

Graduate Course Spring 2010 (Roman Kossak and Lucien Szpiro )

### Model Theory for Arithmetic Geometry

The course will survey basic model theory with an eye on applications in arithmetic geometry. We will focus on the concept of first-order definability and will study in detail first-order definability in classical rings and fields. We will also look at classifications of structures and theories with respect to various dividing lines: decidable/undecidable, categorical/non-categorical, stable/unstable, tame/wild. All model theoretic concepts will be defined, and we will concentrate on the analysis of concrete examples. Since we do not assume that students had prior exposure to mathematical logic, not all results from model theory will be given in full generality and/or with complete proofs. We will use David Marker's "Model Theory: An Introduction" (Springer 2002) as the main text. Covered topics will include.

1. Mathematical structures, languages, theories. First-order definability.
2. Compactness of type spaces with applications.
3. Ultraproduct and ultrapower constructions.
4. A bit of set theory: uniqueness of ultrapowers and Keisler-Shelah Theorem. The Monster model.
5. Incompleteness and undecidability of arithmetic theories.
6. Minimal and strongly minimal sets and structures. Model theory of algebraically closed fields.
7. 0-minimal theories. Model theory of real closed fields.
8. Negative solution of Hilbert's 10th problem and its consequences.
9. Definability of integers in the field of rational numbers.
10. Categoricity.
11. Elements of stability theory. Morley rank.
12. Some model theory of difference fields.

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