Financial Innovation, Collateral and Investment.

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Financial innovation was at the center of the recent financial crisis.
Prices and Investment

Price and Investment in Housing Sector

Source Investment: Construction new privately owned housing units completed. Department of Commerce.
Observe that the Down Payment axis has been reversed, because lower down payment requirements are correlated with higher home prices.

Note: For every AltA or Subprime first loan originated from Q1 2000 to Q1 2008, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13% down payment in Q1 2000 corresponds to leverage of about 7.7, and 2.7% down payment in Q2 2006 corresponds to leverage of about 37.

Note Subprime/AltA Issuance Stopped in Q1 2008.
Leverage, Prices and Investment

The financial crisis was preceded by years in which leverage, prices and investment increased dramatically.

Then all collapsed after the crisis. Leverage Cycle.
Two Financial Innovations: Credit Default Swaps and Leverage

Source: CDS: IBS OTC Derivatives Market Statistics
Credit Default Swaps, Prices and Investments

CDS and Prices

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Source CDS: IBS OTC Derivatives Market Statistics.
Source Investment: Construction new privately owned housing units completed.
Department of Commerce.
Credit Default Swaps (CDS) was a financial innovation that was introduced much later than leverage.

Peak in CDS coincides with lower prices and investment.
We show that financial innovations that change either:

- the set of assets that can be used as collateral
- or the types of promises that can be backed with the same collateralized

affect prices and investment.

We provide precise predictions.
Results

I) The ability to leverage an asset generates higher prices and over-investment compared to the Arrow-Debreu level.

II) The introduction of CDS generates lower prices and under-investment with respect to the Arrow-Debreu level. It can even destroy competitive equilibrium.

III) The ability to leverage an asset never generates marginal under-investment in collateral general equilibrium models.
Literature

- Financial innovations and asset pricing as in Fostel-Geanakoplos (2012b) and Che and Sethi (2011).
- Literature on existence: Polemarchakis and Ku (1990), Duffie and Shaffer (86), Geanakoplos and Zame (1997).
Model Outline

Set-up.

Arrow Debreu Equilibrium.

Financial Innovation and Collateral.
We present a simple GE model with incomplete markets, collateral and production, that we call the C-Model (C*-Model.)

In the paper we present a completely general GE model with collateral.
Time and Assets

Time $t = 0, 1$.

Two states of nature $s = U, D$ at time 1.

Two assets: risky, $Y$, and riskless, $X$. Dividends in consumption good.

$X$ can be thought as a durable consumption good or cash, numeraire. Price of $Y$ at $t = 0$ is $p$. 
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$X$ can be thought as a durable consumption good or cash, numeraire. Price of $Y$ at $t = 0$ is $p$. 
Agents have access to an intra-period production technology at $t = 0$ that allows them to invest the riskless asset $X$ and produce the risky asset $Y$.

$Z_0^h \subset \mathbb{R}^2$ is the set of feasible intra-period production for agent $h \in H$ in state 0 ($Z_0^h$ is convex and compact, $(0,0) \in Z_0^h$ and $Z_0^h = Z_0, \forall h$.)

Inputs appear as negative components, $z_x < 0$ of $z \in Z^h$, and outputs as positive components, $z_y > 0$ of $z \in Z_0^h$.

Investment: $z_x$
Continuum of investors $h \in H = [0, 1]$.

Risk neutral. No discounting. Consumption only at the end.

Expected utility to agent $h$ is

$$U^h(c_U, c_D) = \gamma^h_U c_U + \gamma^h_D c_D$$

Each agent $h \in H$ has an endowment $x_0^*$ of asset $X$ at time 0.

The only source of heterogeneity is in subjective probabilities, $\gamma^h_U$. The higher the $h$, the more optimistic the investor ($\gamma^h_U$ are increasing and continuous).
C and C*-Models

C-Models are very tractable models. In particular, we can represent the equilibrium in an Edgeworth Box even though we have a continuum of agents.

We define a C*-Model as a C-model where:

- the space of agents $H$ can be finite or a continuum.

- the agents’ preferences $U^h = \gamma_U^h u^h(x_U) + \gamma_D^h u^h(x_D)$ can be risk averse.

