit market. As a result, income inequality is endogenously generated among ex-ante identical agents. Equilibrium income inequality disappears when equilibrium wage is sufficiently high.

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\(^1\)See Atkinson and Micklewright (1992), Chhibber et al. (1997), and Makeev and Kharchenko (1998) for more details.

and participation in the credit market is possible for everyone. When credit market
imperfection is sufficiently high then the economy can fluctuate endogenously due to
waves of optimistic and pessimistic expectations. As a result, the model is capable of
accounting for observed excessive volatility and sudden changes in income inequality
without any shocks to fundamentals or without any policy change. As conditions
in the credit market improve, complementarity between wage and aggregate employ-
ment weakens, equilibrium becomes globally unique and the possibility of income
inequality fluctuations due to self-fulfilling expectations disappears.

There are several other well-known mechanisms through which income inequality can
become a self-fulfilling prophecy. For example, Phelps (1972), Arrow (1974), Coate
and Loury (1993), and Acemoglu (1995) argue that in the presence of asymmetric
information between employer and employees, employers’ stereotypes about the pro-
ductivity of workers affect productivity of workers so that negative beliefs become
a self-fulfilling prophecy. Employers who believe that a worker is less productive
are unlikely to assign highly rewarded jobs to that worker. This lowers a worker’s
expected return and thus worker invests less in productivity. For this reason an em-
ployers’ negative beliefs about a worker are confirmed in equilibrium, even when all
workers are ex-ante identical. As a result, inequality is self-fulfilling prophecy because
employers expect agents from a certain group to be less qualified for top jobs and
they promote them less often, so that agents from those groups adopt a behavior that
validates the employers’ expectations. Through this mechanism, self-fulfilling income
inequality can be generated between men and women, employed and unemployed,
black and white, people living in rich and poor neighborhoods, etc.

Piketty (1995) and Hassler et al. (2007) propose another mechanism which works
through the linkage between the income distribution and political attitudes toward
redistribution. Piketty (1995) argues that the long-run evolution of income inequality
depends on the opportunity of social mobility. When individuals learn from their

Figure 1: Time evolution of Gini index of income inequality.
own experience and not from others' beliefs nor from the aggregate income distribution, then persistent income inequality can be generated between dynasties that are intrinsically identical. As a result the economy may attain multiple equilibria, including an equilibrium with a majority of discouraged individuals who exert little effort and support higher taxes and another equilibrium with a majority of successful individuals exerting high work effort and support lower taxes. In contrast, Hassler et al. (2007) argues that income inequality may become a self-fulfilling prophecy if young agents form expectations about future income redistribution policies before undertaking their educational investments. Young agents are not motivated to engage in educational investment if they expect high future redistribution. This results in an equilibrium in which a large number of low-income young individuals support future redistribution through taxes. The opposite holds when young agents form the expectation of lower redistribution policy and thus the economy attains an equilibrium in which there exists a rich-majority and low demand for redistributive policy. Piketty (1998) incorporates social status motive into the agents decision making and demonstrates the existence of multiple equilibria with different levels of work effort. In one equilibrium, individuals with upward (downward) social mobility enjoy social respect (disrespect), while in another equilibrium upward (downward) social mobility is viewed as pure fortune (misfortune) and do not deserve any particular recognition. In both equilibria, lower-class agents loose less than upper-class agents from a weak economic performance. As a result, status motive amplifies the income inequality between agents of different social origins.

Apart from this body of work, the current paper also relates to the stream of literature examining the relationship between credit market imperfections and persistence of income inequality. Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), Matsuyama (2000), Mookherjee and Ray (2002), Mookherjee and Ray (2003), and others argue that imperfections in the credit market can be a root cause of the persistence of income inequality. The most obvious channel explaining why inequality can persist across generations is the transmission of wealth from parents to children through inheritance. Frictions in the credit market generate entry barriers, less opportunities for the neediest, and cause the borrowing rate and access to credit to depend on wealth and social status. Poor individuals under-invest in both physical and human capital. As a result, long run living standards depend on initial inequality which can be magnified and persist over time.

The present work shares the spirit of the above cited works, but it distinguishes itself because we do not discuss imperfections in the labor market, nor introduce imperfect rational learning, or introduce social status motive into the agent’s decision making. The main differences between this paper and the existing ones are: (i) first, agents in this paper face equality of opportunities and income inequality is not a result of historically shaped inequalities of opportunities – transmitted across generations through education, social position, place of birth, etc.; (ii) second, we explicitly incorporate the dynamics of capital formation which is missing in most of the above
cited papers; and (iii) third, we focus on imperfections in the credit market as the main determinant of income inequality. These differences in the specifications lead to different predictions. In particular, we argue that the self-fulfilling expectations of agents as well as the initial conditions matter. Such that income inequality may become indeterminate or income inequality may experience a sudden change and excessive volatility.

The organization of the paper is as follows. Section 2 proposes the model. Section 3 characterizes the equilibrium and demonstrates the existence of multiple equilibria. Section 4 discusses how Gini index of income inequality changes by selecting different equilibria. Section 5 analysis the dynamics of the economy and demonstrates different dynamic behavior that can arise from the model. Section 6 discusses how the present model and main results obtained in this paper differers from ones existing in the literature. Section 7 concludes the paper. Detailed description of the data is located in the Appendix.

2 The Model

Let us consider a discrete time economy populated by an infinite sequence of two period lived overlapping generations. In each period, \( t = 0, 1, \ldots \), there are two generations alive, young and old. Each generation consists of a continuum of agents of unit mass. A single final commodity can be produced either by a large number of competitive firms or by self-employed workers. The final good produced in period \( t \) may be consumed in period \( t \) or may be invested in the production of physical capital, which becomes available in period \( t + 1 \).

