

Durable Goods Production and Inventory Dynamics: An Application to the Automobile Industry

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May 2012

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Abstract

This paper develops a model of the joint determination of production, inventories and pricing of a monopolistically competitive durable good producer. The model gives rise to time-varying markups that interact with the inventory-sales ratio, even with flexible prices. Maximum likelihood estimation with automobile industry data yields plausible parameter estimates and impulse responses. We then apply the model to analyze the impact of the "Cash-for-Clunkers" program, and find that the model predicts a negligible production response; essentially all the action is inventories. This leads us to consider evidence of threshold effects that imply a stronger response very far from the steady state. This results in a modest but more plausible production response to the policy—still modest in comparison to the sales impact, but now at least measurable. Even with some production response, the results still provide a cautionary tale for countercyclical policies that rely on stimulating consumer spending. Even an impact on spending need not translate into a comparable impact on employment and output.

This paper develops a model of durable goods inventories. Evoking the classic Carlton (1973) and Maccini (1973) analyses in which firms use both price and delivery lags to clear the market, each monopolistically competitive producer chooses inventory and price given demand that reflects the benefits of inventories. The result is an endogenous and time-varying markup that interacts with the production and inventory decisions. We estimate key parameters of the model on automobile industry data using maximum likelihood, and then simulate the model’s responses, first to standard cost and demand shocks, and then to a “cash-for-clunkers” type of intervention—a temporary subsidy for purchases. For the latter, consistent with the findings in Copeland and Kahn (2011), we find that there is a very modest impact, as most of the action is in inventories. On the sales side, virtually all of the increase in sales is “borrowed” from the subsequent two months, so the cumulative impact is zero a very short time after the program ends. This suggests that as a stimulus to employment and GDP, such programs are of very limited benefit, if any.

1 Durable Goods Production, Sales, and Inventory

This section introduces the model that we will use to describe the equilibrium response of a durable goods-producing industry to various shocks. It is a partial equilibrium model, since the focus is one industry that produces a variety of goods. We derive consumer demand based on utility maximization, but add stochastic aggregate and idiosyncratic shocks. To make the model tractable, we assume that at each date t , consumers rent a continuum of durables $x_t(i)$, $i \in [0, 1]$ for a price $p_t(i)$ from monopolistically competitive retailers. Each retailer purchases a quantity of a generic durable good from perfectly competitive manufacturers, and costlessly transforms it into a specific good which it adds to its inventory. Rented goods depreciate at rate δ , while goods in inventory do not depreciate.

While this framework is necessary because of the technical difficulties associated with monopolistic pricing of durable goods, it does raise the question of what exactly are “in-

ventories” and “sales” in this context. Let x_t denote the stock of durables in consumers’ possession at date t (in the model, this is the total amount of rentals). Let I_t denote the stock of retail inventories left at the end of period t , and let q_t denote orders of goods by the retailers that arrive during period t .

Given the definition of x_t , a natural definition of “sales” s_t would be

$$s_t \equiv x_t - x_{t-1}(1 - \delta), \quad (1)$$

that is, the gross increase in the stock from $t - 1$ to t . Inventories I_t , defined as unsold (in this case, unrented) goods as of the end of period t , would then obey the usual identity

$$I_t = I_{t-1} + q_t - s_t. \quad (2)$$

As in Kahn (1987) and Bils and Kahn (2000), we will focus on the “stock available” to consumers as of period t , A_t , from which x_t or s_t is chosen. We assume that this includes q_t . With durable goods, there is some ambiguity in this concept, as it may or may not include the stock of existing (or “used”) goods. For example, is the relevant inventory of houses the entire housing stock, including newly built but unsold houses as well as those currently occupied? Just the former? Or the unsold newly built plus those existing homes designated as “for sale” (a rather nebulous concept). Primarily for convenience we will adopt the more expansive (or “gross” stock) view, and define A_t as $I_{t-1} + q_t + (1 - \delta)x_{t-1}$. Thus we can think of the total stock of durables (new and used) in the hands of consumers, x_t , as being chosen from A_t , as opposed to the “net” concept of s_t being chosen from $I_{t-1} + q_t$.¹ The stock available A_t then evolves according to

$$A_t = A_{t-1} + q_t - \delta x_{t-1}. \quad (3)$$

¹Depending on the industry, one or the other may be more realistic. The “gross” definition is most appropriate for goods with high turnover: Rental cars, hotel rooms, perhaps apartments. We adopt it mainly for tractability.

Figure 1 illustrates the timeline behind these definitions.

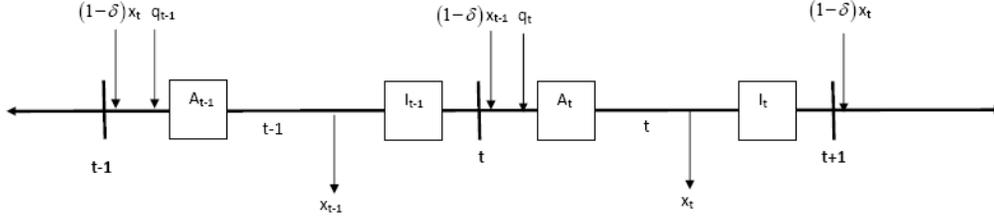


Figure 1: Timeline

1.1 Consumers

Following [Bils and Kahn \(2000\)](#), we will assume that the stock A_t interacts smoothly with the choice of x_t —that A_t provides a service that affects the marginal utility consumers get from x_t . This could be, for example, from greater selection, so that the consumer can get closer to his preferred variety, or better information from being able to see the product. At the same time, the specification permits the possibility, as a limiting case, of pure stockout-avoidance, where the consumer obtains the minimum of x_t and A_t . In particular, we have in mind that each good may come in a variety of sizes or colors, and that the larger the inventory the better “fit” with the consumers’ needs.

