DEBT, DELEVERAGING, AND THE LIQUIDITY TRAP: A FISHER-MINSKY-KOO APPROACH*

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In this article we present a simple new Keynesian–style model of debt-driven slumps—that is, situations in which an overhang of debt on the part of some agents, who are forced into rapid deleveraging, is depressing aggregate demand. Making some agents debt-constrained is a surprisingly powerful assumption. Fisherian debt deflation, the possibility of a liquidity trap, the paradox of thrift and toil, a Keynesian-type multiplier, and a rationale for expansionary fiscal policy all emerge naturally from the model. We argue that this approach sheds considerable light both on current economic difficulties and on historical episodes, including Japan's lost decade (now in its 18th year) and the Great Depression itself. (*JEL* Codes: E32, E52, E62)

I. Introduction

If there is a single word that appears most frequently in discussions of the economic problems now afflicting both the United States and Europe, that word is surely *debt*. As Table I shows, there was a rapid increase in gross household debt in a number of countries in the years leading up to the 2008 crisis. This debt, it is widely argued, set the stage for the crisis, and the overhang of debt continues to act as a drag on recovery. Debt is also invoked—wrongly, we argue—as a reason to dismiss calls for expansionary fiscal policy as a response to unemployment: you cannot solve a problem created by debt by running up even more debt, say the critics.

The current preoccupation with debt harks back to a long tradition in economic analysis. Irving Fisher (1933) famously argued that the Great Depression was caused by a vicious circle in which falling prices increased the real burden of debt, which then led to further deflation. The late Hyman Minsky (1986), whose work is back in vogue thanks to recent events, argued for a recurring cycle of instability, in which calm periods for the

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TABLE I

Household Gross Debt as Percent of Personal Income

	2000	2008
United States	96	128
United Kingdom	105	160
Spain	69	130

Source: McKinsey Global Institute (2010).

economy lead to complacency about debt and hence to rising leverage, which in turn paves the way for crisis. More recently, Richard Koo (2008) has long argued that both Japan's "lost decade" and the Great Depression were essentially caused by balance sheet distress, with large parts of the economy unable to spend thanks to excessive debt. Finally, since the onset of the crisis, Hall (2011) argues that tightening household borrowing constraint is essential to understand the crisis in the United States, whereas Mian and Sufi (2011a, 2011b) suggest that differences in the debt overhang of households across U.S. counties go a long way in explaining why unemployment is higher in some regions than others.

There is also a strand of thinking in international monetary economics that stresses the importance of debt, especially debt denominated in foreign currency. Krugman (1999), Aghion, Bacchetta, and Banerjee (2001) and others have suggested that "third-generation" currency crises—the devastating combinations of drastic currency depreciation and severe real contraction that struck such economies as Indonesia in 1998 and Argentina in 2002—are largely the result of private-sector indebtedness in foreign currency. Such indebtedness, it is argued, exposes economies to a vicious circle closely related to Fisherian debt deflation: a falling currency causes the domestic currency value of debts to soar, leading to economic weakness that in turn causes further depreciation.

Given the prominence of debt in popular discussion of our current economic difficulties and the long tradition of invoking debt as a key factor in major economic contractions, one might have expected debt to be at the heart of most mainstream macroeconomic models—especially the analysis of monetary and fiscal policy. Perhaps somewhat surprisingly, however, it is quite common to abstract altogether from this feature of the economy.[1] Even economists trying to analyze the problems of monetary and fiscal policy at the zero lower bound—and yes, that includes the present authors (see Krugman 1998, Eggertsson and Woodford 2003)—have often adopted representative agent models in which everyone is alike and the shock that pushes the economy into a situation in which even a zero interest rate is not low enough takes the form of a shift in everyone's preferences. Now, this assumed preference shift can be viewed as a proxy for a more realistic but harder to model shock involving debt and forced deleveraging. But as we will see, a model that is explicit about the distinction between debtors and creditors is much more useful than a representative agent model when it comes to making sense of current policy debates.

Consider, for example, the anti–fiscal policy argument we have already mentioned, which is that you cannot cure a problem created by too much debt by piling on even more debt. Households borrowed too much, say many people; now you want the government to borrow even more?

What is wrong with that argument? It assumes, implicitly, that debt is debt—it does not matter who owes the money. Yet that cannot be right; if it were, debt would not be a problem in the first place. After all, to a first approximation debt is money we owe to ourselves—yes, the U.S. has debt to China and so on, but that is not at the heart of the problem. Ignoring the foreign component, or looking at the world as a whole, the overall level of debt makes no difference to aggregate net worth—one person's liability is another person's asset.

It follows that the level of debt matters only if the distribution of that debt matters, if highly indebted players face different constraints from players with low debt. This means that all debt is not created equal—which is why borrowing by some actors now can help cure problems created by excess borrowing by other actors in the past. In particular, deficit-financed government spending can, at least in principle, allow the economy to avoid unemployment and deflation while highly indebted private sector agents repair their balance sheets.

1. Important exceptions include Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Considerable literature has sprung from these papers; for a comprehensive review, see Gertler and Kiyotaki (2010). For another recent contribution that takes financial factors explicitly into account, see Christiano, Motto, and Rostagno (2009).

This is, as we will see, just one example of the insights we can gain by explicitly putting private debt in our model.

In what follows, we begin by setting out a flexible-price endowment model in which “impatient” agents borrow from “patient” agents but are subject to a debt limit. If this debt limit is, for some reason, suddenly reduced, the impatient agents are forced to cut spending; if the required deleveraging is large enough, the result can easily be to push the economy up against the zero lower bound. If debt takes the form of nominal obligations, Fisherian debt deflation magnifies the effect of the initial shock. The type of liquidity constraints we consider are relatively standard, see,for example, Bewley (1977), Aiyagari (1994), and Huggett (1993). In these papers, however, borrowing is motivated by uninsurable idiosyncratic shocks, whereas we motivate it from difference in impatience to consume which simplifies the analytics considerably. We then turn to a sticky-price model in which the deleveraging shock affects output instead of, or as well as, prices. In this model, a shock large enough to push the economy up against the zero lower bound also lands us in a topsy-turvy world, in which many of the usual rules of macroeconomics are stood on their head. The familiar but long-neglected paradox of thrift emerges immediately; there are other perverse results as well, including both the “paradox of toil” (Eggertsson 2010b)—increasing potential output may reduce actual output—and the proposition that increasing price flexibility makes the real effect of a debt shock worse, not better.

Finally, we turn to the role of monetary and fiscal policy, where we find, as already indicated, that more debt *can* be the solution to a debt-induced slump. We also point out a possibly surprising implication of any story that attributes the slump to excess debt: precisely because some agents are debt-constrained, Ricardian equivalence breaks down, and old-fashioned Keynesian-type multipliers in which current consumption depends on current income reemerge. At the end of the article we summarize the results of various extensions of the model (mostly relegated to the appendices) to suggest further empirical prediction of the model. We show, for example, that a deleveraging shock implies a larger drop in investment and durable consumption than in regular consumption in our model, a pattern observed during the crisis. We also illustrate that our basic findings are not affected by various modeling details and how the deleveraging can be incorporated into much richer models such as the medium-scale dynamic stochastic general equilibrium (DSGE) models now common in the literature.

The article relates to several strands of the literature that has started emerging following the current crisis. We have already mentioned work by Hall (2011) and Mian and Sufi (2011a, 2011b) which we view as complementary to our findings. Curdia and Woodford (2009, 2010, 2011) and Del Negro et al. (2011) also draw the connections that disturbances in the financial markets can trigger a drop in the natural rate of interest and focus on the policy response of the Federal Reserve to these disturbances. The main difference relative to our model is that in our case there is a sudden reduction in the quantity of debt the household can borrow, which we argue corresponds well with a "Minsky moment," whereas in the other papers the source of the shock is a drop in resellability of financial assets in secondary markets (Del Negro et al. 2010) or various types of disturbances that directly affect the financial intermediaries (Curdia and Woodford 2009). Independently, Guerrieri and Lorenzoni (2010) study a Bewley-Aiyagari-Hugget model and subject the households to a reduction in the borrowing limit as we do,[2] and Goldberg (2010) conducts a similar exercise with firms that are also subject to idiosyncratic shocks. One important difference is that these authors do not incorporate the nominal debt deflation mechanism, which is central to our analysis. Another difference is that in these papers borrowing and lending is motivated by idiosyncratic shocks, rather than differences in preferences, which precludes a simple closed-form solution.[3]

II. Debt and Interest in an Endowment Economy

Imagine a pure endowment economy in which no aggregate saving or investment is possible, but individuals can lend to or borrow from each other. Suppose, also, that although individuals

2. One interesting difference in Guerrieri and Lorenzoni (2010) relative to this article is that the borrowing limit is not strictly binding for most consumers due to precautionary savings. Yet a tightening in the borrowing limit has qualitatively the same effect in their model as in our framework.

3. Our closed-form solution is what allows us to show several of the paradoxes and key results via simple aggregate supply–aggregate demand diagrams. Our assumption allows us to show how deleveraging can be put into a medium-scale DSGE model and facilitates various extensions of the model (see Section VIII).

all receive the same endowments, they differ in their rates of time preference. In that case, "impatient" individuals will borrow from "patient" individuals. We assume, however, that there is a limit on the amount of debt any individual can run up. Implicitly, we think of this limit as being the result of some kind of incentive constraint; however, for the purposes of this article we take the debt limit as exogenous.

Specifically, assume for simplicity that there are only two representative agents, each of whom gets a constant endowment $\left(\frac{1}{2}\right)Y$ each period. They have log utility functions:

$$E_t \sum_{t=0}^{\infty} \beta(i)^t \log C_t(i) \; with \, i = s \, or \, b,$$

where $\beta(s) = \beta > \beta(b)$—that is, the two types of individuals differ only in their rates of time preference. We assume initially that borrowing and lending take the form of risk-free bonds denominated in the consumption good. In that case the budget constraint of each agent is

$$D_t(i) = (1 + r_{t-1})D_{t-1}(i) - \frac{1}{2}Y + C_t(i) \, with \, i = s \, or \, b,$$

using the notation that a positive D means debt, and a negative D means a positive asset holding. As for example in Aiyagari (1994) and Huggett (1993), agents need to respect a borrowing limit D^{high} so that at any date t (here we include the next period interest rate payments for analytic simplicity)

$$(1 + r_t)D_t(i) \leq D^{high} > 0.$$

We assume that this bound is at least strictly lower than the present discounted value of output of each agent, that is, $D^{\text{high}} < \left(\frac{1}{2}\right)(\beta/(1-\beta))Y$. Because one agent ($b$) is more impatient than the other (s), the steady-state solution of this model is one in which the impatient agent will borrow up to his borrowing limit so that

$$C^b = \frac{1}{2}Y - \frac{r}{1+r}D^{high},$$

where r is the steady-state real interest rate. All production is consumed so that

$$Y = C^s + C^b,$$

implying

$$C^s = \frac{1}{2}Y + \frac{r}{1+r}D^{high}.$$

Consumption of the saver satisfies a consumption Euler equation in each period:

$$\frac{1}{C_t^s} = (1+r_t)\beta E_t \frac{1}{C_{t+1}^s},$$

implying that in the steady state the real interest rate is given by the discount factor of the patient consumer so that

$$r = \frac{1-\beta}{\beta}.$$

III. The Effects of a Deleveraging Shock

We have not tried to model the sources of the debt limit, nor will we try to in this article. Clearly, however, we should think of this limit as a proxy for general views about what level of leverage on the part of borrowers is "safe," posing an acceptable risk either of unintentional default or of creating some kind of moral hazard.