The production technology employed by firms is described by a constant elasticity of substitution (CES) production function

\[
F(K_t, L_t) = \begin{cases} 
\min \{\theta K_t, L_t\} & \text{if } \sigma = 0 \\
\left[\frac{\alpha(\theta K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L_t^{\frac{\sigma-1}{\sigma}}}{\theta K_t^{\frac{\sigma-1}{\sigma}}}\right]^\frac{\sigma}{\sigma-1} & \text{if } \sigma \in (0, 1) \cup (1, \infty) \\
(\theta K_t)^{\alpha} L_t^{1-\alpha} & \text{if } \sigma = 1.
\end{cases}
\]  

(1)

In this formulation, \( K_t \) and \( L_t \) represent the aggregate supplies of capital and labor inputs respectively, while parameters \( \alpha \in (0, 1) \), \( \theta > 0 \), and \( \sigma \geq 0 \) represent the capital share in production, relative efficiency of capital input, and elasticity of substitution between capital and labor factors respectively. Total factor productivity of each firm is normalized to unity. The factor markets are competitive and rewards on capital and labor are paid according to a marginal product rule, i.e., \( \rho_t = F_1(K_t, L_t) \) and \( w_t = F_2(K_t, L_t) \) denote the rental rate of capital and wage rate paid in terms of final good. Capital invested in firm depreciates fully in production. The production technology employed by self-employed sector is \( G(N_t) = AN_t \), where \( N_t \) represents
the employment in the sector, and the parameter \( A \in (0, 1) \) represents the productivity of each self-employed worker. I assume that the productivity of each self-employed worker is smaller than the total factor productivity of the firm.

Each young agent is endowed with two units of labor.\(^3\) Young agents decide how much labor to supply and in which sector to work. By supplying \( \ell_t \in [0, 2] \) units of labor, young agents receive the labor income \( \ell_t w_t \). Young agents do not consume during first period, that’s why \( \ell_t w_t \) also represents the level of wealth young agents hold at the end of period \( t \). Young agents allocate their wealth in order to finance their old age consumption. They have two options. First, they may become depositors by lending their entire wealth to the competitive credit market at the gross interest rate \( r_{t+1} \) and consume \( c_{t+1} = \ell_t w_t r_{t+1} \) in the second period. Second, they may become entrepreneurs by borrowing funds in the competitive credit market at the gross interest rate \( r_{t+1} \) and starting an investment project. The project comes in a discrete, nondivisible units, and each young agent can run only one project.\(^4\) The project transforms one unit of the final commodity in period \( t \) into one unit of capital in period \( t + 1 \). Produces capital can be rented to the final commodity producing firm. Entrepreneurs can not produce capital if they invest less than one unit of final commodity. If entrepreneur’s first period wealth is less then the minimum investment requirement, \( \ell_t w_t < 1 \), then the entrepreneur borrows \( 1 - \ell_t w_t \) units of final commodity in the competitive credit market and \( c_{t+1} = \rho_{t+1} - (1 - \ell_t w_t) r_{t+1} \) during second period. \( \rho_{t+1} \) represents the rental rate of capital in period \( t + 1 \) while \( (1 - \ell_t w_t) r_{t+1} \) represents the amount of debt each entrepreneur carries from the previous period.\(^5\) Young agents choose their labor supply decision in order to maximize the following intertemporal utility function

\[
(\ell_t, c_{t+1}) \mapsto \ln(2 - \ell_t) + \ln c_{t+1},
\]

where \( 2 - \ell_t \) is the young age leisure and \( c_{t+1} \) is the old age consumption.

The credit market is competitive in the sense that both lenders and borrowers take the equilibrium interest rate \( r_{t+1} \) as given, however, it is not competitive in the sense that one cannot borrow any amount at the equilibrium rate. The borrowing limit exists because agents can credibly pledge only a \( \lambda \in (0, 1] \) fraction of their future profit for debt repayment. I.e., young agents are able to borrow and become entrepreneurs when the following borrowing constraint holds

\[
(1 - \ell_t w_t) r_{t+1} \leq \lambda \rho_{t+1}.
\]

Parameter \( \lambda \), in the above borrowing constraint, captures the credit market imperfection in a parsimonious way and allows us to investigate the macroeconomic im-

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\(^3\) The reader will see in the following section that the assumption about two units of labor endowment is made for notational convenience.

\(^4\) Since there is a continuum of agents investing in the same indivisible project, it follows that the aggregate technology is convex despite the assumption of indivisibility of individual investment projects.

\(^5\) Later on, we restrict the model parameters so that \( \ell_t w_t < 1 \) will holds for any \( t \), so that entrepreneurs and depositors co-exist at any time \( t \).
plication of credit market imperfection. What is the micro foundation for the borrowing constraint? One justifying for partial pledgeability would be to assume that entrepreneurs can hide a $1 - \lambda$ fraction of their revenue from financiers. Another justification for partial pledgeability would be the assumption of default cost, which is proportional to the project revenue. In such case entrepreneurs strategically default, whenever the repayment obligation exceeds the default cost.

3 Equilibrium

This section I start by analyzing young agent’s optimal labor supply/investment behavior for a given $w_t$. Then I derive aggregate labor supply curve which is consistent with the equilibrium in the capital market. Finally, I turn my attention to the labor market and demonstrate the possibility of multiple equilibria for a given aggregate capital stock, $K_t$, which is a predetermined variable in the model.