We represent this relationship between the effective service level h , the quantity of the good x , and the available stock A by a CES function

$$h(x_t(i), A_t(i)) = [(1 - \phi)x_t(i)^{1-\gamma} + \phi A_t(i)^{1-\gamma}]^{1/(1-\gamma)}. \quad (4)$$

Since (4) is a key assumption, it is worth justifying. One way to see it is as a generalization of the stockout-avoidance model where actual sales equal the minimum of x and A . This specification has a similar characteristic as γ goes to infinity. With $\phi = 0.5$ we would essentially have $h = \min\{x, A\}$ in the limiting case. At the other extreme, it is possible

that A and x could be substitutes; a high degree of availability could reduce the need to buy or rent x , much as automatic teller machines reduce the demand for money. In fact, as we shall see, the least interesting case is $\gamma = 1$.

We let H denote a Dixit-Stiglitz composite of the underlying goods. There are also other goods summarized by a composite z_t with a constant price that we will normalize to one. The consumer solves the following maximization problem given $\{p_t(i), A_t(i)\}$:

$$\max E_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} [(1 - 1/\sigma)^{-1} H_{\tau}^{1-1/\sigma} + u(z_{\tau})] \right\} \quad (5)$$

where

$$H_t \equiv \left[\int h(x_t(i), A_t(i))^{1-1/\rho} di \right]^{\rho/(\rho-1)} \quad (6)$$

subject to

$$W_t = W_{t-1}(1 + r_t) + y_t - \int p_t(i) x_t(i) di - z_t \quad (7)$$

where W_t is wealth at the end of period t , r_t is the rate of return, and y_t is income.

The first-order condition for $x_t(i)$ is

$$H_t^{-\sigma} \left(\frac{H_t}{h(x_t(i), A_t(i))} \right)^{1/\rho} \left(\frac{h(x_t(i), A_t(i))}{x_t(i)} \right)^{\gamma} (1 - \phi) = \lambda_t p_t(i) \quad (8)$$

where λ_t is the shadow price of the budget constraint at t , using the fact that

$$\frac{\partial h}{\partial x} = (h/x)^{\gamma} (1 - \phi).$$

This yields, for the marginal rate of substitution between goods i and j (suppressing the t subscript and using the shorthand $h(i)$ for $h(x(i), A(i))$):

$$\left(\frac{h(i)}{h(j)} \right)^{-1/\rho} = \frac{p(i)}{p(j)} \left(\frac{h(i)/x(i)}{h(j)/x(j)} \right)^{-\gamma}.$$

This modifies the relative price of i and j by the inverse of the marginal impact on the actual

service flow h . If $\phi = 0$ then $h/x = 1$ and the term multiplying the relative price goes away, as it does if $\gamma = 0$.

Following Dixit-Stiglitz, we can define

$$\begin{aligned} v(i) &= p(i) (h(i)/x(i))^{-\gamma} \\ V &= \left[\int v(i)^{1-\rho} di \right]^{1/(1-\rho)}. \end{aligned}$$

Then we have

$$h(i) = H \left(\frac{v(i)}{V} \right)^{-\rho}. \quad (9)$$

Here we will make the standard Dixit-Stiglitz assumption that agents ignore the impact of changes in $x(i)$, $p(i)$, or $A(i)$ on V . Of course, this is not an explicit expression for $x(i)$ as a function of $p(i)$ and $A(i)$, but it does define an implicit relation that will serve as a “demand curve” for retailers.

In a symmetric equilibrium, $h(i) = H$ and $v(i) = V$. We therefore have

$$\begin{aligned} \xi &\equiv \frac{\partial x/x}{\partial A/A} = \frac{(\gamma\rho - 1) (v/V)^{-\rho} (h/A)^{\gamma-1} \phi}{\gamma\rho (h/H) - (\gamma\rho - 1) (v/V)^{-\rho} (h/x)^{\gamma-1} (1 - \phi)} \\ &= \frac{(\gamma\rho - 1) \phi (h/A)^{\gamma-1}}{\gamma\rho - (h/x)^{\gamma-1} (1 - \phi) (\gamma\rho - 1)} \end{aligned} \quad (10)$$

So sales will depend positively on inventories if $\gamma\rho > 1$. If $\gamma > \rho$ the primary reason is that higher $A(i)$ makes $x(i)$ more attractive, as the consumer does not like the ratio to vary. If $\rho > \gamma$, the primary reason is that higher $A(i)$ induces substitution from other goods toward good i .

For the price elasticity we have

$$\begin{aligned} \epsilon &\equiv -\frac{\partial x/x}{\partial p/p} = \frac{\rho (h/H)}{\gamma\rho (h/H) - (\gamma\rho - 1) \left(\frac{v}{V}\right)^{-\rho} (h/x)^{\gamma-1} (1 - \phi)} \\ &= \frac{\rho}{\gamma\rho - (h/x)^{\gamma-1} (1 - \phi) (\gamma\rho - 1)} \end{aligned} \quad (11)$$

Note that h/x is positively related to A/x , the inventory-sales ratio. Consequently, again assuming $\gamma\rho > 1$, ϵ is increasing in A/x if $\gamma > 1$, and decreasing in A/x if $\gamma < 1$. If A/x is countercyclical (as it is in the data), then the markup is procyclical if $\gamma > 1$, countercyclical if $\gamma < 1$, and constant if $\gamma = 1$. We will focus on the case $\gamma < 1$ and $\rho > 1/\gamma$.