The central idea of debt-centered accounts of economic instability, however, is that views about safe levels of leverage are subject to change over time. An extended period of steady economic growth or rising asset prices will encourage relaxed attitudes toward leverage. But at some point this attitude is likely to change, perhaps abruptly—an event known variously as the Wile E. Coyote moment or the Minksy moment.[4]

In our model, we can represent a Minsky moment as a fall in the debt limit from D^{high} to some lower level D^{low}, which we can think of as corresponding to a sudden realization that assets were overvalued and that people's collateral constraints were too lax. In our flexible price economy, this downward revision of the debt limit will lead to a temporary fall in the real interest rate, which

4. For those not familiar with the classics, a recurrent event in the Warner Bros. Road Runner cartoons is the point when Wile E. Coyote, having run several steps off a cliff, looks down. According to the laws of cartoon physics, it's only when he realizes that nothing is supporting him that he falls. The phrase "Minsky moment" actually comes not from Minsky himself but from Paul McCulley of Pimco, who also coined the phrase "shadow banking."

corresponds to the natural rate of interest in the more general economy we consider shortly. As we will now see, a large enough fall in the debt limit will temporarily make the natural rate of interest *negative*, an observation that goes to the heart of the economic problems we currently face.

Suppose, then, that the debt limit falls unexpectedly from D^{high} to D^{low}.[5] Suppose, furthermore, that the debtor must move quickly to bring debt within the new, lower limit and must therefore "deleverage" to the new borrowing constraint. What happens?

To simplify, divide periods into "short run" and "long run." Denote short run with S and long run with L. Again, as in steady state, in the long run we have for the borrower

$$C_L^b = \frac{1}{2}Y - \frac{r}{1+r}D^{low} = \frac{1}{2}Y - (1-\beta)D^{low},$$

where we substituted for the long-run equilibrium real interest rate. In the short run, however, the borrower needs to deleverage to satisfy the new borrowing limit. Hence his budget constraint in the short run is

$$D_S = D^{high} - \frac{1}{2}Y + C_S^b.$$

Let us assume that he must deleverage to the new debt limit within a single period. We are well aware that this assumption sweeps a number of potentially important complications under the rug, and we return to these complications at the end of the article. For now, however, assuming that the borrower must deleverage within a single period to the new debt limit, we have $D_S = \frac{D^{low}}{1+r_S}$, so his consumption is given by

$$C_S^b = \frac{1}{2}Y + \frac{D^{low}}{1+r_S} - D^{high}.$$

The long-run consumption of the saver is

$$C_L^s = \frac{1}{2}Y + \frac{r}{1+r}D^{low} = \frac{1}{2}Y + (1-\beta)D^{low}.$$

5. We do not address the possibility that consumers may anticipate this reduction, which may be quantitatively important; see, for example, Kiyotaki, Michaelides, and Nikolov (2010).

Again, recall that all production in the short run is consumed so that

$$C_S^s + C_S^b = Y.$$

Substituting for the consumption of the borrower we get

$$C_S^s = \frac{1}{2}Y - \frac{D^{low}}{1+r_S} + D^{high}.$$

The optimal consumption decision of the saver satisfies the consumption Euler equation

$$C_L^s = (1+r_S)\beta C_S^s.$$

Substitute the short- and long-run consumption of the saver into this expression and solve for $1+r_S$ to obtain

$$1+r_S = \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta D^{high}}.$$

Now all we need for a deleveraging shock to produce a potentially nasty liquidity trap is for the natural rate of interest r_S to go negative, that is,

$$(C1)\ \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta D^{high}} < 1 \text{ or}$$

$$\beta D^{high} - D^{low} > \frac{1}{2}(1-\beta)Y.$$

This condition will apply if $\beta D^{\text{high}} - D^{\text{low}}$ is big enough, that is, if the debt overhang is big enough. The intuition is straightforward: the saver must be induced to make up for the reduction in consumption by the borrower. For this to happen, the real interest rate must fall, and in the face of a large deleveraging shock it must go negative to induce the saver to spend sufficiently more.

IV. Determining the Price Level, with and without Debt Deflation

We have said nothing about the nominal price level so far. To make the price level determinate, assume that there is a nominal government debt traded in zero supply so that we also have an arbitrage equation that needs to be satisfied by the savers:

$$\frac{1}{C_t^s} = (1 + i_t)\beta E_t \frac{1}{C_{t+1}^s} \frac{P_t}{P_{t+1}},$$

where P_t is the price level and i_t is the nominal interest rate. We need not explicitly introduce the money supply; the results that follow hold for a variety of approaches, including the "cashless limit" as in Woodford (2000), a cash-in-advance constraint as in Krugman (1998), and a money in the utility function approach as in Eggertsson and Woodford (2003).

We impose the zero bound

$$i_t \geq 0.$$

Let us now follow Krugman (1998) and fix $P_L = P^*$, that is, assume that after the deleveraging shock has passed the zero bound will no longer be binding, and the price level will be stable. We can think of this long-run price level as being determined either by monetary policy, as explained later, or by an exogenously given money supply, as in Krugman (1998). Then we can see that in the short run,

$$1 + r_S = (1 + i_S)\frac{P_S}{P^*}.$$

If the zero bound was not a problem, it would be possible to set $P_S = P^*$. But if we solve for the nominal interest rate under the assumption that $P_S = P^*$, we get

$$1 + i_S = (1 + r_S)\frac{P^*}{P_S} = \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta D^{high}} < 1.$$

That is, maintaining a constant price level would require a negative nominal interest rate if condition $C1$ is satisfied. This cannot happen; so if we substitute $i_S = 0$ instead and solve for the price level, we get

$$\frac{P_S}{P^*} = \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta D^{high}} < 1.$$

As pointed out in Krugman (1998), then, if a shock pushes the natural rate of interest below zero, the price level must drop now so that it can rise in the future, creating the inflation necessary to achieve a negative real interest rate.

This analysis has assumed, however, that the debt behind the deleveraging shock is indexed, that is, denominated in terms of the consumption good. Suppose instead that the debt is in nominal terms, with a monetary value B_t. In that case, deflation in the short run will increase the real value of the existing debt. Meanwhile, the debt limit is presumably defined in real terms, because it is ultimately motivated by the ability of the borrower to pay in the future out of his endowment. So a fall in the price level will increase the burden of deleveraging. Specifically, if debt is denominated in dollars, then $D^{\mathrm{high}} = B^{\mathrm{high}}/P_S$, and the indebted agent must make short-run repayments of

$$\frac{B^{high}}{P_S} - \frac{D^{low}}{1+r_S}$$

to satisfy the debt limit. Hence, as the price level drops he must pay more. Thus the natural rate of interest becomes

$$1 + r_S = \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta\frac{B^{high}}{P_S}}.$$

What this tells us is that the natural rate of interest is now *endogenous*: as the price level drops, the natural rate of interest becomes more negative, thus making the price level drop even more, and so on. This is simply the classic Fisherian debt deflation story.

V. Endogenous Output

We now move to an economy with production. To do this we assume that C_t now does not refer to a single good but is a Dixit-Stiglitz aggregate of a continuum of goods giving the producer of each good market power with elasticity of demand given by θ. Our representative consumers thus have the following utility function:

$$\sum_{t=0}^{\infty} \beta(i)^t [u^i(C_t(i)) - v^i(h_t(i))] \text{ with } i = s \text{ or } b,$$

where consumption now refers to $C_t = \left[\int_0^1 c_t(j)^{\frac{(\theta-1)}{\theta}}\right]^{\frac{\theta}{\theta-1}}$ and P_t is the corresponding price index $P_t = \left[\int_0^1 p_t(i)^{(1-\theta)}\right]^{\frac{1}{1-\theta}}$, while and h_t is the number of hours the agents work. We also make a slight generalization of our previous setup. We assume that there is a continuum of consumers of measure 1, and that an arbitrary fraction χ_s of these consumers are savers and a fraction $\chi_\mathrm{b} = 1 - \chi_s$ are borrowers. Aggregate consumption is thus

$$C_t = \chi_s C_t^s + \chi_b C_t^b,$$

where C_t has the interpretation of being per capita consumption in the economy, and C_t^s is per capita savers' consumption, and C_t^b per capita borrowers' consumption. Instead of receiving an endowment income, the agents now receive their income through wage, W_t, paid for each hour they work and through the profits of the firms in the economy. Aside from this different source of income, they face the same budget constraint as before.

There is a continuum of firms of measure 1, each of which produce one type of the varieties the consumers like. We assume all firms have a production function that is linear in labor. Suppose a fraction $1-\lambda$ of these monopolistically competitive firms keep their prices fixed for a certain planning period, while the λ fraction of the firms can change their prices all the time. We assume the firms are committed to sell whatever is demanded at the price they set and thus have to hire labor to satisfy this demand.

In the Appendix we put all the pieces of this simple general equilibrium model together. After deriving all the equilibrium conditions, we approximate this system by a linear approximation around the steady state (denoted by a bar) of the model where $D_t = D^{low} = \bar{D}$ and report the linearized equations of the model below.[6]

There are two main new elements. The first is that because production is endogenous agents are not only choosing consumption, they are also choosing how much to work. This gives rise to an optimal work and consumption choice, so that each type satisfies a new first-order condition given by (in a linearized form)

$$\hat{W}_t = \omega \hat{h}_t^i + \sigma^{-1} \hat{C}_t^i \text{ where } i = b \text{ or } s,$$

6. See Eggertsson and Woodford (2003) for a discussion of the conditions under which the linearization is valid.

where $\sigma \equiv -\frac{\bar{u}_c^b}{\bar{u}_{cc}^b \bar{Y}} \equiv -\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \bar{Y}} > 0$, $\omega \equiv \frac{\bar{v}_{hh}^b \bar{h}}{\bar{v}_h^b} \equiv \frac{\bar{v}_{hh}^s \bar{h}}{\bar{v}_h^s} > 0$, $\hat{C}_t^i \equiv \frac{C_t^i - \bar{C}^i}{\bar{Y}}$, and $\hat{W}_t \equiv log \frac{W_t}{\bar{W}}$.