3.1 Agent’s Optimal Behavior

Perfect mobility of young workers between sectors implies that $w_t \geq A$. If $w_t = A$ then young agents are indifferent between working either for a firm or becoming a self-employed worker. In contrast, if $w_t > A$ then all young agents strictly prefer to work for a firm. If the old agent becomes a depositor then her optimization problem is

$$\max_{\ell_t \in [0,2]} \ln(2 - \ell_t) + \ln(\ell_t w_t r_{t+1}).$$

(4)

This implies that a depositor’s optimal labor supply is $\ell_t^d = 1$. If the old agent becomes an entrepreneur then her optimization problem is

$$\max_{\ell_t \in [0,2]} \ln(2 - \ell_t) + \ln (\rho_{t+1} - (1 - \ell_t w_t) r_{t+1}).$$

(5)

Since entrepreneurs are subject to a borrowing constraint, it follows from (3) and (5) that entrepreneur’s optimal labor supply is

$$\ell_t^e = \max \left\{ 1 - \frac{\rho_{t+1} - r_{t+1}}{2w_t r_{t+1}}, \frac{1}{w_t} \left( 1 - \frac{\lambda \rho_{t+1}}{r_{t+1}} \right) \right\}.$$  

(6)

I consider three cases separately.

Case 1: Suppose, $r_{t+1} > \rho_{t+1}$. Then it follows from (4) and (5) that for a given $\ell_t \in (0,2)$, depositors achieve higher second period consumption. As a result, no one would become an entrepreneur, there will be no demand for external funds, and thus the gross interest rate will fall to zero. This implies a contradiction with $r_{t+1} > \rho_{t+1}$. That is, $r_{t+1} > \rho_{t+1}$ could not happen in equilibrium.
Case 2: Suppose, \( r_{t+1} = \rho_{t+1} \). Then depositors and entrepreneurs enjoy the same levels of second period consumption. If \( w_t < 1 - \lambda \) then it follows from (6) that \( \ell_t^e = \frac{1 - \lambda}{w_t} > 1 = \ell_t^d \). That is, entrepreneurs consume less leisure than depositors do, while they both enjoy the same levels of second period consumption. As a result, no one would become an entrepreneur and there will be no demand for external funds. This, as above, implies a contradiction with \( r_{t+1} = \rho_{t+1} \). If \( w_t \geq 1 - \lambda \) then everyone supplies the same amount of labor,

\[
\ell_t^e = \ell_t^d = 1,
\]

and everyone earns the same amount of labor income.

Case 3: Suppose, \( r_{t+1} < \rho_{t+1} \). Then entrepreneurs enjoy higher level of second period consumption. If \( \ell_t^e = \ell_t^d = 1 \) then agents strictly prefer to become entrepreneurs rather than depositors because entrepreneurs and depositors consume the same amount of leisure. Competition between agents will drive interest rate up, so that agents are indifferent between becoming entrepreneurs or depositors. That is, in equilibrium, entrepreneurs should supply more labor, save more and rely less on external funding so that they can offer a higher interest rate to lenders without violating the borrowing constraint. In equilibrium, entrepreneurs’ and depositors’ optimal labor supplies are

\[
\ell_t^d = 1 \quad \text{and} \quad \ell_t^e = \frac{1}{w_t} \left( 1 - \frac{\lambda \rho_{t+1}}{r_{t+1}} \right) \in (1, 2)
\]

respectively. In order such allocation to be stable, entrepreneurs and depositors should achieve the same levels of lifetime utilities. This happens when

\[
\ln(2 - \ell_t^e) + \ln (\rho_{t+1} - (1 - \ell_t^e) w_t r_{t+1}) = \ln(w_t r_{t+1}).
\]

After substituting (8) in (9) and arranging terms, we obtain that the equilibrium ratio, \( \frac{\rho_{t+1}}{r_{t+1}} > 1 \), should solve a quadratic equation.

\[
\left( \frac{2w_t - 1}{\lambda} + \frac{\rho_{t+1}}{r_{t+1}} \right) \frac{\rho_{t+1}}{r_{t+1}} = \frac{w_t}{\lambda(1 - \lambda)}.
\]

It follows from (10) that

\[
\frac{\rho_{t+1}}{r_{t+1}} = -\frac{2w_t - 1}{2\lambda} + \sqrt{\left( \frac{2w_t - 1}{2\lambda} \right)^2 + \frac{w_t^2}{\lambda(1 - \lambda)}} > 1 \quad \text{if} \quad w_t < 1 - \lambda.
\]

This with (8) implies that

\[
\ell_t^e = 1 + \frac{1}{2w_t} - \sqrt{\left( 1 - \frac{1}{2w_t} \right)^2 + \frac{\lambda}{1 - \lambda}} \in (1, 2) \quad \text{if} \quad w_t < 1 - \lambda.
\]

(11) and (12) guarantee that entrepreneurs are willing and at the same time are able to run an investment project. An interesting reader should have immediately
realize that equilibrium quantities $\ell_t^e$ and $\frac{\rho_{t+1}}{r_{t+1}}$ should decline strictly with $w_t$ only for $w_t \in (0, 1 - \lambda)$. This is so because when the wage is sufficiently low then entrepreneurs supply relatively more labor compared to depositors in order to accumulate more wealth and guarantee the participation in the credit market. In this case, the rental rate of capital exceeds the interest rate and entrepreneurs earn the rent by producing capital. The equilibrium rent compensates entrepreneurs from obtaining additional disutility from extra supply of labor so that entrepreneurs and depositors achieve the same levels of lifetime utilities. In contrast, when $w_t \in [1 - \lambda, 1)$ then $\ell_t^e = 1$ and $\frac{\rho_{t+1}}{r_{t+1}} = 1$. Figure 2 visualizes both cases. A minimum investment requirement plays an important role for obtaining this result. Since self-employed worker faces a low wage and the alternative is to work in another sector that also pays a low wage, then the labor supply rises in order to save and overcome the borrowing constraint.\(^6\)