Income or wealth uncertainty makes λ_t , and hence H_t , stochastic. In a symmetric equilibrium (where $h_t = H_t$), (8) becomes

$$h_t^{-\sigma} \left(\frac{h_t}{x_t} \right)^\gamma (1 - \phi) = \lambda_t p_t. \quad (12)$$

This suggests that an increase in p_t holding λ (inversely related to wealth) fixed would cause both H_t and x_t to fall, but x_t would fall by more. On the other hand, a pure wealth effect (an increase in λ_t) would cause H_t and x_t to fall in proportion. While a full solution for λ would require a general equilibrium model, we can capture the essential implications of (??) as follows. Suppose

$$\begin{aligned} h_t &= \lambda_t^{-1/\sigma} \\ x_t &= \lambda_t^{-1/\sigma} \left(\frac{1 - \phi}{p_t} \right)^{1/\gamma} \end{aligned}$$

What the partial equilibrium framework cannot determine is the relationship between p_t and λ_t , i.e. the wealth effect of changes in p_t . We parameterize this as

$$\lambda_t = \kappa_t p_t^\psi$$

where κ_t is an exogenous stochastic process with a mean of one, and p_t is the endogenous equilibrium common rental price of each durable good i .

So to summarize, ϵ is the own price elasticity of $x(i)$ with respect to $p(i)$, holding fixed $A(i)$ and all other prices and inventory stocks. $1/\gamma$ is the “ λ constant” elasticity of aggregate x with respect to the aggregate price p when the consumer is compensated so that λ_t (and

hence H_t) remains the same. Finally, $-(1 + \psi) / \gamma$ is the uncompensated elasticity of x with respect to p .

It is worth emphasizing a key feature of the model: The interaction of inventories with demand results in a variable elasticity in a natural way, and hence variable markups. Earlier work (Bils and Kahn, 2001) emphasized the role of time-varying markups in accounting for cyclical dynamics, but did not have a model of markups, and the literature has few other examples. Also, the model is perfectly adaptable to the case of nondurable goods, simply by setting $\delta = 1$, and retains the same features, just with different dynamic responses to shocks.

1.2 Retailers

Retailers purchase generic durable goods and hold them in stock for consumers to rent. We can assume a continuum of retailers indexed by i . Each retailer costlessly transforms the generic durable good into a unique good indexed by i . Given a price process for the generic durable $\{\pi_t\}$ and a stock of goods carried over from $t - 1$, $A_{t-1}(i) - \delta x_{t-1}(i)$, retailer i chooses to purchase $q_t(i)$ goods from producers and to charge price $p_t(i)$ to solve the problem

$$\max E_t \left\{ \sum_{\tau=t}^{\infty} \beta_{t,\tau} [p_{\tau}(i) x_{\tau}(i) - \pi_{\tau} q_{\tau}(i)] \right\} \quad (13)$$

subject to

$$A_t(i) = A_{t-1}(i) + q_t(i) - \delta x_{t-1}(i) \quad (14)$$

and to the implicit demand for $x_t(i)$ derived in the previous section, where $\beta_{t,\tau}$ is the τ -period ahead discount factor at date t . Thus the choice of q is equivalent to the choice of A . In what follows we will suppress the subscripts on β , as we will be assuming a constant real interest rate.

As is standard in models with inventories and demand uncertainty, we will assume that q_t and p_t are chosen knowing π_t , but without knowing the realization of $x_t(i)$, which will be

affected by demand shocks. The first-order condition for that choice is

$$\begin{aligned} E_t \left\{ p_t(i) \frac{\partial x}{\partial A} - \pi_t + \beta \pi_{t+1} \left(1 - \delta \frac{\partial x}{\partial A} \right) \right\} &= \\ E_t \left\{ p_t(i) \xi_t \frac{x_t}{A_t} - \pi_t + \beta \pi_{t+1} \left(1 - \delta \xi_t \frac{x_t}{A_t} \right) \right\} &= 0. \end{aligned} \quad (15)$$

This is similar in form to the condition in Bils and Kahn (2000), but the dependency of x on A here is derived from utility maximization rather than posited directly. (Also, Bils and Kahn assumed $\delta = 1$.) As for price, we have a condition similar to the standard monopolistic pricing formula, modified by consideration of uncertainty, durability, and inventories:

$$E_t \{ x_t [p_t (\epsilon_t - 1) - \epsilon_t \beta \delta \pi_{t+1}] \} = 0 \quad (16)$$

where $\epsilon_t \equiv - (p_t/x_t) \partial x_t / \partial p_t$ is assumed to exceed one. As we have seen, in general it will vary over time with the level of A/x . Price is set relative to discounted one-period-ahead cost because that is the replacement cost for a good sold out of inventory. And because the good is durable and only rented, the cost is deflated by $\delta < 1$. Note that if x_t were known when p_t is chosen, we would have the familiar result that the markup (defined as $p_t / (\beta_{t,t+1} \delta \pi_{t+1})$) is $\epsilon_t / (\epsilon_t - 1)$.