Note that because the borrower can get more income by working, he can now deleverage by either cutting consumption or increasing hours worked. In equilibrium he will do both. Because production is linear in labor, we have

$$\hat{Y}_t = \chi_s \hat{h}_t^s + \chi_b \hat{h}_t^b.$$

where $\hat{Y}_t \equiv log \frac{Y_t}{\bar{Y}}$.

The second main new element comes from the fact that production is endogenous and that some prices are rigid. This implies a new classical Phillips curve, or aggregate supply (AS) equation, of the following form:

$$\pi_t = \kappa \hat{Y}_t + E_{t-1} \pi_t,$$

where $\kappa \equiv \left(\frac{\lambda}{1-\lambda}\right)\left(\omega + \sigma^{-1}\right)$, and $\pi_t \equiv log \frac{P_t}{P_{t-1}}$. The key point is that if inflation is higher than expected in the short run, output will be above potential.

We are now also a bit more specific about how monetary policy is set. In particular, we assume that the central bank follows a Taylor rule of the following form:

$$i_t = \max(0, r_t^n + \phi_\pi \pi_t),$$

where $\phi_\pi > 1$ and r_t^n is the natural rate of interest (defined shortly) and i_t now refers to $\log(1+i_t)$ in terms of our previous notation. The rest of the model is the same as we have already studied, with minor adjustments due to the way we have normalized our economy in terms of per capita consumption of each group and the different sources of income (which are no longer derived from an endowment but through wages and profits). Linearizing the consumption Euler equation of savers gives

$$\hat{C}_t^s = E_t \hat{C}_{t+1}^s - \sigma(i_t - E_t \pi_{t+1} - \bar{r}),$$

where $\sigma \equiv -\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \bar{Y}}$, $\hat{C}_t^s \equiv \frac{C_t^s - \bar{C}^s}{\bar{Y}}$, and $\bar{r} \equiv \log \beta^{-1}$. Linearizing the resource constraint yields

$$\hat{Y}_t = \chi_s \hat{C}_t^s + \chi_b \hat{C}_t^b.$$

To close the model, it now remains to determine the consumption behavior of the borrowers, which is again at the heart of the action. To simplify exposition, let us split the model into short run and long run with an unexpected shock occurring in the short run. We can then see immediately from the AS equation that $\hat{Y}_L = 0$, so that the economy will revert back to its "flexible price" equilibrium in the long run because this model has long-run neutrality. The model will then (with one caveat) behave exactly like the flexible price model we just analyzed. We have already seen that in the long run $\hat{C}^b_L = \hat{C}^s_L = 0$ in the model with flexible prices. Also note that away from the zero bound the policy rule implies a unique bounded solution for the long run in which $i_L = r^n_L = \bar{r}$ and $\pi_L = 0$.[7] Again, then, all the action is in the short run. The caveat here involves the determination of the long-run price level. Given the Taylor rule we have just specified, prices will *not* revert to some exogenously given P^*. Instead, they will be stabilized after the initial shock, so prices will remain permanently at the short-run equilibrium level P_S. It would be possible to write a different Taylor rule that implies price level reversion; as we will see shortly, the absence of price level reversion matters for the slope of the aggregate demand curve.

Back to the model: in the short run, the borrower once again needs to deleverage to satisfy his borrowing limit. His consumption is now given by

$$\hat{C}^b_S = \hat{I}^b_S - \hat{D} + \gamma_D \pi_S - \gamma_D \beta (i_S - \pi_L - \bar{r}),$$

where $\gamma_D \equiv \frac{\bar{D}}{\bar{Y}}$, $\hat{D} \equiv \frac{D^{high} - \bar{D}}{\bar{Y}}$, and $\hat{I}^b_S$ refers to the income of the borrower (but before we had the endowment in its place). Note that this is a "consumption function" in which current consumption is in part determined by current income (recall that in our current notation $\hat{I}^b$ is output per capita in percentage deviation from steady state)—not, as has become standard in theoretical macroeconomics, solely by expectations of future income. The explanation is simple: by assumption, the borrower is liquidity-constrained, unable to borrow and paying down no more debt than he must. In fact, the marginal propensity to consume out of current income on the part of borrowers is 1.

7. See Woodford (2003), but the key condition is that the Taylor principle is satisfied according to our policy rule. Here we abstract from the possibility that the zero bound can be binding due to self-fulfilling expectations; see, for example, discussion in Eggertsson and Woodford (2003) about how this sort of equilibrium can be excluded in the long run.

Meanwhile, the saver's consumption is given by

$$\hat{C}_S^s = \hat{C}_L^s - \sigma(i_S - \pi_L - \bar{r}).$$

By using the aggregate resource constraint, the consumption of the saver, and the optimal labor decisions of each household and $\hat{C}_L^b = \hat{C}_L^s = \pi_L = 0$, we can solve for the income of the borrower as

$$\hat{I}_S^b = \hat{W}_S + \hat{h}_S^b = \mu \hat{Y}_S - \chi_s \omega^{-1} \chi_b^{-1}(i_S - \bar{r}),$$

where $\mu \equiv (1+\omega^{-1})(\omega + \sigma^{-1}) - \sigma^{-1}\omega^{-1}\chi_b^{-1}$. Now substitute the two consumption functions into the resource constraint to obtain

$$\hat{Y}_S = \chi_s\{-\sigma(i_S - \bar{r})\} + \chi_b\left\{\hat{I}_S^b - \hat{D} + \gamma_D \pi_S - \gamma_D \beta(i_S - \bar{r})\right\},$$

and substituting for $\hat{I}_S^b$ with the expression from above and rearranging to obtain

$$\hat{Y}_S = -\frac{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}{1 - \chi_b \mu}(i_S - \bar{r}) - \frac{\chi_b}{1 - \chi_b \mu}\hat{D} + \frac{\chi_b \gamma_D}{1 - \chi_b \mu}\pi_S \text{ or} \tag{2}$$

$$\hat{Y}_S = -\frac{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}{1 - \chi_b \mu}(i_S - r_S^n) \tag{3}$$

where in the last line we have used the definition of the natural rate of interest (i.e., the real interest rate if prices were fully flexible) given by

$$r_S^n \equiv \bar{r} - \frac{\chi_b}{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}\hat{D} + \frac{\chi_b \gamma_D}{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}\pi_S.$$

Here once again, the nature rate of interest depends directly on the amount of deleveraging just as in the endowment economy. As the borrower cuts back on his spending, then the real interest rate needs to decline for the saver to pick up the slack. Observe that the strength of this effect depends on how much the borrower delevers by increasing work instead of cutting consumption. In the extreme where labor is perfectly elastic, that is, $\omega \to 0$, this effect disappears, whereas the endowment economy we just studied corresponded to the other extreme where $\omega \to \infty$. In what follows, we only consider the empirically relevant case in which ω

is positive but finite, and we also impose the restriction that $1-\chi_b\mu > 0$ in equation (2).[8]

What does equation (3) mean? It is an IS curve, a relationship between the interest rate and total demand for goods. The underlying logic is very similar to that of the old-fashioned Keynesian IS curve.[9] Consider what happens if i_S falls, other things equal. First, savers are induced to consume more than they otherwise would. Second, this higher consumption leads to higher income for both borrowers and savers. Because borrowers are liquidity-constrained, they spend their additional income, which leads to a second round of income expansion, and so on.

Once we combine this derived IS curve (3) with the assumed Taylor rule, we obtain aggregate demand relationship or an aggregate demand (AD) curve. It is immediately clear that there are two possible regimes following a deleveraging shock. If the shock is relatively small, so that the natural rate of interest remains positive, the actual interest rate will fall to offset any impact on output. If the shock is sufficiently large, however, the zero lower bound will be binding, and output will fall below potential.

The extent of this fall depends on the aggregate supply response, because any fall in output will also be associated with a fall in the price level, and the natural rate of interest is endogenous thanks to the Fisher effect. Because the deleveraging shock is assumed to be unanticipated, so that E-π_S = 0, the aggregate supply curve may be written as

$$\pi_S = \kappa \hat{Y}_S.$$

Substituting this into equation (2), and assuming the shock to D is large enough so that the zero bound is binding, we obtain

$$\hat{Y}_S = \Gamma - \frac{\chi_b}{1-\chi_b(\mu+\kappa\gamma_D)}\hat{D} < 0,$$
$$\pi_S = \kappa\Gamma - \frac{\chi_b\kappa}{1-\chi_b(\mu+\kappa\gamma_D)}\hat{D} < 0.$$

8. If this condition were not satisfied, demand would fall as interest rates are cut. Note that this condition is always satisfied for small enough χ_b. Hence, this condition requires that there is a sufficient number of agents that are unconstrained and thus respond directly to interest rate variations through the standard consumption Euler equation, the is, the number of constrained agents cannot be "too big." More precisely, the condition is satisfied if $\chi_b < \frac{1}{1+\omega}$.

9. Provided $\mu > 0$, which is the case as long as there are enough constrained players, or more specifically provided $\chi_b > \frac{\omega^{-1}\sigma^{-1}}{1+\omega^{-1}+\sigma^{-1}+\omega^{-1}\sigma^{-1}}$.

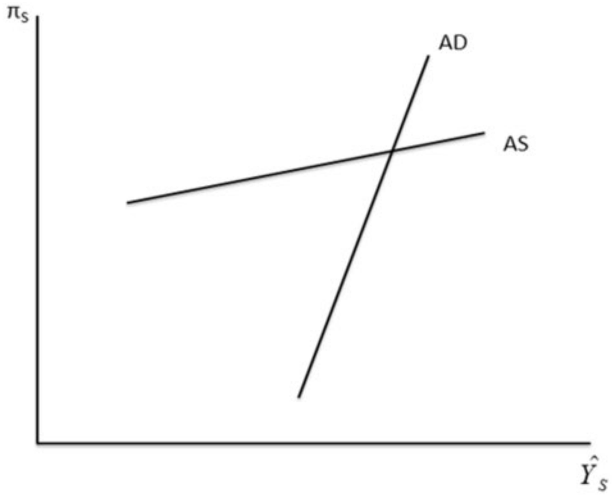

FIGURE I
Topsy-Turvy Economics

where $\Gamma > 0$.[10] So the larger the debt shock, the larger the fall in both output and the price level. But the really striking implications of this model come when one recasts it in terms of a familiar framework, that of aggregate supply and aggregate demand. The basic picture is shown in Figure I. The short-run AS curve is, as we have already seen, upward sloping. The surprise, however, is the AD curve: in the aftermath of a large deleveraging shock, which puts the economy up against the zero lower bound, it is *also* upward sloping—or, if you prefer, backward bending.[11] The reason for this seemingly perverse slope should be obvious from

10. $\Gamma = \frac{\chi_s(\omega^{-1}+\sigma)+\chi_b\gamma_D\beta}{1-\chi_b(\mu+\kappa\gamma_D)}\bar{r}$. The denominator has to be positive due the restriction discussed in note 7 and the first term has to be less than the second under the assumption that the natural rate of interest is negative.

