Titles and Abstracts

Matt Baker (University of California at Berkeley)

Linear series on metrized complexes of algebraic curves

A metrized complex of algebraic curves over a field K is, roughly speaking, a finite edge-weighted graph G together with a collection of marked complete nonsingular algebraic curves C_v over K, one for each vertex; the marked points on C_v correspond to edges of G incident to v. We will present a Riemann-Roch theorem for metrized complexes of curves which generalizes both the classical Riemann-Roch theorem and its combinatorial analogue. As an application, we formulate a partial generalization of the Eisenbud-Harris theory of limit linear series to semistable curves which are not necessarily of compact type. This is joint work with Omid Amini.

Jean-Benoît Bost (Université de Paris 11)

Algebraization of vector bundles on surfaces over number fields

Classical algebraization results in SGA2 deal with the algebraization of formal vector bundles over projective schemes of (relative) dimension at least 3. This talk will be devoted to algebraization results concerning vector bundles on surfaces over number fields, that may be established by using techniques from Arakelov geometry.

José Ignacio Burgos Gil (Instituto de Ciencias Matemáticas)

Arakelov theory of toric varieties, positivity properties

Arakelov theory is one of the fields where L. Szpiro has made significant contributions. In this talk we will I will explain several results in the Arakelov theory of toric varieties: I will give characterizations of arithmetically ample, nef big and pseudo-effective metrized toric divisors and we will discuss the Dirichlet unit theorem, Fujita aproximation and Zariski decomposition for metrized toric divisors. This is joint work with A. Moriwaki, P. Philippon and M. Sombra.

Antoine Chambert-Loir (Université de Rennes 1)

Differential forms and currents on Berkovich spaces

Abstract: I will describe a construction of differential forms and currents on Berkovich spaces which is parallel to the classical theory on complex spaces. This is joint work with Antoine Ducros (Paris).

Huayi Chen (Université Paris 7)

Limit theorems in Arakelov geometry

We show that some arithmetic invariants can be interpreted as the limit measure of a sequence of random variables and we explain different consequence of such limit theorems.

Henri Gillet (University of Illinois at Chicago)

Higher Derivations and Descent in Characteristic p

In characteristic 0 the Kodaira-Spencer class is the obstruction to a family of curves being isotrivial, and more generally one knows that if X a projective variety over an algebraically closed field K of characteristic zero, then it is defined over the fixed field of all the derivations of K which lift to X. I shall describe how one may use higher derivations, in the sense of Hasse-Schmidt, to obtain a similar result in positive characteristic.

Walter Gubler (Universität Tübingen)

Canonical measures and the geometric Bogomolov conjecture

Chambert-Loir measures occur as equidistribution measures of non-archimedean dynamical systems. In the case of a subvariety X of an abelian variety, we get canonical measures on the Berkovich space associated to X. In this talk, we will give a precise description of these canonical measures and we will show that this has applications to the geometric Bogomolov conjecture.

Luc Illusie (Université de Paris 11)

Quotient stacks and equivariant cohomology algebras : Quillen's theory revisited

This is joint work with W. Zheng. Around 1970 Quillen studied the structure of equivariant cohomology algebras $H^*_G(X, \mathbf{F}_\ell)$ for a compact Lie group G acting on a space X. We establish analogues and variants of his results for the mod ℓ étale cohomology of certain Artin stacks over an algebraically closed field k and ℓ a prime invertible in k, especially Artin stacks of the form $[X/G]$ for a k-scheme X separated and of finite type acted on by an algebraic k-group G.

David Masser (Universität Basel)

Unlikely intersections for abelian surfaces

A couple of years ago Umberto Zannier and I proved that there are at most finitely many complex numbers t such that two points with abscissas 2 and 3 on the Legendre elliptic curve $E(t)$ parametrised by t are both torsion. This can be interpreted in terms of unlikely intersections on the square of $E(t)$. We describe some subsequent generalizations to arbitrary abelian surface schemes.

Laurent Moret-Bailly (Université de Rennes 1)

Greenberg's approximation theorem and the topology of varieties over valued fields

In the first part of this talk, I will generalize Greenberg's strong approximation theorem to more general (i.e. not necessarily discrete) valuations, using the ultraproduct method of Becker-Denef-Lipshitz-van den Dries. Then I will report on a joint work (in progress) with Philippe Gille, where we apply this result (and others) to the topology of (morphisms of) varieties over henselian valued fields, in particular morphisms of algebraic groups.

Reference for the first part: " An extension of Greenberg's theorem to general valuation rings", to appear in Manuscripta Mathematica; available at springerlink
http://www.springerlink.com/content/60106284n373382w/?MUD=MP
or on my web page
http://perso.univ-rennes1.fr/laurent.moret-bailly/publis.html

Atsushi Moriwaki (Kyoto University)

Characterization of nef arithmetic divisors on arithmetic surfaces

In this talk, we will give a characterization of nef arithmetic $\mathbf{R}$-divisors of continuous type in terms of the arithmetic volume and the arithmetic self-intersection number. This is a generalization of the equality condition of Faltings-Hriljacs Hodge index theorem.

Christian Peskine (Université Paris 6)

Smooth linear congruences of lines

A congruence of lines in the projective complex space $\mathbf{P}^N$ is an $(N-1)$-dimensional subvariety of the Grassmann variety. The *order* $o(\Sigma)$ of a congruence $\Sigma \subset G(1, N)$ is the number of its lines passing through a general point of $\mathbf{P}^N$. The points of $\mathbf{P}^N$ through which there pass infinitely many lines of Σ form the fundamental locus $X(\Sigma)$.