In a steady state with β , π , and ϵ constant, we have

$$\xi = (\epsilon - 1) \frac{1 - \beta}{\beta \delta} \frac{A}{x} \quad (17)$$

which becomes

$$(\gamma \rho - 1) \phi (h/A)^{\gamma-1} = (\rho(1 - \gamma) + (h/x)^{\gamma-1} (1 - \phi) (\gamma \rho - 1)) \frac{1 - \beta}{\beta \delta}$$

If $\gamma = 1$ this simplifies to

$$A/x = \left(\frac{\phi}{1 - \phi} \right) \frac{\delta}{r}$$

where $\beta = 1/(1+r)$. More generally, there is no closed form solution for steady state A/x . Simulations indicate that steady state A/x is increasing in both γ and ρ (at least for $\gamma < 1$, $\rho\gamma > 1$), i.e. as $A(i)$ and $x(i)$ become less substitutable, and as the different service flows $H(i)$ become more substitutable. Intuitively, retailer i will hold more inventory of good i the more it enhances the attractiveness of $x(i)$ to the consumer, and the more willing the consumer is to substitute other varieties for i . The price elasticity of x increases with ρ and decreases slightly with γ . The steady state price elasticity is increasing in ρ , not surprisingly, but slightly decreasing in γ . Presumably the latter occurs because A/x is increasing and $\gamma < 1$.

Given (10) and (15) it is clear that demand shocks taking the form of multiplicative shifts in the expected value of H_t will, holding prices fixed, increase each $x(i)$, $A(i)$, and hence $h(i)$ by the same factor as the change in H . There is no direct impact on the elasticity of demand ϵ_t or on $\partial x/\partial A$. Any changes in these variables result from changes in the other elements of conditions (12) and (13)—for example, changes in production costs or interest rates.

Since this is a partial equilibrium model, we treat π_t and $\beta_{t,t+1}$ as exogenous. For simplicity we will assume $\beta_{t,t+1}$ is constant, while π_t is positively related to the level of aggregate orders q_t relative to some “trend” level of output \bar{q}_t :

$$\pi_t = (q_t/\bar{q}_t)^\theta w_t \tag{18}$$

where w_t is a disturbance term. In a stationary model \bar{q}_t would just be the steady state level of q , but we will have a unit root process. Hence we will define \bar{q}_t as the level that q would have obtained in the absence of shocks:

$$\bar{q}_t = \lim_{j \rightarrow \infty} E_t \{q_{t+j} e^{-gj}\}$$

Note that retailers are price takers, but use (18) in forming expectations of π_{t+1} . We will

see that θ will be crucial along with γ and ρ in determining the relative importance of production-smoothing versus stockout-avoidance in the industry.

We assume that cost shocks are a stationary $AR(1)$ process:

$$\ln w_t = \chi \ln w_{t-1} + u_t$$

We will also make κ_t follow an exogenous growth process:

$$\Delta \ln \kappa_t = g(1 - \omega) + \omega \Delta \ln \kappa_{t-1} + \eta_t.$$

The parameters and shock volatilities will be calibrated to match the properties of the data. With these unit root processes it is necessary to construct variables that are stationary. One such variable is the ratio A_t/x_t . Also, from the specification of λ_t , we have

$$\kappa_t p_t^{1+\psi} = h_t^{-\sigma} (h_t/x_t)^\gamma (1 - \phi).$$

This will serve as the aggregate “demand curve” for the industry.

2 Industry Equilibrium

Now we have the following system:

$$h_t = [(1 - \phi) x_t^{1-\gamma} + \phi A_t^{1-\gamma}]^{1/(1-\gamma)} \quad (19)$$

$$\kappa_t p_t^{1+\psi} = h_t^{-\sigma} (h_t/x_t)^\gamma (1 - \phi) \quad (20)$$

$$A_t = A_{t-1} + q_t - \delta x_{t-1} \quad (21)$$

$$0 = E_t \{x_t [p_t (\epsilon_t - 1) - \epsilon_t \beta \delta \pi_{t+1}]\} \quad (22)$$

$$0 = E_t \left\{ p_t \xi_t \frac{x_t}{A_t} - \pi_t + \beta \pi_{t+1} - \beta \delta \pi_{t+1} \xi_t \frac{x_t}{A_t} \right\} \quad (23)$$

$$\epsilon_t \equiv \frac{\rho}{\gamma \rho - (h_t/x_t)^{\gamma-1} (1 - \phi) (\gamma \rho - 1)} \quad (24)$$

$$\xi_t = \frac{(\gamma \rho - 1) \phi (h_t/A_t)^{\gamma-1}}{\gamma \rho - (h_t/x_t)^{\gamma-1} (1 - \phi) (\gamma \rho - 1)} = \epsilon_t (\gamma - 1/\rho) \phi (h_t/A_t)^{\gamma-1} \quad (25)$$

$$\pi_t = (q_t/\bar{q}_t)^\theta w_t \quad (26)$$

$$\Delta \ln \kappa_t = g(1 - \omega) + \omega \Delta \ln \kappa_{t-1} + \eta_t \quad (27)$$

$$\ln w_t = \chi \ln w_{t-1} + u_t \quad (28)$$

We can also add equations for the observables I_t and s_t :

$$I_t = A_t - x_t$$

$$s_t = x_t - x_{t-1} (1 - \delta)$$

or for stationary transformations I_t/s_t and s_t/s_{t-1} .

Before solving the equilibrium we need to make more specific assumptions about what is known when the retailers make pricing and inventory decisions. We will assume that in choosing date t variables they do not know the realization of demand shocks (i.e. the value of κ_t and hence x_t), but they do know the wholesale cost π_t and tax/subsidy τ_t . Because of these assumptions we need to introduce several additional artificial variables: $\pi_t^* \equiv \pi_{t+1}$, $p_t^* \equiv p_{t+1}$, $A_t^* \equiv A_{t+1}$, and $q_t^* = q_{t+1}$, and similarly for the exogenous disturbances w_t^* and τ_t^* .