11. We assume that the AD curve, while backward sloping, remains steeper than the AS curve. Otherwise the short-run equilibrium will be unstable under any plausible adjustment process. This amounts to the assumption that $1-\chi_b\mu > \kappa\chi_b\gamma_D$. Note that if $\chi_b = 0$ then the AD is vertical. As we increase the number of constrained people it starts sloping backward, eventually so far that the AS and AD become closed to parallel, the model explodes, and our approximation is no longer valid. Our assumption guarantees that this is not the case. In the absence of a deleveraging shock, then the zero bound is not binding, and we have a regular looking AD curve that is downward sloping and a well-behaved unique equilibrium.

the preceding exposition: because a lower price level increases the real value of debt, it forces borrowers to consume less; meanwhile, savers have no incentive to consume more, because the interest rate is stuck at zero.

We next turn to the seemingly paradoxical implications of a backward-sloping AD curve for some key macroeconomic issues.

VI. Topsy-Turvy: Paradoxes of Thrift, Toil, and Flexibility

The paradox of thrift is a familiar proposition from old-fashioned Keynesian economics: if interest rates are up against the zero lower bound, a collective attempt to save more will simply depress the economy, leading to lower investment and hence (through the accounting identity) to lower savings. Strictly speaking, our model cannot reproduce this paradox, because it is a pure consumption model without investment. However, it does give a plausible mechanism through which the economy can find itself up against the zero lower bound. So this model is, in spirit if not precisely in letter, a model of a paradox-of-thrift type world.[12]

Beyond this, there are two less familiar paradoxes that pop up thanks to the backward-sloping AD curve.

First is the "paradox of toil," first identified by Eggertsson (2010b) but appearing here in a starker, simpler form than in the original exposition, where it depended on expectation effects. Suppose that aggregate supply shifts out, for whatever reason—a rise in willingness to work, a change in tax rates inducing more work effort, a rise in productivity, and so on.[13] As shown in Figure II, this shifts the AS curve to the right, which would ordinarily translate into higher actual output. But the rise in aggregate supply leads to a fall in prices—and in the face of a

12. See Eggertsson (2010b) for an example of how the paradox occurs with endogenous investment but through preference shocks, which also show up as a decline in the natural rate of interest.

13. For simplicity we are considering a shock that only shifts out the AS curve without any effect on the AD schedule. Most shocks, however, shift both curves at the same time because they also affect the labor income of the borrower (positively or negatively). It can be shown, for example, that a uniform increase in willingness to work across the two types will not only shift out the AS curve, it will also shift the AD curve *backward* because it lowers real wages, which translates into lower income for the borrowers. This type of shock thus makes the paradox of toil even more severe in the model than what we consider in the figure.

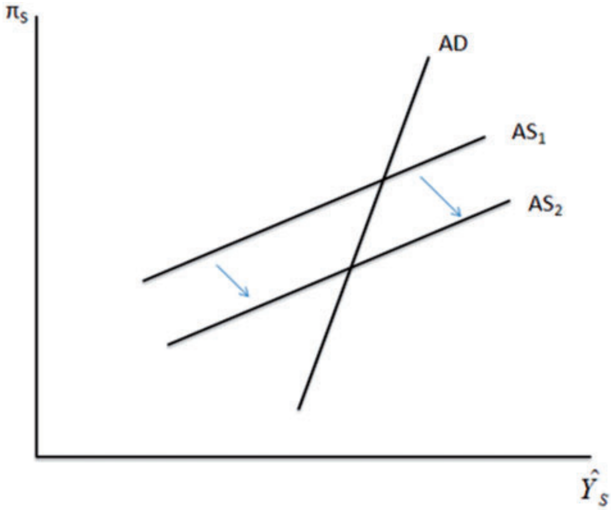

FIGURE II
The Paradox of Toil

backward-sloping AD curve, this price decline is contractionary via the Fisher effect. So more willingness or ability to work ends up reducing the amount of work being done.

Second, and of considerable relevance to the ongoing economic debate, is what we call the "paradox of flexibility." It is commonly argued that price and wage flexibility helps minimize the losses from adverse demand shocks. Thus Hamilton (2007), discussing the Great Depression, argues that "What is supposed to help the economy recover is that a substantial pool of unemployed workers should result in a fall in wages and prices that would restore equilibrium in the labor market, as long as the government just keeps the money supply from falling." The usual criticism of New Deal policies is that they inhibited wage and price flexibility, thus blocking recovery.

Our model suggests, however, that when the economy is faced by a large deleveraging shock, increased price flexibility—which we can represent as a steeper aggregate supply curve—actually makes things worse, not better. Figure III illustrates the point. The shock is represented as a leftward shift in the AD curve from AD_1 to AD_2; we compare the effects of this shock in the face of a flat AS curve AS_{sticky}, corresponding to inflexible wages and

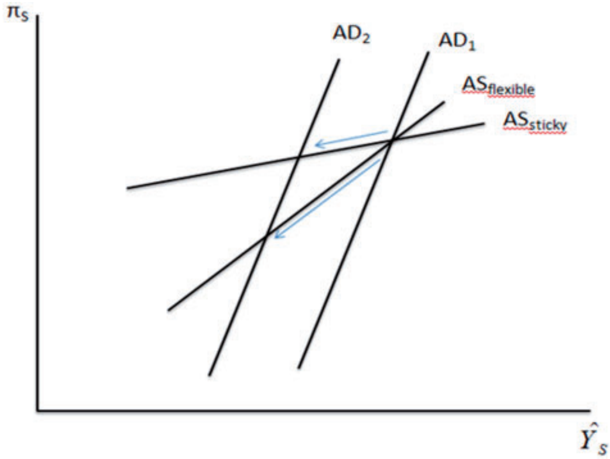

FIGURE III
The Paradox of Flexibility

prices, and a steeper AS curve $AS_{flexible}$, corresponding to more responsive wages and prices. The output decline in the latter case is larger, not smaller, than in the former. Why? Because falling prices do not help increase demand, they simply intensify the Fisher effect, raising the real value of debt and depressing spending by debtors.[14]

VII. MONETARY AND FISCAL POLICY

What can policy do to avoid or limit output loss in the face of a deleveraging crisis? Our model has little new to say on the monetary front, but it offers some new insights into fiscal policy.

On monetary policy: as pointed out by Krugman (1998) and reiterated in Section II of this article, expected inflation is the "natural" solution to a deleveraging shock, in the sense that it

14. A similar paradox is documented in a general equilibrium model in Eggertsson (2010b), but unlike here, there it relies on an expectation channel. Earlier literature documenting that increased price flexibility can increase output volatility includes Tobin (1975) and De Long and Summers (1986); see Bhattarai, Eggertsson, and Schoenle (2011) for a more recent exposition.

is how the economy can achieve the negative natural real interest rate even though nominal rates are bounded at zero. In a world of perfect price flexibility, deflation would “work” under liquidity trap conditions, if it does, only by reducing the current price level relative to the expected future price level, thereby generating expected inflation. It is therefore natural, in multiple senses, to think that monetary policy can deal with a deleveraging shock by generating the necessary rise in expected inflation directly, without the need to go through deflation first.

In the context of the model, this rise in expected inflation could be accomplished by changing the Taylor rule; this would amount to the central bank adopting, at least temporarily, a higher inflation target. As is well understood, however, this would only work if the higher target is credible—that is, if agents expect the central bank to follow through with promises of higher inflation even after the deleveraging crisis has passed. Achieving such credibility is not easy, because central bankers normally see themselves as defenders against rather than promoters of inflation and might reasonably be expected to revert to type at the first opportunity. So there is a time consistency problem.[15]

Where this model adds something to previous analysis on monetary policy is what it has to say about an incomplete expansion—that is, one that reduces the real interest rate, but not enough to restore full employment. The lesson of this model is that even such an incomplete response will do more good than a model without debt suggests, because even a limited expansion leads to a higher price level than would happen otherwise, and therefore to a lower real debt burden.

Where the model really suggests new insights, however, is on fiscal policy. It is a familiar proposition, albeit one that is strangely controversial even within the macroeconomics community, that a temporary rise in government purchases of goods and services will increase output when the economy is up against the zero lower bound; Woodford (2010) offers a comprehensive account of what representative agent models have to say on the subject. Contrary to widely held belief, Ricardian equivalence, in which consumers take into account the future tax liabilities created by current spending, does not undermine this

15. See Eggertsson (2006) for a formalization of this dynamic inconsistency problem, building on Krugman (1998).

proposition. In fact, if the spending rise is limited to the period when the zero lower bound is binding, the rise in income created by that spending fully offsets the rise in future taxes; the multiplier on government spending in a simple one-period liquidity trap consumption-only model like the one considered here, but without debt, ends up being exactly 1 (once multiple periods are studied, and expectations taken into account, this number can be much larger, especially at the zero bound as for example shown in Christiano, Eichenbaum, and Rebelo 2009 and Eggertsson 2010a).

What does modeling the liquidity trap as the result of a deleveraging shock add? First, it gives us a reason to view the liquidity trap as temporary, with normal conditions returning once debt has been paid down to the new maximum. This in turn explains why more (public) debt can be a solution to a problem caused by too much (private) debt. The purpose of fiscal expansion is to sustain output and employment while private balance sheets are repaired, and the government can pay down its own debt after the deleveraging period has come to an end.

Beyond this, viewing the shock as a case of forced deleveraging suggests that fiscal policy will, in fact, be more effective than standard models suggest—because Ricardian equivalence will not, in fact, hold. The essence of the problem is that debtors are liquidity-constrained, forced to pay down debt; this means, as we have already seen, that their spending depends at the margin on current income, not expected future income, and this means that something resembling old-fashioned Keynesian multiplier analysis reemerges even in the face of forward-looking behavior on the part of consumers.[16]

Let us revise the model slightly to incorporate government purchases of goods and services on one side and taxes on the other. For now, let us assume that future taxes on the borrowers stay constant across our policy experiments, so that any fiscal adjustment is offset by current or future taxes for the saver

16. The closest parallel to our debt-constraint consumers in studies of fiscal policy in new Keynesian models are the "rule-of-thumb" consumers in Gali, Lopez-Salido, and Valles (2007). In their work a fraction of workers spend all their income (because of rules of thumb or because they do not have access to financial markets). This gives rise to a multiplier of a similar form as we study here because in their model aggregate spending also depends in part directly on income as in old Keynesian models.