- initial endowments of $X$ at time 0 $x^h_0$ can be arbitrary.
Model Outline

Set-up.

*Arrow Debreu Equilibrium.*

Financial Innovation and Collateral.
Before focusing on financial innovation, let us consider the Arrow-Debreu economy with production, without any type of collateral considerations.

This will be an important benchmark throughout the paper.
Arrow-Debreu Equilibrium

Since $Z^h_0 = Z_0$, $\forall h$, then $\Pi^h = \Pi$. Because of convexity, wlog we may assume that production plans are the same across agents.

Then $(z_x, z_y)$ is also the aggregate production.

Arrow Debreu equilibrium is easy to solve.
The Arrow Debreu Equilibrium
The Arrow Debreu Equilibrium: Edgeworth Box
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The Arrow Debreu Equilibrium: Edgeworth Box

Graph showing the Edgeworth Box with the axes labeled as \( c_U \) and \( c_D \). The graph includes points labeled as \( (1-h_1)Q \), \( x_0 = (1, 1) \), \( Q \), and \( Y(d^U, d^D) \). The slope of the line is \(-q^{h_1}_D / q^{h_1}_U\). The price line is equal to the indifference curve of \( h_1 \).

The equation \( Y(d^U, d^D) \) is also mentioned along with the expression \( (1-h_1)Q \).
The Arrow Debreu Equilibrium: Edgeworth Box

\[ Y(d_{U}, d_{D}) \]

Slope \(-q^{h_{1}}_{D} / q^{h_{1}}_{U}\)

Price line equal to Indifference curve of \(h_{1}\)

\((1-h_{1})Q\)

\(x_{0^*}(1,1)\)

\(45^\circ\)
The Arrow Debreu Equilibrium: Edgeworth Box

Price line equal to Indifference curve of $h_1$
The Arrow Debreu Equilibrium: Edgeworth Box

Production Possibility Frontier

Slope \(-q^{h_1}_D / q^{h_1}_U\)

Price line equal to Indifference curve of \(h_1\)
The Arrow Debreu Equilibrium: Edgeworth Box
Arrow Debreu Equilibrium Summary

Optimists consume only in the $U$ state.

Pessimists consume only in the $D$ state.

The marginal buyer determines state prices.
Model Outline

Set-up.

Arrow Debreu Equilibrium.

Financial Innovation and Collateral.
Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral.

In Arrow Debreu the question of why agents honor their promises is ignored.

We explicitly incorporate in our model repayment enforceability problems.

Collateral is the only enforcement mechanism: agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance.
A financial contract is an ordered pair

\[(A, C)\]

**Promise:** \(A = (A_U, A_D)\) denotes the promise in units of consumption good in each final state.

**Collateral:** \(C \in \{X, Y\}\) asset used as collateral.
Actual delivery in each state is (no-recourse):

\[(\min(A_U, C_U), \min(A_D, C_D))\]

We are explicitly assuming repayment enforceability problems.
But crucially, we are assuming away cash flow problems:

Every agent knows exactly how the future cash flow depends on the exogenous state of nature. This eliminates any issues associated with managerial hidden effort or unobserved firm quality.

\[ C = (C_U, C_D) \] does not depend on the size of the promise or on who owns the asset at the end.
We shall suppose every contract is collateralized either by one unit of $X$ or by one unit of $Y$.

Let $J = J^X \cup J^Y$ be the total set of contracts.
Financial Contracts and Borrowing

Price of contract \( j \in J \) is \( \pi_j \).

An investor can borrow \( \pi_j \) today by selling the contract \( j \) in exchange for a promise tomorrow.

Let \( \varphi_j > 0 \) \((< 0)\) be the number of contracts \( j \) sold (bought) at time 0.
Budget Set

\[
B^h(p, \pi) = \\
\{(x, y, z_x, z_y, \varphi, c_U, c_D) \in \mathbb{R}_+^2 \times \mathbb{R}_- \times \mathbb{R}_+ \times \mathbb{R}_J \times \mathbb{R}_+^2 : \\
(x - z_x - x_0^*) + p(y - z_y) \leq \sum_{j \in J} \varphi_j \pi_j \\
\sum_{j \in J^x} \max(0, \varphi_j) \leq x, \sum_{j \in J^y} \max(0, \varphi_j) \leq y \\
z = (z_x, z_y) \in \mathbb{Z}_0
\]
Collateral Equilibrium