So, for example, at date t the retailer will choose p_t^* (i.e. p_{t+1}) knowing all t -dated variables including π_t^* and τ_t^* (i.e. π_{t+1} and τ_{t+1}).

2.1 Policy Intervention

We will also consider temporary subsidies on new sales (as, for example, with the “cash-for-clunkers” program). To capture this in a framework where consumers rent the durables, the subsidy needs to be on $s_t = x_t - x_{t-1}(1 - \delta)$, which we can assume is received by dealers. That is, a dealer receives a subsidy not for renting any unit, but for increasing the stock of rented units. Let τ_t denote the subsidy on s_t received by the dealer, so the dealer’s period t revenue is $p_t x_t - \pi_t q_t + \tau_t (x_t - x_{t-1}(1 - \delta))$. The first-order conditions (15) and (16) then become

$$E_t \left\{ (p_t + \tau_t) \xi_t \frac{x_t}{A_t} - \pi_t + \beta \pi_{t+1} - \beta (\delta \pi_{t+1} + (1 - \delta) \tau_{t+1}) \xi_t \frac{x_t}{A_t} \right\} = 0 \quad (29)$$

$$E_t \{ x_t [p_t (\epsilon_t - 1) - \epsilon_t (\beta [\delta \pi_{t+1} + \tau_{t+1} (1 - \delta)] - \tau_t)] \} = 0. \quad (30)$$

The subsidy τ may be thought of as a shock or as a deterministic variable lasting a specified number of periods.

2.2 Data

The data used in our study come from Ward’s Communications, a leading source of news and statistics on the automobile industry. The data consist of total monthly U.S. sales and the stock of dealer held inventories, and are expressed in physical units—specifically, the number of automobiles. The data cover the period from January 1994 to February 2010.

We begin by looking at sales over our sample period, plotting total light vehicles in Figure 2², illustrating how total U.S. sales reflects general macroeconomic trends. In particular, plummeting sales in 2007 and 2008 illustrate the dramatic impact of the latest recession on

²Light trucks are composed of Vans, Sport utility vehicles (SUV), and Pickups.

the automobile sector. Recall that in early 2009, both GM and Chrysler declared bankruptcy. The cash-for-clunkers program occurs in July and August of 2009, towards the end of the sample, and is visible as a large spike in sales in August 2009.

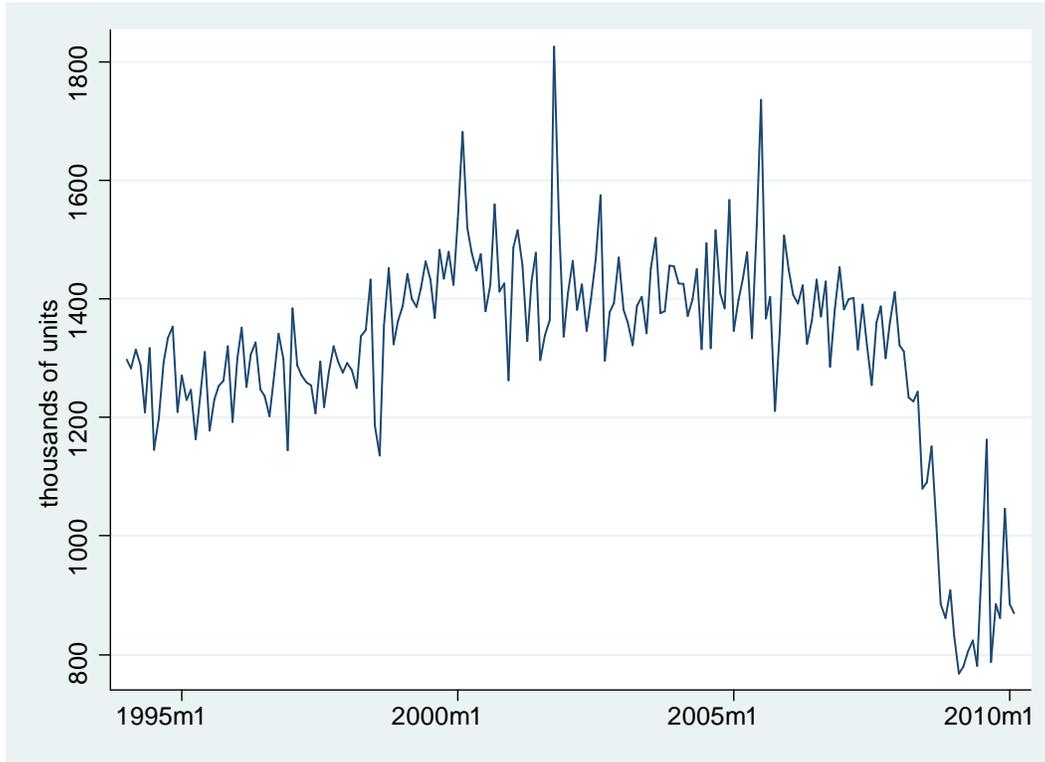


Figure 2: Monthly Automobile Sales

In Figure 3 we plot inventories for all light vehicles. While there is substantial variation, inventories are rising from 1994 through 2004. Starting in 2005, inventories start to gradually taper, before dramatically plunging at the start of 2009. The lowest point in this figure occurs on August 2009, coinciding with the cash for clunkers program. From this figure, the cash for clunkers program simply looks like the tail end of a year-long drop in inventories. But a better metric to observe how automakers are managing their inventories is to adjust their stock of inventories relative to the flow of sales, or to consider the ratio of inventories to sales, a.k.a. months' supply.