(we relax this assumption in the next section).[17] We assume that the government purchases the same composite good consumed by individuals, but uses that good in a way that, although it may provide utility to consumers, is separable from private consumption and therefore does not affect intertemporal choices of the savers. We also assume that taxation takes a lump-sum form. The budget constraint for borrowers may now be written

$$\hat{C}^b_S = \hat{I}^b_S - \hat{D} + \gamma_D \pi_S - \gamma_D \beta(i_S - \bar{r}) - \hat{T}^b_S,$$

whereas the savers' consumption Euler equation remains the same. The income of the borrower is now given by

$$\hat{I}^b_S = \mu \hat{Y}_S - \omega^{-1}\chi_b^{-1}\chi_s(i_S - \bar{r}) + \sigma^{-1}\left[\omega^{-1}\chi_b^{-1}\chi_s - 1\right]\hat{G}_S.$$

The AS equation is now

$$\pi_S = \kappa \hat{Y}_S - \varphi\kappa \hat{G}_S.$$

The resource constraint is now given by

$$\hat{Y}_S = \chi_s \hat{C}^s_S + (1-\chi_s)\hat{C}^b_S + \hat{G}_S.$$

Substituting the AS equation into the consumption function of the borrower, and substituting the resulting solution into the resource constraint, together with the consumption of the saver, and solving for output, we obtain an expression for output as a function of the fiscal instruments:

$$\hat{Y}_S = \Gamma - \frac{\chi_b}{1-\chi_b(\mu + \kappa\gamma_D\kappa)}\hat{D} - \frac{\chi_b}{1-\chi_b(\mu + \kappa\gamma_D\kappa)}\hat{T}^b_s + \frac{1+\omega^{-1}\sigma^{-1}\chi_s - \sigma^{-1}\chi_b - \chi_b\kappa\gamma_D\varphi}{1-\chi_b(\mu + \kappa\gamma_D\kappa)}\hat{G}_S. \tag{4}$$

To understand this result, it is helpful to focus first on a special case, that of a horizontal short-run aggregate supply

17. Because the saver is unconstrained, Ricardian equivalence applies, so it does not matter if the fiscal adjustment takes place by taxes in the short or the long run. The formal way of verifying this is to observe that the set of equations that pin down the equilibrium in the short run do not depend on the tax rates of the saver, see the Appendix.

curve, that is, $\kappa = 0$.[18] In that case the third term simplifies to

$$\left(\frac{1+\omega^{-1}\sigma^{-1}\chi_s - \sigma^{-1}\chi_b}{1-\chi_b\mu}\right)\hat{G}_S.$$

With just a little bit of algebra, using the expression for μ. is easy to show that a temporary rise in government spending always has a multiplier greater than 1 (the condition is that $\omega \geq -1$, which is always satisfied because this parameter must be positive), with the size of that multiplier depending positively on the share of debt-constrained borrowers in the economy. Consider for example the commonly used parameter values σ. 2 and $\omega = \frac{1}{2}$. Then the multiplier is $\frac{2-\frac{3}{2}\chi_b}{2-3\chi_b}$, so if constrained borrowers receive one-third of income, for example, the multiplier would be 1.5; if they receive half of income, it would be 2.5, and so on.

If we now reintroduce an upward-sloping aggregate supply curve, so that $\kappa > 0$, the multiplier is affected by two forces. First, the fiscal expansion has the additional effect of raising the price level above what it would have been otherwise, and hence reducing the real debt burden. Second, the increase in spending increases aggregate supply,[19] which now has a price effect and works in the opposite direction due to the paradox of toil. By taking a partial derivative of the multiplier with respect to κ we can see that the first effect will always dominate, so that the multiplier is increasing in κ (because $\varphi < 1$). Overall this model suggests a relatively favorable view of the effectiveness of fiscal policy after a deleveraging shock.

Also note the middle term: in this model, tax cuts and transfer payments are effective in raising aggregate demand, as long as they fall on debt-constrained agents. In practice, of course, it is presumably impossible to target such cuts entirely on the debt-constrained, so the old-fashioned notion that government spending gets more bang for the buck than taxes or transfers survives. The model also suggests that if tax cuts are the tool chosen, it matters greatly who receives them.

18. In this case we abstract from the "Fisher effect" of inflation reducing real debt and thus creating more expansion, but we also abstract from the fact that an increase in government spending increases AS, which works in the opposite direction due to the paradox of toil.

19. It makes people work more due to an increase in the marginal utility of private consumption.

The bottom line, then, is that if we view liquidity trap conditions as being the result of a deleveraging shock, the case for expansionary policies, especially expansionary fiscal policies, is substantially reinforced. In particular, a strong fiscal response not only limits the output loss from a deleveraging shock; it also, by staving off Fisherian debt deflation, limits the size of the shock itself.

VIII. Extensions

VIII.A. Deleveraging, Endogenous Debt Limits, and Long-Run Taxes

How important is it that we assumed that current or future taxes on the saver financed the fiscal policy experiments in the last section? How important is it for our conclusion about fiscal policy that the debt limit is exogenous? Here we show that fiscal policy can be even more expansionary if it is financed by taxes on future borrowers instead of the savers. Yet though the results from the last section did not depend on the debt limit being exogenous, we show that this new conclusion depends critically on the debt limit being fixed.

Consider first the possibility that a tax cut on the borrower is not met by current or future tax increases on the saver, but instead by higher long-run taxes on the borrower. This makes the tax cuts even more expansionary under our assumption about an exogenous debt limit (as the reader can confirm in equation (5) shortly). The reason for this is that although the borrower will spend every additional dollar of income in the short run, the saver's spending depends on his current and expected future income. Because an increase in taxes on the borrower in the future (relative to our previous policy experiment) moves money from the borrower to the saver in the long run, the saver's short-run consumption increases. In fact, the deleveraging shock can in principle be completely undone by a tax cut targeted at the borrower in the short run that is large enough to fully offset the amount of deleveraging needed, as long as this tax cut is financed by a corresponding increase in long-run taxes on the borrower. Intuitively, the governments "circumvent" the change in the private borrowing limit this way by using its own ability to borrow.

That shifting the future tax burden from the saver to the borrower is expansionary due to higher short-run consumption of the saver is a rather special result. It relies heavily on the assumption that the borrowing limit is exogenous.[20] Consider a slight generalization of our previous debt limit so that

$$D_t^b = (1-\gamma^l)D^l + (1+r_t)\gamma^l E_t \sum_{j=1}^{\infty} R_{t,t+j}(I_{t+j}^b - T_{t+j}^b),$$

where $R_{t,t+j} \equiv \frac{1}{1+r_t}\frac{1}{1+r_{t+1}}\cdots\cdots\frac{1}{1+r_{t+j}}$ is the real discount factor and $0 \leq \gamma^l \leq 1$. Here the debt limit does not depend only on an exogenous constant, it also depends on the net present value of future income net of taxes ($\gamma^l = 1$ corresponds to the "natural borrowing limit," and $\gamma^l = 0$ is our previous case). We now think of a reduction in D_t^b as being due to an unexpected drop in D^l(as before), a drop in expected future disposable income or a change in the level of future interest rates. Importantly, a change in expectation about the future tax can also reduce the debt limit, as higher future taxes reduce the borrower ability to pay back.

Denote taxes on the borrower in the short run by $\hat{T}_S^b$ and long-run taxes by $\hat{T}_L^b$, and the (now endogenous) debt limit is $\hat{D}_S^b$.[21] Going through the same steps as when deriving equation (4), we obtain a solution for output in the short run as

$$\begin{aligned} \hat{Y}_S = \Gamma &- \frac{\chi_b}{1-\chi_b(\mu+\kappa\gamma_D\kappa)}\hat{D} - \frac{\chi_b}{1-\chi_b(\mu+\kappa\gamma_D\kappa)}\hat{T}_s^b \\ &+ \frac{1+\omega^{-1}\sigma^{-1}\chi_s - \sigma^{-1}\chi_b - \chi_b\kappa\gamma_D\varphi}{1-\chi_b(\mu+\kappa\gamma_D\kappa)}\hat{G}_S \\ &+ \frac{\chi_b\beta}{1-\chi_b(\mu+\kappa\gamma_D\kappa)}\hat{D}_S + \frac{\chi_b+\omega^{-1}\sigma^{-1}}{1-\chi_b(\mu+\kappa\gamma_D\kappa)}\hat{C}_L^s, \end{aligned} \tag{5}$$

where the last two terms are new relative to expression (4) and are determined by long-term taxes on the borrower as follows:

$$\hat{C}_L^s = \frac{\chi_b}{\chi_s}\frac{1-\gamma^l}{1+\omega^{-1}\sigma^{-1}(1-\gamma^l)}\hat{T}_L^b$$

20. This is one reason we assumed long-run taxes on the borrower were constant in our previous experiment, that is, we did not want to exaggerate the effect of fiscal policy.

21. Again we assume that any shortfall or surplus in the government budget is made up by current or future lump-sum taxes on the saver.

and

$$\hat{D}_S^b = -\frac{(1-\beta)^{-1}\gamma^l}{1+\omega^{-1}\sigma^{-1}(1-\gamma^l)}\hat{T}_L^b.$$

Observe first that under this more general specification of the borrowing limit, the results from the last section are unchanged as long as the government spending or tax cuts on borrowers are financed by current or future taxes on the saver (because in this case $\hat{T}_L^b = 0$ and the new terms drop out). It now becomes clear, however, that it is no longer the case that an increase in future taxes of the borrower necessarily increases demand in the short run: An increase in future taxes of the borrower reduces the debt limit of the borrower, thus requiring an ever bigger reduction in his spending due to deleveraging. We can see that in the limit as γ^l gets closer to 1 there is a Ricardian equivalence result of sorts that enters the picture: if a short-run tax cut on the borrower is financed by a higher long-run taxes on the borrower, it has no effect due to the endogenous effect on the borrowing limit. But note that even in this limiting case, our result from last section survives, that is, the tax cut on the borrower is still expansionary (because it was being financed by current and future taxes on the saver), but in this limiting case it is *only* expansionary due to the fact that it reflects a redistribution of the tax burden from the borrower to the saver.

Before concluding this section, it is worth noting that there are several quite reasonable cases in which an endogenous debt limit amplifies the effect of fiscal policy beyond what we have already studied. Consider, for example, a debt limit that depends also on the current level of income of the borrower, for example $D_S^b = f(Y_S)$ with $f'(Y_S) > 0$. In this case, a tax cut that increases demand will have a “second-round effect” due to the fact that it increases aggregate income and hence increases his debt limit, thus increasing demand even further, and so on. There are many reasons to expect the borrowing limit to depend to some extent on current condition, for example, if the collateral value of the borrowers assets depend on current market conditions (such as the price of houses), or if current income of the borrower is used by banks to predict his future income.