\[ \left( (p, \pi), (x^h, y^h, z^h, \varphi^h, c^h_U, c^h_D) \right)_{h \in H} \text{ such that} \]

\[ \int_0^1 x^h dh = \int_0^1 (x^h_0 + z^h_x) dh \]

\[ \int_0^1 y^h dh = \int_0^1 z^h_y dh \]

\[ \int_0^1 \varphi^h_j dh = 0, \quad \forall j \in J \]

\[ (x^h, y^h, z^h, \varphi^h, c^h_U, c^h_D) \in B^h(p, \pi), \forall h \]

\[ (x, y, z, \varphi, c_U, c_D) \in B^h(p, \pi) \Rightarrow U^h(c_U, c_D) \leq U^h(c^h_U, c^h_D), \forall h. \]
We regard the use of new kinds of collateral, or new kinds of promises that can be backed by collateral, as financial innovation.

Hence, financial innovation in our model is a different set $J$.

We will show how different financial innovations, such as leverage, and CDS can be cast within our model with collateral.
Outline

1. Introduction
2. Model
3. Leverage
4. CDS
5. Endogenous Leverage
6. Over Investment
7. Conclusion
In this case $J = J^Y$, and each $A_j = (j, j)$ for all $j \in J = J^Y$.

Traded instruments:

- risky asset $Y$ and cash $X$
- non-contingent promises $j$ (debt contracts or loans) using the asset $Y$ as collateral.
What does it mean to leverage Y?
What does it mean to leverage Y?

![Diagram showing leverage and its components](image)
What does it mean to leverage $Y$?
The only contract traded in equilibrium is \( j^* = (d^Y_D, d^Y_D) \).

The equilibrium regime is the following:
L-Economy: Equilibrium

- Optimists leverage $Y$ using max min bond. They buy Arrow $U$.
- Pessimists lenders buy max min bond

Marginal buyer
Numerical Example

We solve for equilibrium the Arrow Debreu and Leverage economies just described for the following:

Production: \( Z_0 = \{ z = (z_x, z_y) \in R_- \times R_+ : z_y = -kz_x \}, \ k \geq 0 \)

Beliefs: \( \gamma^h_U = 1 - (1 - h)^2 \)

Parameter values: \( x_0^* = 1, \ d^U_Y = 1, \ d^Y_D = .2. \)
Numerical Example: Investment

Investment in Y: -z_x

Investment L-economy
Investment AD

k

1 1.1 1.2 1.3 1.4 1.45 1.5 1.55 1.6 1.65 1.7

0 0.2 0.4 0.6 0.8 1 1.2
Numerical Example: Welfare

![Graph showing welfare with different values for h and L. The graph compares L and AD economies with various h values and corresponding welfare outcomes.]
Theoretical Results: Over Valuation and Investment

Proposition: Over-Valuation and Investment compared to Arrow Debreu in C-Models.

In C-Models $p^L \geq p^A$, and $z^L_y \geq z^A_y$. 
Proposition: Over-Valuation and Investment compared to Arrow Debreu in C*-Models.

In C*-Models under constant return to scale, \( p_L \geq p_A \), and \( z^L_y \geq z^A_y \).
Proposition: Welfare in C*-Models

In C*-Models under constant return to scale, Arrow Debreu equilibrium Pareto-dominates Leverage equilibrium.
Theoretical Results: Intuition

When $Y$ can be used as collateral, its cash flows are split into Arrow $U$ and a riskless bond.

$X$ cannot be used as collateral, hence its cash flows cannot be split.

This splitting gives $Y$ additional value (collateral value) beyond its payoff value. This gives agents more incentive to produce $Y$.

Agents are worse off over-investing.
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What is a CDS?
CDS and Collateral

A seller of a CDS must post collateral typically in the form of money that is worth $d_U^Y - d_D^Y$ when $Y$ pays only $d_D^Y$ in the down state.