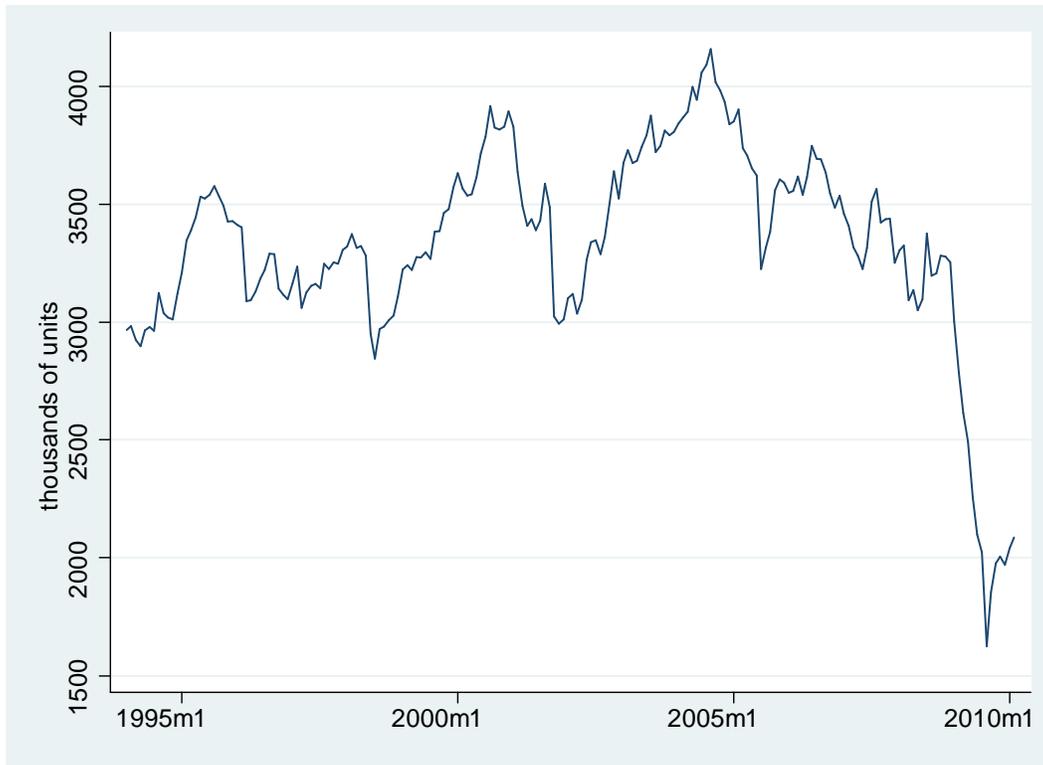


Figure 3: Automobile Inventories

In figure 4 we plot months' supply. Here, we see that months' supply from 1994 through the end of 2007 fluctuated around 2.5. Inventories in stock, then, were roughly two and a half times bigger than the current period flow of sales. In 2008, months' supply began to increase, reaching an extraordinary 3.8 in November 2008, and highlighting the perilous state of the US motor vehicle industry at this time. Reflecting the fall in inventories in 2009, months' supply decreases in 2009. Interestingly, in May and June of 2009, the two months before the cash-for-clunkers program went into effect, months' supply had been brought down to 2.7. The implementation of the cash-for-clunkers program then coincided with a further decline in months' supply, which reached a low point of 1.4 in August, before jumping back up to 2.4 in October.