\subsection{3.2 Capital Market Equilibrium}

Since each entrepreneur produces one unit of capital, it follows that $K_{t+1}$ and $1 - K_{t+1}$ represent the sizes of young agents who become entrepreneurs and depositors respectively. Each entrepreneur borrow $1 - \ell_t^e w_t$, while each depositor lends $w_t$. This implies that the capital market equilibrium is established when aggregate borrowing is equal to aggregate lending,

$$
(1 - \ell_t^e w_t) K_{t+1} = w_t (1 - K_{t+1}).
$$

\(^6\)One possible justification for existence of minimum investment requirement is the housing market. Houses tends to be by far the largest asset owned by households in both developed and developing countries. Of course, houses can always be scale down in size/value but households choice is discrete and indivisible. The mechanism through which indivisibility of household investment affects household’s labor supply has been proposed and then empirically tested with micro data, by Campbell and Hercowitz (2011).
Let \( L^s(w_t) \) denotes the aggregate supply of labor. The aggregate saving is equal to the aggregate investment along the assumption of no first period consumption implies that the aggregate capital stock in period \( t + 1 \) is \( K_{t+1} = L^s(w_t)w_t \). This along with (7), (12), and (13) implies that the aggregate supply of labor, consistent with equilibrium in the capital market, is

\[
L^s(w_t) = \begin{cases} 
\frac{2}{1 + \sqrt{1 - 4w_t + \frac{4w_t^2}{1-\lambda}}} & \text{if } w_t < 1 - \lambda \\
1 & \text{if } w_t \geq 1 - \lambda.
\end{cases}
\]

(14)

The careful reader must have undoubtedly noticed that the assumption about the logarithmic utility function of consumers, made in (2), implies that depositor’s labor supply is constant while entrepreneur’s labor supply depends only on wage but not on interest rate. On the one hand this significantly reduces the complexity of the dynamical system describing the evolution of the economy, and on the other hand, allows us to visualize the results in a more intuitive/transparent way. In this sense we are considering a benchmark case.

### 3.3 Labor Market Equilibrium

In this section we treat the aggregate supply of capital \( K_t \in [0,1] \), as given and identify conditions under which the labor market clears. The reader can easily observe that \( w_t < A \) can not happen in equilibrium because young agents can always becoming self-employed workers. Let \( L^d(w_t, K_t) \) denotes the aggregate demand of labor created by all firms. Then \( L_t = L^d(w_t, K_t) \) solves \( F_2(K_t, L_t) = w_t \) for \( w_t \geq A \). Since \( F \) is homogeneous of degree one it follows that \( F_2 \) is homogeneous of degree zero and thus the aggregate demand of labor created by all firms can be represented as \( L^d(w_t, K_t) = \theta K_t \psi(w_t) \), where \( \psi \) satisfies \( F_2(1, \psi(w_t)) \equiv w_t \). With some algebra, it can be shown that if \( \sigma = 0 \) then

\[
\psi(w_t) = \begin{cases} 
1 & \text{if } w_t \in [A, 1) \\
1 & \text{if } w_t = 1 \\
0 & \text{if } w_t > 1,
\end{cases}
\]

(15)

and if \( \sigma > 0 \) then

\[
\psi(w_t) = \begin{cases} 
(\frac{1 - \alpha}{\alpha})^\frac{1 - \sigma}{\sigma} \left( \frac{1 - \sigma}{1 - \alpha} \right)^\frac{1 - \sigma}{\sigma} & \text{if } \sigma \neq 1 \\
(\frac{1 - \sigma}{w_t})^\frac{1}{\sigma} & \text{if } \sigma = 1
\end{cases}
\]

(16)

for \( w_t \geq A \).
Aggregate demand of labor created by self-employed sector is \( N^d(w_t) \in [0, \infty) \) if \( w_t = A \) and \( N^d(w_t) = 0 \) if \( w_t > A \). The aggregate demand of labor is equal to the aggregate supply of labor implies that the labor market clearing wage solves

\[
L^d(w_t, K_t) + N^d(w_t) = L^s(w_t). \tag{17}
\]

In the rest of this section I will argue that for a given \( K_t[0, 1] \), there might exists multiple \( w_t \) solving (17). To demonstrate this, I consider two cases separately. A benchmark case is one when the production function is Leontief, \( \sigma = 0 \). More general case is one when the production function is CES with positive substitutability between capital and labor inputs, \( \sigma > 0 \). The advantage of analyzing a benchmark case is its anaclitic tractability. That’s why I analyze this case with great details. The case with \( \sigma > 0 \) will be analyzed only numerically.

### 3.3.1 Multiplicity of Equilibria: Benchmark Case with \( \sigma = 0 \)

If \( \sigma = 0 \) then it follows from (15) that the labor market equilibrium condition, given in (17), can be rewritten as

\[
\frac{L^s(w_t)}{\theta K_t} = \begin{cases} 
[1, \infty) & \text{if } w_t = A \\
1 & \text{if } w_t \in (A, 1) \\
[0, 1] & \text{if } w_t = 1 \\
0 & \text{if } w_t \in (1, \infty).
\end{cases} \tag{18}
\]

In order to demonstrate multiplicity of equilibria I define functions, \( W_0 \) and \( W_2 \) as follows

\[
W_{0,2}(K_t) = \frac{1 - \lambda}{2} + \frac{1}{2} \sqrt{4(1 - \lambda) \frac{1 - \theta K}{\theta K} + (1 - \lambda)^2}. \tag{19}
\]

The careful reader must have undoubtedly noticed that \( W_0(K_t) \in (0, 1-\lambda) \), \( W_2(K_t) \in (0, 1-\lambda) \) and \( W_3(K_t) = 1 \) are three distinct solutions of \( \theta K_t \psi(w_t) = L^s(w_t) \). \( W_0(K_t) \) and \( W_2(K_t) \) exist if \( \theta K_t \in (1, L^s(\frac{1-\lambda}{2})) \) and \( W_3(K_t) \) exist if \( \theta K_t > 1 \).