We can therefore incorporate CDS into our economy by taking $J^X$ to consist of one contract called $c$ promising $A_c = (0, 1)$. 
The CDS-Economy

In this case $J = J^X \cup J^Y$ where:

- $J^X$ consists of the single contract called $c$ promising $A_c = (0, 1)$

- $J^Y$ consists of contracts $A_j = (j, j)$ as described in the leverage economy.

Agents can leverage $Y$ and also can tranche $X$ into Arrow securities.
What does it mean to tranche X?

Selling a CDS on $Y$ collateralized by $X$ is like selling an Arrow $D$ promise:

Sellers of promise $A_c = (0, 1)$ get the residual which is like the Arrow $U$ which pays 1.

We call it Tranche X because $X$ is perfectly split into Arrow securities.
What does it mean to tranche X?

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We call it Tranche X because X is perfectly split into Arrow securities.
The CDS-Economy

Traded instruments:
- risky asset $Y$ and cash $X$.

- non-contingent promises (debt contracts) using the asset $Y$ as collateral.

- contingent promises (CDS) using the asset $X$ as collateral.

The equilibrium regime is as follows:
CDS-Economy: Equilibrium

Optimists: buy all remaining X and Y. Issue bond and CDS (holding the Arrow U)

Moderates: hold the bond

Pessimists: buy the CDS

Marginal buyer
Numerical Example

We solve for equilibrium in the Arrow Debreu, Leverate and CDS economies just described for the following:

Production: \( Z_0 = \{ z = (z_x, z_y) \in R_- \times R_+ : z_y = -kz_x \}, \ k \geq 0 \)

Beliefs: \( \gamma_U^h = 1 - (1 - h)^2 \)

Parameter values: \( x_0^* = 1, \ d_U^y = 1, \ d_D^y = .2. \)
Numerical Example: Investment

Investment in $Y: -z_x$

- Investment L-economy
- Investment AD
- Investment CDS-economy
Numerical Example: Welfare

The diagram illustrates the welfare function for different scenarios. The x-axis represents different values of a variable, labeled as $h$, ranging from 0 to 1. The y-axis represents the welfare level. The graph shows three different economies: L economy, AD economy, and CDS economy. Each curve on the graph corresponds to one of these economies, with distinct markers for $h=0$, $h^{LT}_2=.348$, $h^{AD}=h^L=.3545$, $h^{LT}_1=.388$, and $h=1$. The welfare levels are plotted for these scenarios, showing how welfare changes with $h$ for each economy type.
Under Valuation and Investment

Proposition: Under-Investment compared to First Best in C-Models.

In C-Models $p^A \geq p^{CDS}$, and $z_y^A \geq z_y^{CDS}$ provided that $\gamma^h_U$ is concave in $h$. 
Under Valuation and Investment

Using $X$ as collateral to sell a CDS splits its cash flows into Arrow securities.

Using $Y$ as collateral splits its cash flows into Arrow $U$ and a riskless bond.

The collateral value of $X$ is higher than the collateral value of $Y$.

This gives agents less incentive to use $X$ to produce $Y$ in the CDS economy than in Arrow Debreu.

There is no welfare domination: moderate agents in the CDS economy are better off than in the Arrow Debreu economy.
CDS and Robust Non-Existence

We saw that selling a CDS on $Y$ using $X$ as collateral is like selling an Arrow $D$ using $X$ as collateral.

The only difference between a CDS and an Arrow $D$ is that when $Y$ is not produced the CDS is no longer well-defined.

It is precisely this difference that can bring about interesting existence problems: introducing CDS can robustly destroy collateral equilibrium in economies with production.
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All contracts $j \in J = J^Y$ with $A_j = (j, j)$ have a price in equilibrium, $\pi_j$. Hence:

All contracts define a gross interest rate $1 + r_j = j / \pi_j$.

All contracts have a well defined $LTV^j = \frac{\pi_j}{p}$.
G (97) introduced the concept of Credit Surface: the equilibrium relationship between \( LTV_j \) and \( 1 + r_j \).

Borrowers can choose any contract on the Credit Surface provided they put up the corresponding required collateral.

In the Arrow-Debreu budget set, borrowers face in equilibrium a flat Credit surface.
G (97) introduced the concept of **Credit Surface**: the equilibrium relationship between $LTV_j$ and $1 + r_j$.

