CUNY Ph.D. Program in Mathematics
Fall 2015
MATH 70500: Algebra I
W & F, 10:00 am - 11:30 am, Rm. TBA
Prof. O. Kharlampovich
4.5 cr.,

TEXT: Abstract Algebra, D. Dummit and R. Foote

(1) **Introduction to groups:** group, subgroup and order, examples, fields, the dihedral and symmetric groups D_n and S_n, matrix groups and quaternions, direct products, cosets and Lagrange's theorem, cyclic groups, $\mathbb{F}^\times$ is cyclic, commutator, centralizer, normalizer, normal subgroups and quotient groups.
(2) **The isomorphism theorems:** homomorphisms, behavior of subgroups under homomorphisms, the three isomorphism theorems.
(3) **Group actions on sets:** basic definitions and properties, Cayley's theorem, the coset representation, a formula for the number of orbits, applications, the game of 16 squares, Rubik's cube.
(4) **The symmetric group:** conjugacy classes, simplicity of A_n.
(5) **Sylow's theorems:** p-groups, groups of order p, p^2, p^3, Cauchy's theorem, S_3, S_4, groups of order pq and p^2q, $GL_n(\mathbb{F})$.
(6) **Composition series, the Jordan-Holder theorem, solvable groups, nilpotent groups.**
(7) **Free groups:** properties, reduced words, generators and relations, famous problems.
(8) **Finitely generated abelian groups, semi-direct products, groups of order 12:** the structure theorem for f.g. abelian groups, application of semi direct products.
(9) **Rings:** the integers and the integers modulo n, matrices over a ring, polynomial and power series rings, the ring of quotient, ring homomorphisms and isomorphism theorems, the Chinese remainder theorem.
(10) **Euclidean, principal ideal and unique factorization domains:** division and gcd"s. Euclidean algorithm, irreducible and prime elements, UFD, arithmetic in UFD, Gauss' lemma, the ring of polynomials over a UFD.
(11) **Introduction to the module theory including tensor products and basic homological algebra.**