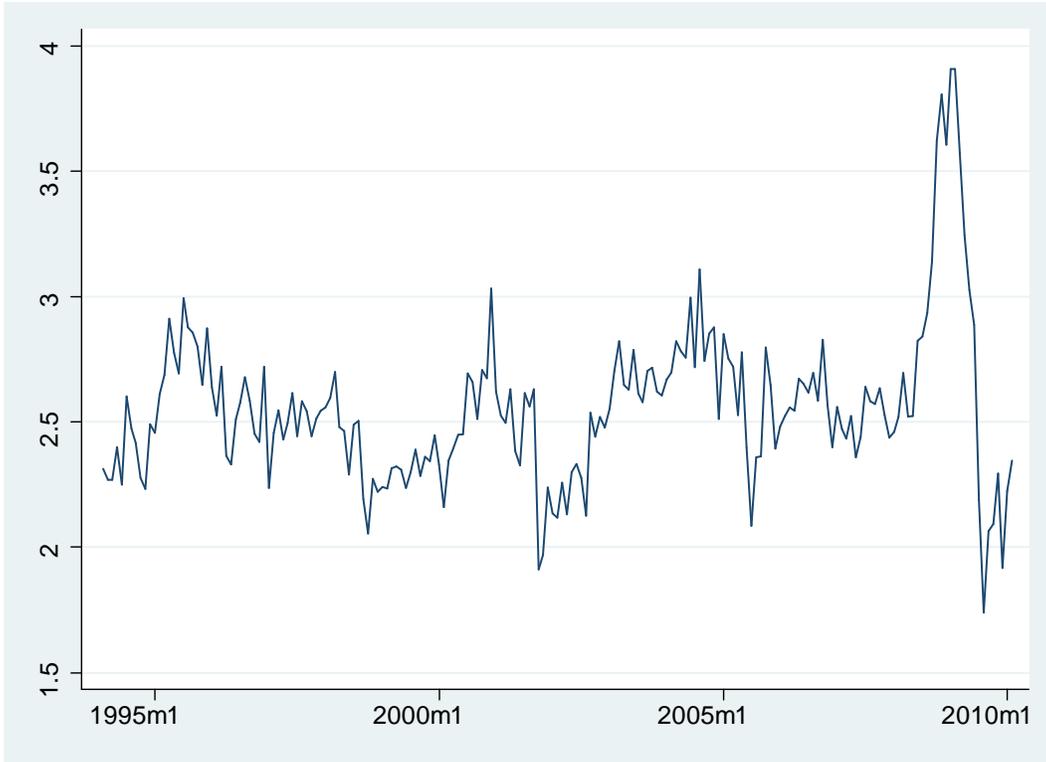


Figure 4: Automobile Months' Supply

2.3 Estimation and Simulation

Many of the key variables in the model such as A and x are not directly measured (or at least not well enough for the purposes of this paper) because they include the stock of used vehicles. But we can nonetheless estimate the model based on the behavior of the observable counterparts of A and x , namely $I_t = A_t - x_t$ and $s_t = x_t - x_{t-1} (1 - \delta)$.

We have estimated most of the parameters by maximum likelihood (ML) using DYNARE on monthly automobile industry data described above, using the sample 1994-2007. (We have also used Bayesian methods, thus far primarily to help with assessing which parameters are identified, and to assist with finding initial values for the ML maximization routine.) The parameters that were set rather than estimated were ρ and ψ , which are not identified, and ϕ , which we constrain (as a function of the other parameters) to mimic the average inventory-sales ratio in the data of 2.4. We also set τ , the steady state “subsidy,” to be -0.0025 (i.e.

a tax), which is approximately 8% of the steady state rental price. The other parameters are all pretty precisely estimated, although γ is only marginally significantly different from one. Both the unit root demand shock and the stationary cost shock are persistent, and δ is estimated surprisingly precisely at about an 18 percent annual rate. Given these parameter values, ϕ is set to 0.184.

Table 1: Maximum Likelihood Estimates from monthly U.S. Automobile Data, 1994-2007

γ	θ	δ	ω	χ	$g \times 10^2$	$\sigma_\eta \times 10^2$	$\sigma_u \times 10^2$
1.3267	0.0110	0.01388	0.3332	0.3263	0.0309	0.0884	0.0872
(0.1946)	(0.0019)	(0.0006)	(0.0697)	(0.0729)	(0.0012)	(0.0071)	(0.0015)

Likelihood Value: 595.42. We set $\rho = 3$, $\tau = -0.0025$, $\phi = 0.184$.

These results are just a first take on the model. We now have data going back to the 1970s. We can consider more general shock specifications. We also will examine identification more closely, and undertake other robustness checks.

To get a sense of how the endogenous variables respond to shocks with these parameters, some simulation results are shown below. We simulate the demand shock η_t and the cost shock u_t , shown below. These have the expected pattern, but with surprisingly persistent deviations of inventories from their steady state in both cases. Intuitively it seems that the size of γ (which affects the elasticity of demand in response to inventories) relative to θ (the slope of marginal cost) is insufficient to bring about a speedy adjustment of inventories back to their “target” level.

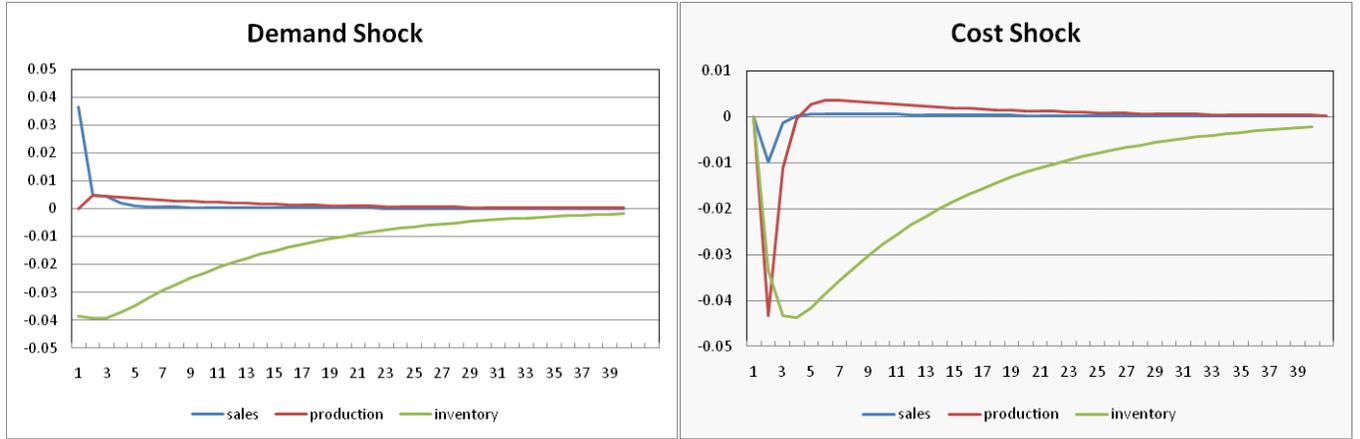


Figure 5: Impulse Responses

This in itself is interesting, in light of the long-standing puzzle (e.g. Feldstein and Auerbach, 1976) about slow estimated adjustment speeds for inventories. As these are based on a structural model, there may be no paradox, though it is also possible, as discussed below, that the slow adjustment reflects the relatively tranquil period over which the parameters were estimated.