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Endogenous Leverage

But because collateral is scarce, only few contracts will be actively traded in equilibrium. So agents will choose only a few points on the credit surface.

In this sense leverage is endogenous.
But which contracts are traded in equilibrium?
**Proposition:**

The only contract traded in equilibrium is $j^* = d_Y^Y$ and the risk-less interest rate is equal to zero, so $\pi j^* = j^* = d_Y^Y$.

**Proof:** Fostel-Geanakoplos (2011).
Fostel-Geanakoplos (2014) provide a complete characterization showing that in all binomial economies with financial assets we can always assume that the max min contract is the only contract traded.

So actual default is never observed, but potential default sets a hard limit on borrowing.

Experimental evidence (Cipriani, Fostel, Houser work in progress)
The only contract traded is \( j^* \) corresponding to point A in the Credit Surface.

Notice that borrowers can leverage as much as they want.

They are still constrained: if they want to borrow more than \( \pi_j^* \) on the same collateral they will face a higher interest rate.
Endogenous Leverage and Credit Surface

The only contract traded is $j^*$ corresponding to point $A$ in the Credit Surface.

Notice that borrowers can leverage as much as they want.

They are still constrained: if they want to borrow more than $\pi j^*$ on the same collateral they will face a higher interest rate.
The only contract traded is \( j^* \) corresponding to point \( A \) in the Credit Surface.

Notice that borrowers can leverage as much as they want. They are still constrained: if they want to borrow more than \( \pi j^* \) on the same collateral they will face a higher interest rate.
Leverage and Down Risk

Leverage in equilibrium is given by:

\[ LTV = \frac{d_D^Y}{p} \cdot \frac{1}{1 + r} = \frac{\text{worst case rate of return}}{\text{riskless gross rate of interest}}. \]

In some special cases this formula can be expressed in terms of volatility of the asset payoffs: the higher volatility, the lower leverage.

However, this link is not general. What matters in general is down risk.
Leverage and Down Risk

Though simple and easy to calculate, our formula provides interesting insights:

-it explains which assets are easier to leverage (the ones with low down risk).

-it explains why changes in the bad tail can have such a big effect on equilibrium even if they hardly change expected payoffs: they change leverage.
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Outline

Geometrical Proof of the Over-Investment Result.

Discussion: over investment without cash flow problems.

Marginal Over-Investment and Collateral Value.
L-Economy: Equilibrium

Optimists leverage $Y$ using max min bond. They buy Arrow $U$.

Pessimists lenders buy max min bond.

$h=1$

$h=0$

$h_1$

Marginal buyer
L-Economy: Edgeworth Box
L-Economy: Edgeworth Box

- Price line equal to indifference curve of $h_1$
- Intra-Period Production Possibility Frontier
- Y($d_Y^U, d_Y^D$)

45° lines represent equal production possibility frontiers.
L-Economy: Edgeworth Box

Production Possibility Frontier

Price line equal to indifference curve of $h_1$

Slope $-\frac{q^{h_1_D}}{q^{h_1_U}}$

$Y(d^Y_U, d^Y_D)$

$z_Yd^Y_D$

$z_Y(d^Y_U-d^Y_D)$

$x_0^*+z_X$

$(1-h_1)Q$

$x_0^*(1,1)$

$C_U$

$O$

$C_D$
Over Valuation and Investment Geometrical Proof

Price line equal to Indifference curve of $h_1$

Slope $-q^{h_1}_D/q^{h_1}_U$

Proof
Over Valuation and Investment Geometrical Proof

Price line equal to Indifference curve of \( h_1 \)

Slope \(-q^{h_1_U}/q^{h_1_D}\)

Proof
Over Valuation and Investment Geometrical Proof

Price line equal to Indifference curve of $h_1$

Slope $-q^{h_1_D}/q^{h_1_U}$

LEVERAGE ECONOMY

Proof
Over Valuation and Investment Geometrical Proof

Y(d'\_U,d'\_D)

Price line equal to Indifference curve of h\_1
Outline

Geometrical Proof of the Over-Investment Result.

Discussion: over investment without cash flow problems.

Marginal Over-Investment and Collateral Value.
Discussion: over investment without cash flow problems

Over-valuation and over-investment due to leverage may seem surprising.