VIII.B. Deleveraging, the Trade-off between Inflation and Output, and Inefficient Policy Responses

Here we outline how a deleveraging cycle can matter even when the interest rates are positive. We also clarify that our theory does not imply that there is *always* deflation at that zero bound. To show this, we need to abandon a simplifying assumption we made in our earlier sections, which was that the deleveraging shock had no effect on marginal costs of firms, and thus it did not shift the aggregate supply relationship between inflation and output in the short run. More generally, however, this short-run relationship is given by

$$\pi_S = \kappa_1 \hat{Y}_S + \kappa_2 (i_S - \bar{r}),$$

where κ_1 and κ_2 are defined in the note.[22] The new term comes about due to the fact that a variation in the interest rate can increase or decrease firms' marginal costs through the effect it has on labor supply and thus the real wage rate. Hence a deleveraging shock can trigger a variation in the natural rate of interest and at the same time create a trade-off between output and inflation. This effect can be positive or negative, depending on how the model is parameterized, although we have not found this to be quantitatively important in numerical experiments. It is not difficult to imagine, however, other quantitatively relevant channels through which a deleveraging shock can affect firms' marginal costs,[23] and hence we think the foregoing specification can illuminate at least two general lessons that go beyond the specifics of the current model and apply more broadly to deleveraging cycles.

First, a "cost push" term of this kind suggests that even in the absence of the zero bound, a deleveraging shock could be relevant for the policy stabilization problem of the government, to the extent it creates not only a need to reduce the nominal interest rate but also a trade-off between inflation and output. Second, this specification illustrates that our theory does not necessarily

22. Here $\kappa_1 \equiv \left(\frac{\lambda}{1-\lambda}\right)\frac{\omega^b + \sigma^{b^{-1}}}{1-\chi_s\left(1-\frac{\omega^b}{\omega^s}\right)}$ and $\kappa_2 \equiv \left(\frac{\lambda}{1-\lambda}\right)\frac{\chi_s\left(1-\frac{\omega^b}{\omega^s}\right)-\chi_s\left(1-\frac{\sigma^s}{\sigma^b}\right)}{1-\chi_s\left(1-\frac{\omega^b}{\omega^s}\right)}$. If $\omega^b = \omega^s$ and $\sigma^s = \sigma^b$, we obtain our previous equation.

23. If the firms need to deleverage, for example, this would show up in a cost push shock under some plausible specifications. Within the context of the current crisis, of course, we have seen other sources of increases in marginal costs due to disturbances in the commodity markets.

imply that there needs to be deflation in equilibrium once the economy is up against the zero bound or even any reduction in current or expected inflation. All that is needed is for the natural rate of interest to be below what the central bank wants it to be (leading to an output slack). If there are strong enough "cost push" shocks triggered by a deleveraging disturbance, then a liquidity trap[24] can very well happen at positive inflation. Nevertheless, at positive or negative inflation, the slope of the AD and AS curves we have derived is exactly the same, and all the results we have derived remain unchanged (i.e., the Fisher effect of inflation reducing real value of debt remains there, the paradox of thrift and toil, large multipliers, and so on).

One other mechanism through which deleveraging cycles can matter for business cycle fluctuation away from the zero bound is if deleveraging shocks are not fully offset by monetary policy. We already noted that this will happen if deleveraging shock triggers simultaneous cost push pressures even if the central bank offsets the drop in the natural rate of interest. It is straightforward to think of many other reasons monetary policy does not fully offset a deleveraging cycle, for example, due to suboptimal monetary policy, additional constraints on what policy can do (e.g., due to concerns about unanchoring inflation expectations). Finally, consider a large currency area in which the deleveraging cycle hits some regions of the currency union harder than others. In this case, policy aimed at offsetting the cycle in one region will not offset it in the other.

VIII.C. Deleveraging, Durable Goods, and Investment

So far we assumed only one type of private spending: a good that was consumed within one period. One interesting aspect of a deleveraging shock, compared to the reduced-form preference shock common in the literature, is that it also has predictions about how the deleveraging takes place, that is, through cutbacks of which *component* of aggregate spending, such as perishable consumption, durable goods, or productive investment. We now briefly summarize what our model has to say about this (although most of the derivation and numerical examples are left to the Online Appendix).

24. Which we define, simply, as the central bank policy to be constrained by the zero bound.

Broadly speaking, our analysis suggests that in response to a deleveraging shock, the main adjustment should take place via cuts in spending on investment or durable goods rather than on perishable consumption goods. The reason for this is simple: if agents need to cut their spending in the short run to satisfy a debt limit, it makes sense to do so by cutting those spending components that yield benefit over a long period of time (if a borrower is trying to deleverage, for example, that would not be a good time to upgrade his car at the expense of buying food). In the case of investment in capital that enters as a factor of production, there is the additional effect that a short-run recession reduces the incentives of people to invest because the factors of production already in existence are being underused.

Consider first durable goods. The utility function now not only involves one-period consumption but also durable consumption that enters utility additively separately: $u^i(C_t(i)) + d^i(K_t(i))$, where $K_t(i) = I_t(i) + (1-\delta)K_{t-1}(i)$. The household needs to decide between purchasing the perishable consumption good $C_t(i)$ and the durable consumption good $I_t(i)$ that yields a flow utility over time trough the capital stock $K_t(i)$. Both the saver and the borrower will now satisfy an additional first-order condition

$$\hat{C}_t^i = (1-\beta_i(1-\delta))\frac{\sigma_C}{\sigma_K}\hat{K}_t^i + \beta_i(1-\delta)E_t\hat{C}_{t+1}^i \text{ with } i = s, b. \quad (6)$$

Our characterization nests our previous specification: if $\delta = 1$ then the two goods are identical so that the model is the same as before and the deleveraging consumers cut down on each consumption item by the same amount. As δ decreases, the deleveraging consumers start cutting back more and more on durables relatively to the perishable goods. This is illustrated by a numerical example in Table II. With $\delta = 1$ the cutback in consumption of the two goods is the same, but with $\delta = 0.25$ the borrower cuts spending on durables two to three times than his spending on perishable consumption (the Online Appendix describes in detail the numerical example and some variations). Under flexible prices this is offset in the aggregate by an increase in the savers' spending on durables (as in the case of nondurables that we established earlier in the article), leaving aggregate spending unchanged as seen in the column labeled "Flexible prices." Once nominal frictions are added, however, the fall in the spending by the borrower carries over to the aggregate, because a substantial reduction in the real interest rate is need to stimulate spending

TABLE II
DURABLE GOODS EXTENSION

	$\delta = 1$ Flexible prices	$\delta = 1$ Sticky	$\delta = 0.25$ Flexible prices	$\delta = 0.25$ Sticky
$\hat{Y}_S (= \hat{C}_S)$	0	−5.4	0	−8.2
π_S	0	−5.2	0	−8.5
r_S	−1.7	0	−1.7	0
$\hat{C}_S^b$	−6.7	−20.5	−6.2	−23.3
$\hat{I}_S^b$	−6.7	−20.5	−20.5	−78
$\hat{C}_S^s$	3.4	2	3	0.3
$\hat{I}_S^s$	3.4	2	12.2	4.5

Note: See Online Appendix for parameter values assumed and further discussion. All variables are expressed in annual percentage terms.

by the saver as can be seen in the column labeled "Sticky." That cannot be achieved due to the zero bound. Overall these examples suggest that the results are even stronger once durable goods enter the picture.

Moving to productive investment, the same basic insight applies, although the details differ. We outlined the main differences shortly. Now capital does not add anything to utility but is instead a factor of production according to the standard Cobb-Douglas function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. There is an adjustment cost of investment given by $\gamma\left(\frac{I_t}{\bar{I}}\right)$ so that the capital owners pay a convex cost of investing differently from steady state $\bar{I}$. This cost allows us to approximate our previous specification. If the cost of adjusting the capital stock is infinite, this variation of the model approximates the one without capital, because then all the deleveraging takes place through the consumption and labor margin. As the cost of adjusting is reduced, more and more of the deleveraging takes place via drop in investment.

A simple way to motivate lending is to assume that the borrower has access to "investment opportunities" while the saver can only invest in the risk-free bond.[25] The extension thus makes clear that we should not only think of "borrowers" as corresponding to "liquidity-constrained poor," as is common in the literature. More generally, the borrowers correspond to those that—for whatever reason—are in need of funding, for example, due to investment opportunities, but their lending is constrained by a

25. One could also include age dynamics and housing to motivate lending.

debt limit.[26] The saver's problem is the same as before, but relative to our previous model the borrower satisfies an additional Euler equation, analogous to (6), given by

$$(7)\qquad \hat{C}_t^b = -\sigma^b \chi_q \hat{q}_t + E_t \hat{C}_{t+1}^b + \frac{\sigma^b \gamma_{II}}{\beta_b (1-\delta)} \hat{I}_t - \sigma^b \gamma_{II} E_t \hat{I}_{t+1},$$

where $\chi_q \equiv \frac{1-\delta\beta}{\delta\beta}$, $\hat{q}_t$ is the rental rate of capital in deviation from steady state, and γ_{II} reflects the investment adjustment cost. Table III shows a numerical example (assuming rigid prices) comparing the model with and without variable investment. The first column, corresponding to the case in which the adjustment cost is infinite, is an approximation to the model with fixed capital stock (similar to our previous example in Table II with $\delta = 0$). Here the deleveraging happens via a drop in aggregate consumption driven by the deleveraging of the borrower. The next column shows how the model behaves once the cost of adjusting investment is relaxed. We see that the deleveraging is no longer achieved by a drop in consumption to the same extent but by an even larger drop in investment. Relative to the model with durable goods, the drop in aggregate demand is bigger for a given deleveraging shock because investment responds even more aggressively than durable consumption, leading to an ever bigger contraction in demand. When investment is productive, there is even less reason to spend on capital goods than on regular consumption because production is already well below capacity, giving rise to even greater demand instability. In the Online Appendix we show the full underlying model, discuss the extension in more detail, and illustrate more numerical examples.

VIII.D. Dynamic Deleveraging

How important was it that we assumed the deleveraging happened only in one condenses time period that we labeled as "short run?" We briefly outline how the model can be extended so that the period of deleveraging stretches out over several time periods. In this case, the duration of the liquidity trap becomes

26. An important implication of our assumption that only borrowers have investment opportunities is that q_t, the rental rate of capital the borrower obtains, can be different from the risk-free borrowing rate. The link between the risk-free real interest rate and marginal productivity of capital is thus broken, and a negative real interest rate in the bond market is perfectly consistent with positive marginal rate of return of capital (even in the absence of adjustment costs).

TABLE III
INVESTMENT DYNAMICS (ZERO BOUND)

	$\gamma_{II} = \infty$, $\chi_s = 0.7$	$\gamma_{II} = 0.5$, $\chi_s = 0.9$
$\hat{Y}_S$	−7.6	−8.6
$\hat{C}_S$	−10.3	−4
$\hat{I}_S$	0	−21.7
π_S	−8.1	−3.3
r_S	0	0

Note: See Online Appendix for parameter values assumed and further discussion. All variables are expressed in annual percentage terms.

an endogenous variable. This, then, may provide even stronger case for policy intervention, as policy now not only alleviates the effects of the crisis itself, it can also reduce its duration.