The congruence is linear if it is cut out, in $G(1, N)$, by a linear subspace of the ambient Plücker space. Clearly, the order of a linear congruence is either 1 or 0.

The fundamental locus of a linear congruence has a natural scheme structure. If a smooth (and connected) linear congruence of order 1 has a smooth (and con- nected) fundamental locus we say that it is twice smooth (and connected). For such congruences we prove that $k = (N-1)/(N-\dim(X)-1)$ is an integer and that the lines of the congruence are the k-secant lines to its fundamental locus. We define k as the secant index of the twice smooth and connected, order 1, linear congruence.

We classify all twice smooth and connected linear congruences of order 1 whose secant index is ≤ 3. We note that there are only four with secant index 3 (their fundamental loci are projected Severi Varieties). We present two such congruences with secant index 4 (their fundamental loci are Palatini Varieties). Finally we note that the fundamental locus of any other twice smooth and connected linear congruence of order 1 would be a counter-example to the celebrated conjecture of Hartshorne concerning low codimenson varieties.

Bjorn Poonen (Massachusetts Institute of Technology)

Random maximal isotropic subspaces and Selmer groups

We show that the p-Selmer group of an elliptic curve is naturally the intersection of two maximal isotropic subspaces in an infinite-dimensional locally compact quadratic space over $\mathbf{F}_p$. By modeling this intersection as the intersection of a random maximal isotropic subspace with a fixed compact open maximal isotropic subspace, we can explain the known phenomena regarding distribution of Selmer ranks, such as the theorems of Heath-Brown, Swinnerton-Dyer, and Kane for 2-Selmer groups in certain families of quadratic twists, and the average size of 2- and 3-Selmer groups as computed by Bhargava and Shankar. The random model is consistent with Delaunays heuristics for $Sha[p]$, and predicts that the average rank of elliptic curves is at most 1/2. This is joint work with Eric Rains.

Joseph H. Silverman (Brown University)
Szpiro's Conjecture with Prime Depletion and Lang's Height Conjecture

Let E/K be an elliptic curve defined over a number field, and let $P \in E(K)$ be a nontorsion point. Lang conjectured that the canonical height of P satisfies

$$\widehat{h}_E(P) c_1 \log N_{K/Q} \mathrm{Disc}_{E/K} + c_2$$

for positive constants c_1 and c_2 depending only on K. Hindry and I proved that Szpiros conjecture

$$\log N_{K/Q} \mathrm{Disc}_{E/K} << \log N_{K/Q} \mathrm{Cond}_{E/K}$$

implies Langs conjecture. It is natural to try to prove the opposite implication. In this talk I will explain how Langs conjecture actually follows from a significantly weakened version of Szpiros conjecture, thus making it unlikely that Szpiros original conjecture is a straightforward consequence of Langs conjecture.

Thomas Tucker (University of Rochester)

Orbits modulo primes and a dynamical Mordell-Lang conjecture

The theorem of Skolem-Mahler-Lech shows that if $f : \mathbf{C}^g \longrightarrow \mathbf{C}^g$ is a linear map and z is any point in $\mathbf{C}^g$, then for any subvariety V of $\mathbf{C}^n$, then the set of n such that $f^n(z)$ is in V forms a finite union of arithmetic progressions. Recently, it has been asked if the same result holds for any morphism $f : X \longrightarrow X$ from a variety to itself. When the map is defined over a number field, the theorem will hold whenever there is a prime p such that the orbit of z under f avoids the ramification locus of f modulo p. A "random map" heuristic indicates that one may not be able to find such a prime in dimension 4 or more.

Emmanuel Ullmo (Université de Paris 11)

The hyperbolic Ax-Lindemann conjecture in the compact case and some applications to the André-Oort conjecture.

We will explain the statement of the hyperbolic Ax-Lindemann conjecture for Shimura varieties. The main result is a proof of this conjecture for projective Shimura varieties (joint work with Andrei Yafaev). We will also give some applications to the André-Oort conjecture. We'll explain a proof of the André-Oort conjecture for projective Shimura varieties contained in an arbitrary power of the moduli space of principally polarized abelian varieties of dimension 6.

Xinyi Yuan (Princeton University)

Effective bound of linear series on arithmetic surfaces

In this talk, I will present an upper bound of number of effective sections for Hermitian line bundles on arithmetic surfaces. It is an effective version of the arithmetic HilbertSamuel formula. It is a joint work with Tong Zhang.

Umberto Zannier (Scuola Normale Superiore di Pisa)

Abelian surfaces and Pell's Equation over polynomial rings

In this talk I shall illustrate joint work with David Masser, concerning an application of the results on abelian surfaces (described in his talk) to a variant of the classical Pell equation $X^2 - DY^2 = 1$ for polynomial rings, studied already by Abel, where now D is a complex polynomial of even degree in a single variable and we seek solutions in nonzero complex polynomials X, Y in this variable. In this context solvability is no longer ensured by simple conditions on D. In the talk we shall let $D = D_t$ vary in a pencil. When D_t has degree ≤ 4, it may be seen that for infinitely many t there are nontrivial solutions. On the other hand, it is not so when D_t has degree 6 (provided natural assumptions are verified).