Many macro models (like Kiyotaki-Moore (97), Bernanke-Gertler (89), Mendoza (10)) with financial frictions get the opposite result: lower price and investment with respect the first best allocation.

Intuitive: one would expect that the need for collateral would restrict borrowing and hence investment.

Why do we get different results?
Discussion: over investment without cash flow problems

The reason for the discrepancy is that in the macro-corporate finance literature it is assumed that there are cash flow problems:

\[-C = (C_U, C_D)\] depends on the size of the promise or on who owns the asset at the end. Hence agents cannot pledge the whole future value of the assets they produce.

This naturally imposes a limit on borrowing and hence depresses investment.

We can clearly see this looking at the credit surface implied by models with cash flow problems.
Discussion: over investment without cash flow problems
Discussion: over investment without cash flow problems

$p$ is fixed at the value of the firm without external financing
Discussion: over investment without cash flow problems

\[ 1 + r \]

\[ j \]

\[ A \]

\[ B \]

\[ p \]

\[ \text{Borrowing } \pi_j \]

\( p \) is fixed at the value of the firm without external financing
Discussion: over investment without cash flow problems

In a family of models (C and C*) we show that when we disentangle cash flow problems from repayment enforcement problems we always get over valuation and over investment compared to the Arrow Debreu level.
Outline

- Geometrical Proof of the Over-Investment Result.

- Discussion: over investment without cash flow problems.

- Marginal Over-Investment and Collateral Value.
Marginal Over Investment and Collateral Value

Investment and prices can be above or below Arrow Debreu levels in GE collateral models.

We show that in GE collateral models there is never marginal under investment in equilibrium due to the presence of collateral value.
Kiyotaki-Moore (97) claimed that the asset price is below its payoff value.

\[ \text{Payoff Value} = \text{PV} = \frac{\text{Expected } \text{MU} \text{ of Asset Payoffs}}{\text{MU of Consumption Today}} \]
Kiyotaki-Moore (97) claimed that the asset price is below its payoff value.

\[
\text{Payoff Value} = \text{PV} = \frac{\text{Expected MU of Asset Payoffs}}{\text{MU of Consumption Today}}
\]
Kiyotaki-Moore (97) claimed that the asset price is below its payoff value.
Collateral Equilibrium
K-M (97).

PV = \frac{\text{Expected MU of Asset Payoffs}}{\text{MU of Money}}
PV \neq \frac{\text{Expected MU of Asset Payoffs}}{\text{MU of Consumption Today}}

KM mistake: consumption is zero, so incorrect PV.
Marginal Over Investment and Collateral Value

Collateral Equilibrium
K-M (97).

Payoff Value
Price
Correct Payoff Value

PV = \frac{\text{Expected MU of Asset Payoffs}}{\text{MU of Money}}

PV \neq \frac{\text{Expected MU of Asset Payoffs}}{\text{MU of Consumption Today}}

KM mistake: consumption is zero, so incorrect PV.
Marginal Over Investment and Collateral Value

Theorem: Price is always above Payoff Value (Fostel-Geanakoplos,08)

Asset pricing: $p = PV + CV$, with $CV \geq 0$.

Experimental evidence for positive $CV$ in Cipriani, Fostel and Houser (12).
Marginal Over Investment

Proposition: No Under-Investment compared to First Best.

*There is never marginal-under investment on assets that serve as collateral in collateral general equilibrium models due to non-negative collateral values.*
Concept of marginal over-investment is a “local” measure of inefficiency.

Given all spot prices, no agent would prefer to invest an extra unit of money in raising production over the equilibrium level, even if he had access to the best technology available in the economy.
Need to post collateral may constrain borrowers in equilibrium.

But when one considers in the same model many durable goods than can be produced with different collateral values, investment migrates to “good” collateral.

Hence, we expose a countervailing force in the incentives to produce:
-when only some assets can be used as collateral, they become relatively more valuable, and are therefore produced more.
Conclusion

We show that financial innovation affect prices and investment.

Leverage can generate higher prices and over-investment compared to the Arrow-Debreu first best level. In C*-models it always does.

Leverage never generates marginal under-investment in assets that can be used as collateral due to the presence of collateral value.