Let

\[
\hat{w} = \max \left\{ A, \frac{1-\lambda}{2} \right\}, \quad K_1^c = \frac{1}{\theta}, \quad \text{and} \quad K_2^c = \frac{L^s(\hat{w})}{\theta}. \tag{20}
\]

Then I can prove the following proposition.

**Proposition 1** If \( A \in (0, 1-\lambda) \) then the equilibrium wage is

\[
W(K_t) = \begin{cases} 
A & \text{if } K_t \in (0, K_1^c), \\
\in [1-\lambda, 1] & \text{if } K_t = K_1^c, \\
\in \{W_1(K_t), W_2(K_t), W_3(K_t)\} & \text{if } K_t \in (K_1^c, K_2^c], \\
1 & \text{if } K_t > K_2^c.
\end{cases} \tag{21}
\]
Proof of this proposition directly follows from the properties of the aggregate labor demand and the aggregate labor supply functions obtained above. Figure 3(a) displays the configurations of aggregate demand and aggregate supply functions when there exists multiple equilibria (case (c) in Proposition 1). Figure 3(b) displays the relationship between equilibrium wage and aggregate capital stock. This figure can be used to verify all three cases considered in the above Proposition.

Figure 3: Existence of multiple equilibria. Both figures are constructed when λ = 0.2, θ = 1.2, and A = 0.2

By carefully inspecting the Figure 3(b), the reader can easily verify that if either the productivity of a self-employed worker is sufficiently high or the imperfection in the credit market is sufficiently high, \( A \in [1 - \lambda, 1] \), then there always exists a unique equilibrium in which the equilibrium wage is \( w_t = A \) if \( K_t < K_1^c \) and is \( w_t = 1 \) if \( K_t > K_1^c \). When \( K_t = K_1^c \) then there exists a continuum of equilibrium wages belonging to the interval \([A, 1]\). In such case the equilibrium of the economy is independent of parameter \( \lambda \). If \( A \in (0, 1 - \lambda) \) then there exists multiple equilibria for \( K_t \in (K_1^c, K_2^c) \).

3.3.2 Multiplicity of Equilibria: General Case with \( \sigma > 0 \)

If \( \sigma > 0 \) then it follows from (16) that the labor market equilibrium condition, given in (17), can be rewritten as

\[
\frac{L^s(w_t)}{\theta K_t} = \begin{cases} 
[\psi(A), \infty) & \text{if } w_t = A \\
\psi(w_t) & \text{if } w_t > A 
\end{cases}
\]  

(22)

Figure 4(a) displays the existence of multiple equilibria when \( \sigma > 0 \). Figure 4(b) shows the relationship between equilibrium wage and the aggregate capital stock. As
in case of $\sigma = 0$, multiple equilibria exists when $A \in (0, 1 - \lambda)$ and $K_t \in (K_t^c, K_t^c)$. As the Figure (4) demonstrates multiplicity of equilibria persists for $\sigma > 0$ as well however the interval $(K_t^c, K_t^c)$ shrinks. When $\sigma = 1$ then the possibility of multiple equilibria entirely disappears for any admissible parameter vector $(\lambda, \theta, A, \alpha)$.

![Figure 4: Existence of multiple equilibria. Both figures are constructed when $\lambda = 0.2$, $\theta = 1.2$, $A = 0.2$, $\alpha = 0.33$ and $\sigma = 0.12$.](image)

**3.3.3 Equilibrium Selection**

At this stage the reader may wonder to what happens when there exists multiple equilibria. When multiple equilibria exist, I assume that the choice of equilibrium, as in complementary games, is made outside the market mechanism. In particular, I define a forward orbit of the economy by employing an iterative function system (IFS) as the set of equilibrium wages and respective probabilities. The probability $p_1$, $p_2$, and $p_3 = 1 - p_1 - p_2$ is attached to each map $W_1(K_t)$, $W_2(K_t)$ and $W_3(K_t)$ respectively. Then the equilibrium aggregate saving is $S[W(K_t)]$, where $S(w) = wL^s(w)$.

Some important properties of the model yielding the multiplicity of equilibria should be highlighted here. First, multiplicity of equilibria does not depend on the existence of self-employment opportunity. As we see below self-employment opportunity for young agents is introduced within the model in order to eliminate the possibility of a corner steady state. Second, multiplicity of equilibria is a consequence of an imperfect credit market (which implies inverted “U” shaped aggregate supply of labor) and locally inelastic demand of labor (which implies a fixed amount of aggregate labor demand for some wage). On the one hand, if imperfection in the credit market is entirely eliminated, $\lambda = 1$, then the aggregate supply of labor becomes $L^s(w_t) \equiv 1$ for any $w_t \geq 0$ and the possibility of multiple equilibria disappears. On the other hand,
if the labor demand becomes sufficiently elastic then the flat part of the aggregate
labor demand function and thus possibility of multiple equilibria jointly disappears.
Multiplicity of equilibria survives as long as one keeps the assumption about imperfect
credit market and keeps the elasticity of substitution between capital and labor inputs
sufficiently low.