The borrower faces a borrowing rate that is a declining function of the "safe" level of debt (thus the higher the "safe" level of debt, the better terms the borrower faces for any given level of debt). Hence, the borrower is no longer at a corner and needs to make a decision about his level of outstanding debt in each period. The basic structure of a loan contract is adopted from Curdia and Woodford (2009) with the main difference being that the borrowing rate is increasing in the borrower's *own* level of debt, in line with our earlier assumption, rather that aggregate debt.[27] In particular, we assume that the borrower faces the borrowing rate $1 + i_t^b = (1 + i_t^d)(1 + \omega_t)$, where i_t^b is the interest rate faced by the borrower, i_t^d is the interest rate faced by the saver ("depositor"), and $1 + \omega_t$ is the spread between the two. We assume the spread is given by the function $\omega_t = \omega(\frac{B_t}{P_t}, b^j)$, where the spread is increasing in both the real value of the debt $\frac{B_t}{P_t}$ and declining in the "safe value of debt" b^j.[28] We now assume that b^j shifts unexpectedly at time 0 from b^{high} to b^{low}, that is, now the safe level of

27. We also keep the same structure of heterogeneity we have earlier, but Curdia and Woodford (2010) instead consider an economy in which the type of the consumers vary stochastically over time. The main advantage of our modeling device is simplicity, but the disadvantage is that it is less suitable for normative analysis of the type found in their paper because our model does not have a natural welfare criterion. Note also that our "Minsky moment" shock represents a permanent shock that Curdia and Woodford (2009) do not consider in their model.

28. We assume that government debt does not affect this relationship, to the extend it does, this may affect the results as we saw in Section VIII.A.

debt each borrower faces has been unexpectedly reduced. In other words, the economy experiences a Minsky moment. As a consequence of the Minsky moment, the borrowing rate increases and the borrower has a strong incentive to "deleverage."[29] Crucially, with this more general structure, the borrower will choose an optimal deleveraging path over time rather than being forced exogenously to cut down spending within one "short run."

Using the same structure of heterogeneity as in the baseline model and assuming that prices are set at a staggered interval as in Calvo (1982) we show in the Online Appendix that the model (abstracting from taxes and government spending) can be summarized as

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(i_t^d - E_t\pi_{t+1} - r_t^e)$$
$$\pi_t = \kappa\hat{Y}_t + \beta E_t\pi_{t+1},$$

where r_t^e is the natural rate of interest given by $r_t^e = -\chi_b(1+\lambda)\hat{\omega}_t = -\chi_b(1+\lambda)\vartheta\hat{b}_t$, where σ, β are defined as in previous sections; $\kappa, \lambda, \vartheta$ are defined in the Appendix; $\hat{b}_t$ is the real value of the debt of the borrower; and the other hatted variables are defined as before. These two equations are the same as in the canonical new Keynesian model, explored, for example, in Eggertsson and Woodford (2003) at the zero bound, with the key difference being that the natural rate of interest is no longer exogenous. Instead, it is an endogenous variable that is a function of the level of private debt in the economy, $\hat{b}_t$, that is optimally chosen by the borrowers. To solve for $\hat{b}_t$, we need to solve an 11-equation dynamic system reported in the Online Appendix. Rather than going into the details, what we want to stress two points here. First, this extension naturally nests the standard new Keynesian model but with an explicit way of thinking about how long the zero bound may be binding due to a negative natural rate of interest. This makes clear that it is relatively straightforward to incorporate a deleveraging mechanism into a larger DSGE model. Second, the dynamic deleveraging mechanism already illustrated can lead to meaningful economic

29. In this variation of the model the inventive to deleverage is driven only by the increase in spreads. A more general environment could allow banks to reduce aggregate borrowing not only via higher spreads but also through margins such as higher "lending standard," that is, such as an increase in "down payments" for house purchases, increased requirements for "collaterals," and so on. All these margins have been exploited by banks following the crisis of 2008.

TABLE IV
DYNAMIC DELEVERAGING

	No $\hat{G}_0$	With $\hat{G}_0$
$\hat{Y}_0$	–7	–0.7
π_0	–2	–0.2
i_0^d	0	0
i_0^b	11.4	10.3
b_{-1}	30	30
G_0	0	1
Duration of ZLB	10	3

Note: See Online Appendix for parameter values assumed and further discussion. All variables are expressed in annual percentage terms.

contraction in a calibrated version of the model and the dynamic deleveraging assumption amplifies the effect of government spending. This is illustrated via numerical example in Table IV, which is discussed in more detail in the Online Appendix. As we see in the table, a debt overhang of 30% leads to a drop in output of about 7% on impact, and the zero bound is binding for 10 quarters. A small increase in government spending almost eliminates the recession. The effect of government spending becomes even larger than in our previous examples; in a dynamic setting, it also has an effect by shortening the expected duration of the zero bound (the duration is reduced from 10 quarters to 3 as shown in Table IV). This effect of government spending comes on top of the effect spending has on inflation expectation during the period in which the zero bound, a major theme of recent work by Eggertsson (2010a) and Christiano, Eichenbaum, and Rebelo (2009). Taken together, the case for fiscal intervention is thus even stronger once deleveraging is modeled in a more dynamic way.

IX. RELATION TO EMPIRICAL FINDINGS

One stark prediction of our theory is relevant for fiscal policy: the model suggests that government spending should have a large effect on output at zero interest rates—because then it will not be offset by monetary policy—and this number should be greater than 1. Meanwhile, at positive interest rates the effects should be much weaker (depending on the policy reaction function of the central bank). It is difficult, therefore, to draw any conclusion about the government spending multipliers in the current crisis in the United States based on cross-country studies (e.g., Alesina and Ardagnia 2010; Guarjardo, Leigh, and Pescatori 2011). Most of the sample in these papers is from periods with positive interest rates. This also applies to some studies of U.S. fiscal policy using post–World War II data (e.g., Blanchard and Perotti 2002; Romer and Romer 2010).

Rather than a cross-country study, a better benchmark may be the United States during the Great Depression or Japan during the Great Recession. Obvious challenges are data limitation and the fact that both episodes had several moving parts. Nevertheless, there is some literature on these events that is largely consistent with our bottom line and can be summarized as "fiscal policy worked when it was tried"; for example, see Brown (1956) for the Great Depression (and Eggertsson and Pugsley (2006) and Eggertsson (2008) and Posen (2010) for the case of the Great Recession in Japan (as well as Kuttner and Posen 2002).

There is also a relatively recent empirical literature that focuses on the effect of variations in defense spending in the United States, where World War II features prominently. During this period, short-term nominal interest rates were close to zero. Hence fiscal policy should have had an effect larger than one according to our theory. Barro and Redlick (2009), Hall (2009), and Ramey (2011) estimate the multiplier of government spending in the range of 0.5–1.0 during this period.[30] Meanwhile, Gordon and Krenn (2010) find that the early part of World War II spending had a large impact but this effect faded as the economy reached capacity constraints in key industries. Overall the U.S. time-series evidence seems mixed with respect to the effect of government spending at zero interest rates. Evidence based on U.S. cross-state evidence, however, is better consistent with larger multiplier in the range of 1.5–2.2 (see Nakamura and Steinsson 2010; Shoag 2010).[31]

30. There is not a consensus on whether the special circumstances of war would lead to an over- or underestimate. Patriotism would tend to lead to an overestimate (see Barro and Redlick 2009), whereas various war time price controls and rationing to an underestimate (see Hall 2009).

31. Our theory does not explicitly allow us to make prediction across states, but Steinsson and Nakamura (2010) suggest that their "open economy" multiplier

Another interesting prediction of our model relates to deleveraging. The basic message of our model was that a debt overhang will increase the amount of the interest rate cut needed for output to stay at potential, and once those interest rates are no longer feasible, the larger the debt overhang, the larger the contraction and the slower the recovery. Although we are not aware of cross-country evidence on this issue, there does exist interesting empirical work on the effect of deleveraging on employment in U.S. counties (where arguably monetary and fiscal policy have been largely the same), see Mian and Sufi (2011a, 2011b). They find that drop in demand has been more pronounced in high household debt counties. Similarly, in those counties, the unemployment rate has remained higher postcrisis (see Midrigan and Philippon 2011 for an alternative model to rationalize these finding).

X. Conclusions

In this article we have sought to formalize the notion of a deleveraging crisis, in which there is an abrupt downward revision of views about how much debt is safe for individual agents to have, and in which this revision of views forces highly indebted agents to reduce their spending sharply. Such a sudden shift to deleveraging, if it is large enough, can create major problems of macroeconomic management. If a slump is to be avoided, someone must spend more to compensate for the fact that debtors are spending less; yet even a zero nominal interest rate may not be low enough to induce the needed spending.

Formalizing this concept integrates several important strands in economic thought. Fisher's famous idea of debt deflation emerges naturally, while the deleveraging shock can be seen as our version of the increasingly popular notion of a Minsky moment. The process of recovery, which depends on debtors paying down their liabilities, corresponds quite closely to Koo's notion of a protracted balance sheet recession.

is a reasonable approximation to a zero bound multiplier due to the unresponsiveness of monetary policy to the state level shocks they consider.

One thing that is especially clear from the analysis is the likelihood that policy discussion in the aftermath of a deleveraging shock will be even more confused than usual, at least when viewed through the lens of the model. Why? Because the shock pushes us into a topsy-turvy world in which saving is a vice, increased productivity can reduce output, and flexible wages increase unemployment. However, expansionary fiscal policy should be effective, in part because the macroeconomic effects of a deleveraging shock are inherently temporary, so the fiscal response need be only temporary as well. The model suggests not only that a temporary rise in government spending will not crowd out private spending, it will lead to increased spending on the part of liquidity-constrained debtors.

The major limitation of this analysis, as we see it, is its reliance on strategically crude dynamics. To simplify the analysis, all the action takes place within a single, aggregated short run, with debt paid down to sustainable levels and prices returned to full ex ante flexibility by the time the next period begins. In the last section we extended the model to incorporate dynamic deleveraging and find that the model can then naturally nest the canonical new Keynesian model. We see further exploration on this variation as one of the major areas of improvements. Moreover, we hope that our simple extension makes clear that incorporating a deleveraging mechanism into medium-scale estimated DSGE models now common in the literature is relatively simple.

We do believe, however, that even the present version of the model sheds considerable light on the problems presently faced by major advanced economies. It does suggest that the current conventional wisdom about what policy makers should be doing now is almost completely wrong.