CDS can generate lower prices and under-investment with respect to the Arrow-Debreu first best level. In C-Models always does. And their introduction can even destroy equilibrium.
Conclusion

We discussed our strategy to endogenize leverage in GE models.

We discussed the relationship between our results and the macro/corporate finance literature: the role of cash flow problems.

This model can provide a framework to think about problems in international finance:
- CDS and Sovereign Debt.
CDS and Robust Non-Existence

The only difference between CDS and Arrow $D$ is that when $Y$ ceases to be produced the CDS is no longer well-defined.

We show how introducing CDS can robustly destroy collateral equilibrium in economies with production.
CDS and Robust Non-Existence

Suppose we introduce into the $L$-economy a CDS. We call this the $LC$-economy.

Equilibrium in the $LC$-economy equals:
- equilibrium in the $LT$-economy if $Y$ is produced.
- equilibrium in the $L$-economy if $Y$ is not produced.

Thus, if all $LT$-equilibria involve no production of $Y$ and all $L$-equilibria involve production of $Y$, then there cannot exist a $LC$-equilibrium.
Constant return to scale production:
\[ Z_0 = \{ z = (z_x, z_y) \in R_- \times R_+ : z_y = -kz_x \}, \ k \geq 0. \]

Consider any \( k \in (1, 1.4) \). Rest of parameters and beliefs as before.

Then \( LC \)-equilibrium does not exist.
CDS and Robust Non-Existence

High CDS volume with low underlying Y volume

L=LT=AD
No production

Non-existence region for CDS

LC=LT with production

CDS and Robust Non-Existence

The equilibrium in the LC economy does not exist for a robust set of parameters.
In the $L$-economy, optimists collectively consume $z_y^L(d_U^Y - d_D^Y)$ in state $U$ while in the Arrow Debreu economy they consume $z_y^A d_U^Y + (x_0^* + z_x^A)$. The latter is evidently much bigger, at least as long as $z_y^A \geq z_y^L$.

So suppose, contrary to what we want to prove, that Arrow-Debreu output were at least as high, $z_y^A \geq z_y^L$ and $p^A \geq p^L$. 
Over Valuation and Investment Geometrical Proof
Over Valuation and Investment Geometrical Proof

\[ Y(d_{U}^{Y},d_{D}^{Y}) \]

Slope \(-q^{hL_{1D}} / q^{hL_{1U}}\)

\[ z_{L}(d_{U}^{Y}-d_{D}^{Y}) \]

\[ A(1-h_{1}^{L})Q^{L} \]

45°
Over Valuation and Investment Geometrical Proof

\[ Y(d^U, d^D) \]

Slope \(-q^{hl_1D}/q^{hl_1U}\)

\[ z^L(y(d^U_d^D)) \]

\[ (1-h^L_1)Q^L \]

\[ x_0^*(1,1) \]

\[ (1-h^L_1)Q^A \]

\[ 45^\circ \]
Over Valuation and Investment Geometrical Proof

\[ Y(d^Y_U, d^Y_D) \]

\[ \text{Slope} = -q^{hL_1}_D / q^{hL_1}_U \]

\[ c^U \]

\[ c^D \]

\[ 45^\circ \]

\[ x_0^*(1,1) \]

\[ (1-h_1^L)Q^L \]

\[ (1-h_1^A)Q^A \]

\[ (1-h_1^L)Q^A \]
Over Valuation and Investment Geometrical Proof
Over Valuation and Investment Geometrical Proof
Over Valuation and Investment Geometrical Proof
Marginal Over Investment and Collateral Value

Will illustrate the concept with our previous numerical example that also has zero consumption at time 0.

Consider our numerical example with production

\[ Z_0 = \{ z = (z_x, z_y) \in R_- \times R_+ : z_y = -kz_x \} , \quad k = 1.5 , \]

beliefs:

\[ \gamma^h_U = 1 - (1 - h)^2 \text{ and } x_{0^*} = 1, \quad d^Y_U = 1, \quad d^Y_D = .2. \]

In the \( L \)-economy equilibrium is given by

\[ h_1 = .35, \quad p = .67, \quad z_x = - .92 \text{ and } z_y = 1.38. \]
To fix ideas let’s consider one of the optimists $h = .9$.