4 Equilibrium Income Inequality

I use the Gini index in order to measure income inequality. We focus on the income
inequality among young agents rather than among young and old agents because
income inequality among young agents provides a quantitatively similar result. In
addition, focusing on the inequality among young agents enables us to illustrate
the numerical result graphically and intuitively. Income inequality among ex-ante
identical agents does not exist if the equilibrium wage satisfies \( w_t \in [1 - \lambda, 1] \). What
is the Gini index of income inequality among young agents if \( w_t \in (0, 1 - \lambda) \)? In
order to answer this question we observe that the share of young agents becoming
depositors is \( 1 - S(w_t) \), while the total income earned by all depositors and all young
agents are \( w_t(1 - S(w_t)) \) and \( S(w_t) \) respectively. This implies that the ratio of total
income earned by all depositors to the total income earned by all young agents is

\[
\frac{w_t(1 - S(w_t))}{S(w_t)} = \frac{1 - S(w_t)}{L^*(w_t)}.
\]

(23)

Since the Gini index of income inequality equals twice the area of the shaded triangle,
depicted in Figure 5(a), it follows from (23) that the Gini index of income inequality

![Figure 5: Configurations of the Lorenz curve and the Gini index of income inequality.](image-url)
Figure 5(b) displays the relationship between the Gini index of income inequality and $K_t$. When there exist multiple equilibria then the value of the Gini index depends on the choice of equilibrium wage. Equilibrium income inequality is highest when the equilibrium wage is smallest.

5 Equilibrium Dynamics

In this section I start to examine the global dynamics of $K_t$ for $\sigma = 0$. At the end, I will discuss the case with $\sigma > 0$.

5.1 Case with $A \in [1 - \lambda, 1)$

If $A \in [1 - \lambda, 1)$ then there always exists a unique equilibrium in which the equilibrium employment is always 1. The equilibrium wage is $A$ if $K_t < K^c_1$ and is 1 if $K_t > K^c_1$. If $K_t = K^c_1$ then there exists a continuum of equilibrium with the equilibrium wage belonging to the interval $[A, 1]$. Aggregate saving is equal to aggregate investment implies that the aggregate capital during the next period is

$$K_{t+1} = \begin{cases} 
A & \text{if } K_t < K^c_1 \\
[A, 1] & \text{if } K_t = K^c_1 \\
1 & \text{if } K_t > K^c_1.
\end{cases} \quad (25)$$

It follows from (25) that the dynamics of the economy is independent from the level of credit market imperfection. Three distinct cases can be identified here. If $\theta < 1 \iff K^c_1 > 1$ then there exists a unique and globally stable steady state in the aggregate capital stock is $K_H = 1$. In such a steady state, all young agents work for a firm, supply one unit of labor, save one unit of final commodity, and become entrepreneurs by self-financing the firm. If $\theta \in (1, \infty) \iff K^c_1 < 1$ then we define parameter regions

$$\Omega_{00} = \left\{ (A, \lambda) | S(A) = A > \frac{1}{\theta} \right\} \quad \text{and} \quad \Omega_{01} = \left\{ (A, \lambda) | S(A) = A < \frac{1}{\theta} \right\}. \quad (26)$$

If $(A, \lambda) \in \Omega_{00}$ then there exists a unique and globally stable steady state in which the aggregate capital stock is $K_L = A$. In such a steady state, the equilibrium wage is $A$, every young agent supplies one unit of labor, the size of young agents working for firms is $A$ and the size of young agent becoming self-employed workers is $1 - A$. If $(A, \lambda) \in \Omega_{01}$ then three steady states with the aggregate capital stock being $K_L$, $K_M = K^c_1$, and $K_H$ co-exist. Among those steady states, $K_L$ and $K_H$ are
asymptotically stable while the steady state \(K_M\) is unstable. Whether the economy converges to \(K_L\) or \(K_H\) depends on the initial condition. If \(K_0 < K_1^c\) then the economy converges to \(K_L\) and if \(K_0 > K_1^c\) then the economy converges to \(K_H\). The reader should keep in mind that there is no income inequality in either of these steady states.

![Figure 6: Configurations of time one correspondence describing the evolution of \(K_t\).](image)

5.2 Case with \(A \in (0, 1 - \lambda)\)

If \(A \in (0, 1 - \lambda)\) then there is a possibility of multiple equilibria. When multiple equilibria exist, then the choice of equilibrium wage (as discussed above) is made outside the market mechanism. In particular, we define a forward orbit of the economy by employing an iterative function system (IFS) as the set of maps and probabilities. The probability \(p_1, p_2,\) and \(p_3 = 1 - p_1 - p_2\) is attached to each map \(S[W_1(K_t)]\), \(S[W_2(K_t)]\) and \(S[W_3(K_t)]\) respectively. Aggregate saving is equal to aggregate investment implies that the evolution of the aggregate capital stock is described by the following time one correspondence.

\[
K_{t+1} = S[W(K_t)] = \begin{cases} 
S[W_1(K_t)] & \text{if } K_t < K_1^c \\
\in S[W_1(K_t)] \cup [1 - \lambda, 1] & \text{if } K_t = K_1^c \\
\in \{S[W_1(K_t)], S[W_2(K_t)], S[W_3(K_t)]\} & \text{if } K_t \in (K_1^c, K_2^c] \\
1 & \text{if } K_t > K_2^c.
\end{cases}
\]

(27)

