Problem 1: Wave Propagation

(a) Derive the Fresnel coefficients for reflection ($r$) and transmission ($t$) for TE polarized light assuming that $\mu_1 = \mu_1 = \mu_0$.

(b) Derive the power reflection ($R$) and transmission ($T$) coefficients for TE polarized light.

(c) Calculate the power reflection ($R$) and transmission ($T$) coefficients for TE polarized light if $\theta_i = 45^\circ$, $n_1 = 1$, and $n_2 = 2$.

Now consider the dielectric stack depicted below:

(d) What is the total transfer matrix connecting the input wave ($E_i$) to the reflected ($E_r$) and transmitted ($E_t$) in terms of the Fresnel coefficients?

(e) What is the expression for the transmission coefficient of the full optical system in terms of the total transfer matrix elements?

(f) What is the expression for the reflection coefficient of the full optical system in terms of the total transfer matrix elements?

(g) If $\theta_i = 0$, $n_1 = 1$, $n_3 = 2$, and $d = \frac{\lambda}{4n_2}$, what is the value of $n_2$ that minimizes the reflected power for TE polarization?
Problem 2: Light-Matter

Based on the diagrams below, classify each material as one of the following:

- Metal
- Semiconductor
- Insulator
- Atomic/Molecular

Pay attention to the labels (they change for each graph)!

a)

b)

c)

d)
At $\lambda = 200 \, \mu m$, a metal has a refractive index $n = 445 + i618$. What is the AC optical conductivity?

At $\lambda = 50 \, \mu m$, a metal has a refractive index $n = 119 + i307$. What is the AC optical conductivity?

Estimate the DC conductivity from the optical constants at 50 $\mu m$ and 200 $\mu m$. Does the estimated DC conductivity values match the measured DC conductivity ($\sim 6.3 \times 10^7 S \, m^{-1}$)?

Using a value for the carrier concentration of $\sim 6 \times 10^{28} \, m^{-3}$, estimate the plasma frequency for this metal.

Compare the optical conductivity at $\lambda = 50 \, \mu m$ and $\lambda = 200 \, \mu m$ (parts a and b) to the DC conductivity (part c). In what ways do they differ and what is the origin of this difference?
Vector Calculus Identities

\[ \nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}. \]

\[ \nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \]

\[ \nabla \times \vec{F} = \hat{x} \left( \frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y} \right) + \hat{y} \left( \frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) + \hat{z} \left( \frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} \right) \]

\[ \nabla \cdot (\nabla \times \vec{F}) = 0 \]

\[ \nabla \times (\nabla f) = 0 \]

\[ \nabla \times (\nabla \times \vec{F}) = \nabla (\nabla \cdot \vec{F}) - \nabla^2 \vec{F} \]

\[ \nabla^2 f = \nabla \cdot \nabla f \]

\[ \nabla^2 \vec{F} = (\nabla^2 F_x) \hat{x} + (\nabla^2 F_y) \hat{y} + (\nabla^2 F_z) \hat{z} \]

Physical Constants

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \quad \text{permittivity of free space} \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \quad \text{permeability of free space} \]

\[ m = 9.11 \times 10^{-31} \text{ kg} \quad \text{mass of an electron} \]

\[ e = 1.60 \times 10^{-19} \text{ C} \quad \text{charge of an electron} \]

\[ k = 1.380650 \times 10^{-23} \text{ J/K} \quad \text{Boltzmann's constant} \]

\[ c_0 = 2.997925 \times 10^8 \text{ m/s} \quad \text{speed of light in free space} \]