**Marginal Utility of Money** for $h = .9$ in equilibrium at time 0:

$$MU_m = \frac{.9(1 - .2)}{.67 - .2} = 1.53.$$  

**Marginal Expected Utility of a dollar invested on $Y$** for $h = .9$ in equilibrium:

$$.9(1)1.5 + .1(.2)1.5 = 1.38.$$  

There is marginal over-investment in equilibrium.

$KM(97)$ despite cash flow problems also had marginal over-investment in equilibrium.
Marginal Over Investment and Collateral Value

There is marginal over-investment on Y since

Marginal Utility of Money for \( h = .9 \) in equilibrium at time 0:

\[
MU_m = \frac{.9(1 - .2)}{.67 - .2} = 1.53.
\]

Payoff value of Y for \( h = .9 \) in equilibrium:

\[
PV = \frac{.9(1) + .1(.2)}{MU_m} = .6
\]

Hence the Collateral Value of Y for \( h = .9 \) in equilibrium:

\[
CV = p - PV = .07.
\]
Fostel-Geanakoplos (08) show in a dynamic setting that movements in collateral values can explain many facts observed in Emerging Markets such as:

- contagion (fixed income in the US and emerging markets are correlated even though their fundamentals are independent).

- flight to collateral (during bad times, emerging market bond prices fall but those with less collateral capacity fall by more).

- capital flows volatility (during bad times, high quality emerging market debt issuance drops drastically).
“Savings glut” story of global imbalances Obstfeld-Rogoff (09), Caballero, Farhi and Gourinchas (08).

Mechanism: through interest rates.

We propose an alternative model (Fostel-Geanakoplos-Phellan, 14)

Mechanism: through leverage.
Based on Willen (96, Yale PhD dissertation).

Suppose two countries: $H$ and $F$.

Both countries have identical preferences and endowments.

There is only idiosyncratic risk. Aggregate endowment in each country is constant.

Utilities are such that $u'''' > 0$ ($u'$ is convex, so volatile future consumption implies higher expected marginal utility).
The only difference between the two countries is in the asset structure.

$H$ complete markets, $F$ only riskless asset.

Hence, $H$ individual consumption constant, $F$ individual consumption volatile.

Hence, expected marginal utility in $F$ higher than in the $H$. High precautionary saving in $F$ and low precautionary saving in $H$ implies that interest rates in $H$ are higher.

This explains direction of capital flow.
International Finance II: The "Savings glut" Story and Financial Crisis

Capital inflows push the $H$ interest to go down.

This in turn generates bubbles in other sectors, like housing, that leads to the financial crisis.
Consider our model with 2 countries: $H$ and $F$.

Both countries are identical in preferences and endowments and assets.

The only difference is in $J$.

$J^H = J^Y$, and each $A_j = (j, j)$ for all $j \in J = J^Y$.

$J^F = \emptyset$
International Finance II: Autarky Equilibrium

Optimists leverage $Y$ using riskless bond. They buy Arrow $U$.

Pessimists lenders buy riskless bond

Optimists buy $Y$

Marginal buyer

Pessimists hold $X$
Suppose now that the $H$ and $F$ can freely trade $X$ and $Y$ and non-contingent financial contracts.

$Y^H$ can serve as collateral but $Y^F$ cannot.
International Finance II: Financial Liberalization

Equilibrium

Optimists leverage $Y^H$ using riskless bond. They buy Arrow U.

Moderates buy $Y^F$

Pessimists lenders

Optimists leverage $Y^H$ using riskless bond. They buy Arrow U.

Moderates buy $Y^F$

Pessimists lenders
The $H$ ability to promise backed by collateral causes the global imbalances: $F$ lends to the $H$.

Trade is not due to interest rate differentials or risk diversification. As we saw the interest rate does not move, and assets are perfectly correlated.

What generates the global imbalance is leverage.

This model would suggest that the domestic bubble generated by leverage creates global imbalances and not the other way around as the Savings Glut story.
CDS on Sovereign Debt have been at the center stage since the European crisis.

CDS-economy is a model of Sovereign CDS: Sovereign bonds are used as collateral (bought on margin). CDS on these bonds must post cash as collateral.

Our Proposition states that Sovereign CDS will increase sovereign borrowing costs and hence affect issuance.