If \(\theta < 1 \iff K_1^c > 1\) then there exists a unique equilibrium for any \(K_0 \in [0, 1]\) and a unique steady state \(K_L\). If \(\theta \in (1, \infty) \iff K_1^c < 1\) then we define parameter regions,
\[ \Omega_{10} = \left\{ (A, \lambda) | S(A) > \frac{L(\hat{w})}{\theta} \right\}, \]
\[ \Omega_{11} = \left\{ (A, \lambda) | S(A) < \frac{1}{\theta} < \frac{L(\hat{w})}{\theta} < 1 \right\}, \quad \Omega_{12} = \left\{ (A, \lambda) | \frac{1}{\theta} < 1 < \frac{L(\hat{w})}{\theta} \right\}, \]
\[ \Omega_{13} = \left\{ (A, \lambda) | \frac{1}{\theta} < S(A) < \frac{L(\hat{w})}{\theta} < 1 \right\}, \quad \Omega_{14} = \left\{ (A, \lambda) | \frac{1}{\theta} < S(A) < 1 < \frac{L(\hat{w})}{\theta} \right\}. \]

\( (A, \lambda) \in \Omega_{10} \) then there exist multiple equilibria for some values of \( K_0 \) but there exists a unique steady state \( K_H \). As a result, the economy will reach to the steady state \( K_H \) either in one or in two iterations. Cases with \( \theta > 1 \) and \( \theta S(A) < L(\hat{w}) \) deserve a particular attention\(^7\) because if \( \theta > 1 \) and \( \theta S(A) < L(\hat{w}) \) then multiple equilibria and multiple steady states co-exist. Let \( K_M \), where \( K_M \) solves \( \theta W_2(K) = 1 \), denotes the aggregate capital stock in the middle steady state. In the rest of this section we consider four cases separately yielding four different dynamics of income inequality.

**Indeterminacy of Long Run Income Inequality:** If \( (A, \lambda) \in \Omega_{11} \) then there exists multiple equilibria only in \( K_M \). This implies that if \( \theta K_0 \in [1, L(\hat{w})] \) then the long run evolution depends not only on the initial condition, \( K_0 \), but also on self-fulfilling expectations. In other words, among two identical economies one might converge to \( K_H \) while the other may converge to \( K_L \) despite the fact that those two economies are structurally identical and share the same initial conditions. This means that the self-fulfilling expectation plays an important role for the long run evolution of income inequality. This is visualized in Figure 6(a).

**Poverty Trap:** If \( (A, \lambda) \in \Omega_{12} \) then multiple equilibria exist around the steady states \( K_M \) and \( K_H \) but not around the steady state \( K_L \). This implies that the economy may fluctuate endogenously due to fluctuations of self-fulfilling beliefs however the economy will eventually converge to \( K_L \) because the positive probability \( p_1 \) is attached to the map \( W_1(K_L) \). Despite the existence of multiple steady states, the lowest steady state (with poverty and income inequality) prevails in the long run. This is visualized in Figure 6(b).

**Poverty Trap as Self-fulfilling Prophecy:** If \( (A, \lambda) \in \Omega_{13} \) then there exits multiple equilibria around steady states \( K_L \) and \( K_M \) but not in the neighborhood of \( K_H \). This implies that the economy can stay in \( K_L \) in the long run only if \( p_L = 1 \), i.e., if the poverty trap can be a self-fulfilling prophecy. The economy will eventually reach \( K_H \) (steady state without poverty and without income inequality) as soon as \( p_L < 1 \). This is visualized in Figure 7(a).

**Endogenous Fluctuation of Income Inequality:** If \( (A, \lambda) \in \Omega_{14} \) then multiple equilibria exists around steady states \( K_L \), \( K_M \), and \( K_H \). Since income inequality is different in each steady state, it follows that the endogenous fluctuation of income inequality is possible due to endogenous fluctuations of self-fulfilling beliefs. This is visualized in Figure 7(b). Figure 8 displays the partition of the parameter space.

\(^7\)\( \theta > 1 \) and \( \theta S(A) < L(\hat{w}) \) corresponds to \( (A, \lambda) \in \Omega_{11} \cup \Omega_{12} \cup \Omega_{13} \cup \Omega_{14} \).
(a) Poverty Trap as Self-fulfilling Prophecy: \((A, \lambda) \in \Omega_{13}\)

(b) Endogenous Fluctuation of Income Inequality: \((A, \lambda) \in \Omega_{14}\)

Figure 7: Configurations of time one correspondence describing the evolution of \(K_t\).

Figure 8: Configuration of parameter regions \(\Omega_{00}, \Omega_{01}, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \text{ and } \Omega_{14}\) when \(\sigma = 0\).

when \(\theta > 1\). Some important properties of the model yielding distinct dynamics of income inequality should be highlighted here. First, multiplicity of equilibria is a necessary condition for obtaining the above considered four cases. The possibility of self-employment opportunity for young agents is not necessary for having either inde-
terminacy of long run income inequality or poverty trap with high income inequality. However, one cannot obtain endogenous fluctuations of income inequality due to the fluctuations of self-fulfilling expectations without assuming self-employment opportunity for young agents. This is so because one cannot guarantee a sufficient aggregate saving without minimum wage. Sufficient aggregate saving is necessary for eliminating the possibility of a corner steady state. At this point it should be straightforward for a careful reader that above described dynamics should hold for $\sigma > 0$ as well because multiplicity of equilibria holds even for $\sigma > 0$.