Appendix

This appendix summarizes the microfoundations of the simple general equilibrium model studied in the article and shows how we obtain the log-linear approximations stated in the text (a textbook treatment of a similar model with the same pricing frictions is found in Woodford 2003, chapter 3).

A. Households

There is a continuum of households of mass 1. th χ^s of type ^{s} and $1-\chi^s$ of type b. Their problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta(i)^t \left[u^i(C_t(i)) - v^i(h_t(i))\right],$$

where $i = s$ or b, s.t.

$$(1+r_t)\frac{B_t(i)}{P_t} \leq D_t(i) > 0,$$

where i_t is the nominal interest rate that is the return on one-period risk-free nominal bond, whereas r_t is the risk-free real interest rate on a one-period real bond. We derive the first-order conditions of this problem by maximizing the Lagrangian

$$\mathcal{L}_0(i) = E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{l} \beta(i)^t \Bigg\{ u^i(C_t(i)) - v^i(h_t(i)) \\ \quad +\phi_{1t}(i) \Bigg[B_t(i) - (1+i_{t-1})B_{t-1}(i) \\ \quad +W_t P_t h_t(i) + \int_0^1 \Pi_t(i) - P_t C_t(i) - T_t(i) \Bigg] \\ \quad +\phi_{2t}(i)\left[D_t - (1+r_t)\frac{B_t(i)}{P_t}\right]\Bigg\}. \end{array} \right\}$$

First-order conditions:

$$\frac{\partial \mathcal{L}_t(i)}{\partial C_t(i)} = u^i_c(C_t(i)) - \phi_{1t}(i)P_t = 0,$$

$$\frac{\partial \mathcal{L}_t(i)}{\partial h_t(i)} = -v^i_h(h_t(i)) + P_t W_t \phi_{1t}(i) = 0,$$

$$\frac{\partial \mathcal{L}_t(i)}{\partial B_t(i)} = \phi_{1t}(i) - \beta(i)E_t\phi_{1t+1}(i)(1+i_t) - \phi_{2t}(i)\frac{(1+r_t)}{P_t} = 0.$$

Complementary slackness condition:

$$\phi_{2t}(i) \geq 0,\ D_t(i) \geq (1+r_t)\frac{B_t(i)}{P_t},\ \phi_{2t}(i)\left[D_t(i) - (1+r_t)\frac{B_t(i)}{P_t}\right] = 0.$$

The $C_t(i)$ refers to the Dixit-Stiglitz aggregator:

$$C_t(i) = \left[\int_0^1 c_t(i,j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)},$$

and P_t to the corresponding price index

$$P_t = \left[\int_0^1 p_t(j)^{(1-\theta)} di\right]^{1/(1-\theta)}.$$

The household maximization problem implies an aggregate demand function of good j given by

$$c_t(j) = C_t\left(\frac{p_t(j)}{P_t}\right)^{-\theta}.$$

B. Firms

There is a continuum of firms of measure 1 with a fraction λ the sets prices freely at all times and a fraction $(1-\lambda)$ that set their prices one period in advance. $y_t(i) = h_t(i)$. We define the average marginal utility of income as $\tilde{\phi}_t = \chi^s \phi^s_{1t} + (1-\chi^s)\phi^b_{2t}$. Firms maximize profits over the infinite horizon using $\tilde{\phi}_t$ to discount profits (this assumption plays no role in our log-linear economy but is stated for completeness):

$$E_t \sum_{t=0}^{\infty} \tilde{\phi}_t \left[(1-\tau)p_t(j)y_t(j) - W_t P_t h_t(j)\right],$$

s.t.

$$y_t(j) = Y_t\left(\frac{p_t(j)}{P_t}\right)^{-\theta},$$

$$y_t(j) = h_t(j),$$

where τ is a subsidy we have introduced for notational convenience (see in linearization). From this problem, we can see that the λ fraction of firms that set eir price freely at all times they set their price so that

$$(1-\tau)\frac{p_t(1)}{P_t} = \frac{\theta}{\theta-1}W_t$$

and each charging the same price $p_t(1)$. Those that set their price one period in advance, however, satisfy

$$E_{t-1}\tilde{\phi}_t P_t^{1+\theta} Y_t p_t(2)^{-\theta-1}\left(\left(\frac{\theta-1}{\theta}\right)(1-\tau)\frac{p_t(2)}{P_t} - W_t\right) = 0.$$

C. *Government*

Fiscal policy is the purchase of G_t of the Dixit-Stiglitz aggregate and the collects taxes T_t^s and T_t^b. For any variations in T_t^b or G_t, we assume that current or future T_t^s will be adjusted to satisfy the government budget constraint (see Online Appendix for more details). Monetary policy is the choice of i_t. We assume it follows the Taylor rule specified in the text.

D. *Log-Linear Approximation*

Aggregate consumption is

$$C_t = \chi^s C_t^s + (1-\chi^s)C_t^b,$$

where C_t^s and C_t^b are the of the consumption levels of each type. Similarly, aggregate hours are

$$h_t = \chi^s h_t^s + (1-\chi^s)h_t^b,$$

and aggregate output is given as:

$$Y_t = C_t + G_t.$$

We consider a steady state of the model in which b borrows up to its limit, while the s does not. Let us start with linearizing the demand side. Assuming type b is up against his borrowing constraint, and aggregating over all types we obtain

$$C_t^b = -\left(\frac{1+i_{t-1}}{1+r_{t-1}}\right)\frac{P_{t-1}}{P_t}D_{t-1} + \frac{D_t}{1+r_t} + I_t^b - T_t^b,$$

where I_t^b is wage income of the borrower given by

$$I_t^b = W_t h_t^b.$$

Log-linearizing this around $D_t = \bar{D}$ (see Online Appendix for detailed description of the steady sate) we obtain

$$\hat{C}_t^b = \hat{I}_t^b + \beta \hat{D}_t - \hat{D}_{t-1} + \gamma_D \pi_t - \gamma_D \beta (i_t - E_t \pi_{t+1} - \bar{r}) - \hat{T}_t^b,$$

where $\hat{C}_t^b \equiv \frac{C_t^b - \bar{C}^b}{\bar{Y}}$ $Y_t = log Y_t / \bar{Y}$, $\hat{D}_t = \frac{D_t - \bar{D}}{\bar{Y}}$, i_t is now $log(1 + i_t)$ in our previous notation, $\bar{r} \equiv log \beta \Delta^1$, $\pi_t \equiv log P_t / P_{t-1}$, $\hat{T}_t^b \equiv \frac{T_t^b - \bar{T}}{\bar{Y}}$, $\gamma_D = (\bar{D}/\bar{Y})$ and $I_t^b = \frac{I_t^b - I}{\bar{Y}}$ and $\hat{I}_t^b = \hat{W}_t + \hat{h}_t^b$. For type s we obtain

$$u_c^s(C_t^s) = \beta(1 + i_t) E_t u_c^s(C_{t+1}^s) \left(\frac{P_t}{P_{t+1}}\right),$$

and log-linearizing this around steady state yields

$$\hat{C}_t^s = E_t \hat{C}_{t+1}^s - \sigma(i_t - E_t \pi_{t+1} - \bar{r}),$$

where $\hat{C}_t^s \equiv \frac{C_t^s - \bar{C}^s}{\bar{Y}}$ and $\sigma \equiv -\left(\frac{u_c^s}{u_{cc}^s \bar{Y}}\right)$. Aggregate consumption is then

$$\hat{C}_t = \chi^s \hat{C}_t^s + (1 - \chi^s) \hat{C}_t^b,$$

where $C_t \equiv \frac{C_t - \bar{C}}{\bar{Y}}$, and

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t,$$

where $\hat{Y}_t \equiv log Y_t / \bar{Y}$ and $\hat{G}_t \equiv \frac{G_t - \bar{G}}{\bar{Y}}$.

Let us now turn to the production side. The pricing equations of the firms imply

$$\log p_t(1) = \log P_t + \hat{W}_t,$$
$$\log p_t(2) = E_{t-1}[\log P_t + \hat{W}_t],$$

where $\hat{W}_t = log W_t / \bar{W}$. This implies that

$$\log p_t(2) = E_{t-1} \log p_t(1).$$

Log-linearizing the aggregate price index implies

$$\log P_t = \lambda \log p_t(1) + (1 - \lambda) \log p_t(2),$$

so it follows (with a few manipulations using the equations above) that

$$\pi_t - E_{t-1}\pi_t = log P_t - E_{t-1} log P_t = \left(\frac{\lambda}{1-\lambda}\right)[\log p_t(1) - \log P_t]$$
$$= \left(\frac{\lambda}{1-\lambda}\right)\hat{W}_t.$$

To solve for $\hat{W}_t$ we linearize each of the optimal labor supply first-order condition for each type to yield

$$\hat{W}_t = \omega^b \hat{h}_t^b(i) + \sigma^{b-1}\hat{C}_t^b,$$
$$\hat{W}_t = \omega^s \hat{h}_t^s(i) + \sigma^{s-1}\hat{C}_t^s,$$

where $\omega^b \equiv \left(\frac{\bar{v}_{hh}^b h}{\bar{v}_h^b}\right)$, $\omega^s \equiv \left(\frac{\bar{v}_{hh}^s \bar{Y}}{\bar{v}_h^s}\right)$ and $\sigma^b \equiv -\left(\frac{\bar{u}_c^b}{\bar{u}_{cc}^b \bar{Y}}\right)$, $\sigma^s \equiv -\left(\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \bar{Y}}\right)$, and $\hat{h}_t^b(i) \equiv \frac{h_t^b - \bar{h}^b}{\bar{Y}}$ and $\hat{C}_t^b \equiv \frac{C_t^b - \bar{C}}{\bar{Y}}$.

Observe that $\hat{h}_t = \hat{Y}_t$. We now assume that $\omega^b = \omega^s = \omega$ and $\sigma^b = \sigma^s = \sigma$. Using this we can combine the labor supply of the two types to yield:

$$\hat{W}_t = \omega\hat{Y}_t + \sigma^{-1}\hat{C}_t.$$

Combine this with our previous result, together with $\hat{Y}_t = \hat{G}_t + \hat{C}_t$, to yield:

$$\pi_t = \left(\frac{\lambda}{1-\lambda}\right)(\omega + \sigma^{-1})\hat{Y}_t - \left(\frac{\lambda}{1-\lambda}\right)\sigma^{-1}\hat{G}_t + E_{t-1}\pi_t,$$
$$\pi_t = \kappa\hat{Y}_t - \kappa\psi\hat{G}_t + E_{t-1}\pi_t,$$

where $\kappa \equiv \left(\frac{\lambda}{1-\lambda}\right)(\omega + \sigma^{-1})$, $\psi \equiv \left(\frac{\sigma^{-1}}{\sigma^{-1}+\omega}\right)$.

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Supplementary Material

An Online Appendix for this article can be found at QJE online (qje.oxfordjournals.org).

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