LOGIC I – MATH 711
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This course is an introduction to the syntax and semantics of first-order logic – the most commonly studied logic. The goal of the course is to explore the fundamental properties of this logic and its application to other areas of mathematics. The material presented in this course provides the basis for the classical subject areas of mathematical logic – model theory, proof theory, recursion or computability theory, and set theory.

First-order logic is the language in which many common axiomatic theories are formulated. These include Peano arithmetic, Zermelo Frankel set theory, non-standard analysis and many further examples. Yet beyond the fact that many common theories are formalized in first-order logic the study of the logic itself is of independent iterest and also has deep applications in the study of other common mathematical objects including linear orderings, graphs, groups, rings, and fields. In this course as I develop the basic properties of first-order logic I will use these latter structures to provide examples and as such a rudimentary knowledge of these mathematical structures is the only prerequisite for this course.

The fundamental tool in the study of first-order logic is the compactness theorem. I will prove this essential result via the method of ultraproducts, which is of interest in its own right.

What follows is a rough list of topics to be covered during the semester. This list is highly subject to change depending on time constraints and student interest.

- Structures – The central objects of study.
- Languages – The basic syntax of first-order logic.
- Semantics – Truth and logical consequence, theories, definability, axiomatizability.
- Compactness – Via the method of ultraproducts.
- Consequences of Compactness – Upward Löwenheim-Skolem, non-standard models.
- Orderings – Cantor's isomorphism theorem, ordinals, and cardinals.
- Elementary Equivalence and Elementary Submodels – Downward Löwenheim Skolem, method of diagrams.
- Elimination of quantifiers and model completeness – Applications to algebraically closed fields and vector spaces.
- Types – The stone space, realizing and omitting types.
- ω-categorical theories – The Ryll-Nardzewski's theorem and examples.
- Preservation theorems

Literature: Rothmaler, "Introduction to Model Theory" (Gordon and Breach); Chang and Keilser, "Model Theory" (Dover).