6 Literature Review

At this point the reader should be able to better compare main results obtained in this paper with the ones existing in the literature. This idea that imperfections in the credit market can create credit cycles through self-fulfilling beliefs is proposed in Reichlin (1986), Woodford (1986), Grandmont et al. (1998), Cazzavillan et al. (1998), and others. A distinctive feature of these models is that the economy is populated by heterogenous agents: workers and capitalists. Workers work in each period; capitalists do not work at all. Workers face a binding cash-in-advance constraint that reflects the difficulty they have in borrowing against labor income. Capitalists do not face any borrowing constraint, because capital is accepted as collateral to secure a loan. The assumption that capitalists discount the future less than workers implies that capitalists will end up owning the entire capital stock and they will never become credit constrained. Conversely, workers save only in the form of money balances and are never able to borrow against future labor income. As a result, workers and capitalists do not compete for credit. The model considered in this paper strengthens result obtained in the above mentioned papers in two directions. First, agents in the present model are ex-ante identical and everyone competes for credit. Second, endogenous income inequality due to self-fulfilling expectations may arise and persist even if agents are ex-ante identical.

Another body of literature investigates how indeterminacy can arise from imperfections in the credit market. For example, Agliari and Vachadze (2014) considers a model similar to one analyzed in this paper. When the credit market is imperfect and saving is endogenous, the aggregate savings plays a dual role. On the one hand high aggregate saving implies a low marginal product of capital which negatively affects the interest rate; on the other high aggregate saving implies more investment in highly productive entrepreneurial activity which positively affects the interest rate. If the former effect is dominant, then the individual and aggregate savings complement each other. Complementarity leads to multiple equilibria and multiple steady states. The existence of multiple steady states entails global indeterminacy and endogenous fluctuations can emerge. The model considered in this paper strengthens result obtained in Agliari and Vachadze (2014) by considering only a stable allocation
of credit. In Agliari and Vachadze (2014) credit is allocated randomly. Of course a random allocation is an internally consistent allocation, however it is not a stable allocation. In a random allocation (symmetric equilibrium), young agents supplies the same amount of labor and earns the same income. Imagine a situation when the aggregate saving is sufficiently low and there exists a rent entrepreneurs can earn. In such case, agents can become better off by supplying more labor, offering a higher interest rate to depositors, and running an investment project. As a result, agents would deviate from a symmetric equilibrium and ex-ante homogenous agents may become ex-post heterogenous (asymmetric equilibrium) with respect to their income. Agliari and Vachadze (2014) analysis only symmetric equilibrium, ignores its instability, and shows that the imperfections in the credit market may create a global and local indeterminacy. In this paper I strengthen results in two directions. First, I show the existence of a stable equilibrium in which there exists income inequality among ex-ante identical agents. Second, I consider only a stable equilibrium and still the possibility of endogenous fluctuations due to a self-fulfilling prophecy.

7 Conclusion

The main goal of this paper is to provide a theoretical explanation of empirically observed excessive volatility of income inequality. The model is based on the standard neoclassical overlapping generations model, with capital accumulation, elastic labor supply, and imperfections in the credit market. Within this framework I derive the necessary and sufficient conditions for (a) indeterminacy of long run income inequality, (b) a poverty trap and income inequality, (c) a poverty trap as self-fulfilling prophecy, and (d) endogenous fluctuations of income inequality to occur. When endogenous fluctuations due to self-fulfilling expectations is possible then one can explain the excessive volatility of income inequality only through a volatility of self-fulfilling expectations. One major advantage of the model presented here is its analytical tractability. Some cautionary remarks should be pointed out about the predictions of the model. I do not argue that the credit market imperfections should be blamed for increased volatility of income inequality or that other sources, like policy change, structural change, globalization, etc., are unimportant for increased volatility of income inequality. Inequality in the model is generated among ex-ante identical agents. This should not be interpreted that exogenous heterogeneity among agents is unimportant factor leading to income inequality. Instead, I argue that a small amount of exogenous heterogeneity can be magnified in order to generate a large income inequality.

In the paper, I made several simplifying assumptions in order to minimize the dimension of the parameter space, and to avoid unnecessary complications while analyzing the model. Of course, some of the results of the paper depend on these simplifying assumptions, but the main result obtained in the paper is robust to alternative spec-
ifications as well. The main features of the model would remain as long as (i) agents face minimum investment requirement for running an investment project, (ii) the credit market is imperfect, (iii) demand for labor is inelastic enough, and (iv) there is self-employment opportunity for young agents. As long as these features of the model are maintained, alternative assumptions about the consumer utility function or the aggregate production function in either sectors would not invalidate the key result, although they might considerably complicate the analysis. This is so because (i) and (ii) imply that aggregate labor supply has inverted “U” shape. This along with (iii), implies the multiplicity of equilibria, and (iv) guarantees a minimum aggregate saving, which with multiple equilibria, yield the main results of the paper.

Some limitations of the above analysis should be pointed out as well. First, agents choose to consume a final commodity only during the second period. It would be interesting to further investigate how the imperfections in the credit market affect the dynamics of income inequality after allowing young agents to consume during both periods. Second, the economy is closed thus does not interact with other economies. It would be more satisfactory to consider a model in which a country is an integral part of the world economy. Third, the model does not allow for sustained growth of the economy. It would be interesting to examine the dynamics of the income inequality in a growing global economy. This would require the model to be modified in such a way that the minimum investment requirement for the project changes with per capita income.

Finally, the model presented above should be useful for policy advice. For example, in the case of endogenous fluctuations of income inequality or in case of income inequality through a self-fulfilling prophecy, government can promote the confidence or optimism within a private sector. This way government can affect the selection of equilibrium so that young agents select a “good” equilibrium (one with the highest wage) and eliminate a “bad” one (one with the lowest wage). It is also hoped that the present paper will stimulate further research on the issue.
References