Finally, we also simulate a “cash-for-clunkers” shock: An unexpected two-period subsidy on sales. For these parameters, as indicated in the chart below on the left, there is almost no production response—nearly all the additional sales come out of inventories, which then revert to normal when sales dip symmetrically below normal after the program is over. Note that the production response is not literally zero, it is just too small to see in the graph. Thus the parameter estimates lead to an even stronger conclusion about the production impact of the policy than the VEC estimates described earlier.

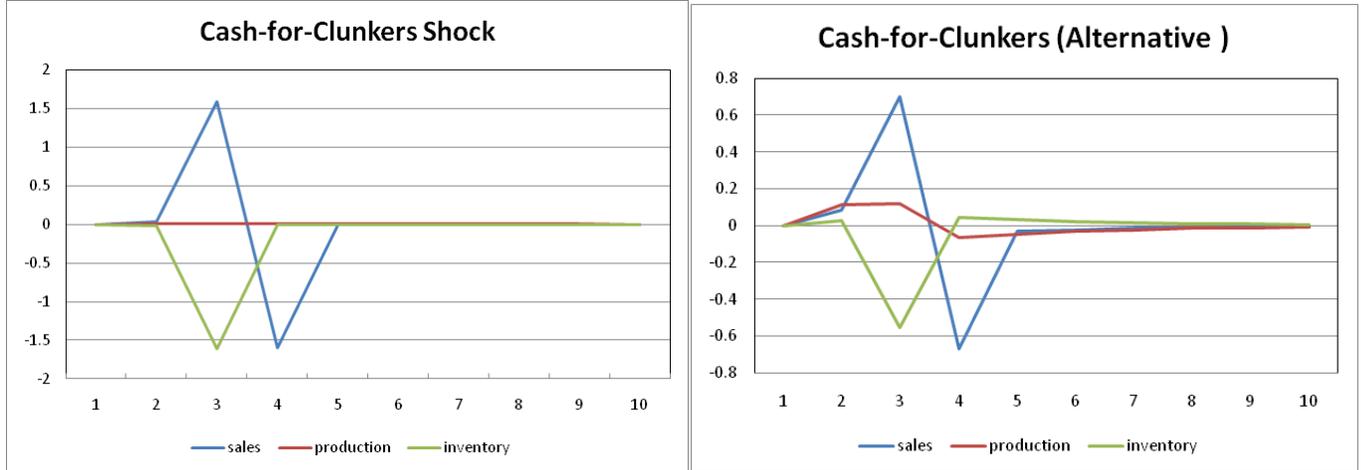


Figure 6: Impulse Responses to Cash-for-Clunkers

Of course for other parameters (larger γ and/or smaller θ) the production response will be larger and the inventory response smaller, as shown on the right (which uses $\gamma = 5$). To get even this large of a production response requires parameters that are significantly different from the ML estimates, however.³ On the other hand, the implications of the ML estimates are very stark. Not only is there almost no production response, but the adjustment back to steady state is very rapid: Inventories are back to normal within one month, and sales the month after that. One could argue that estimation over the relatively tranquil 1994-2007 period is misleading, and that the high volatility from 2008-2010 might provide more meaningful estimates. Further, perhaps the cash-for-clunkers episode provides a sort of controlled experiment that is more informative about the underlying parameters than a decade or more of data.⁴ A larger value of γ in particular adds more “friction” in making sales more sensitive to inventory levels. At the same time, as discussed above, it increases the incentive of firms to quickly get inventories back in line with sales.

To explore this a little further, we redo the maximum likelihood estimation and impulse response functions for $\gamma = 5$. The other parameters did not change meaningfully. With this larger value of γ , it is more important for firms to keep inventories close to sales, so we see,

³One other thing to note: Although the subsidy is for two periods, almost all of the action is in the second period. This is because of perfect foresight. In the more realistic case in which it is not known for sure if the program will go for a second month, there is a more even response.

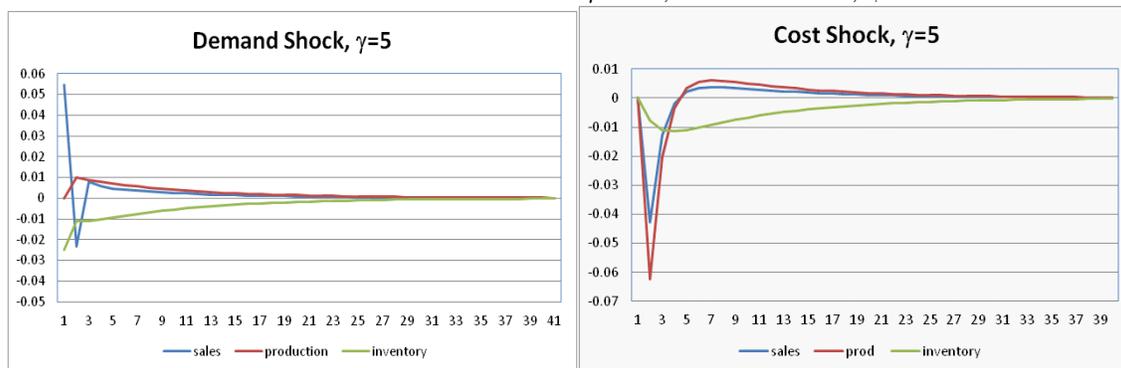
⁴Hendel and Nevo (2006) make a related point regarding econometric estimates of elasticities.

both for demand and cost shocks, larger responses of both production and sales, and smaller deviations of inventories from from their long-run value, as well as more rapid adjustment. These responses seem more in keeping with what we observe with large shocks, as discussed below.

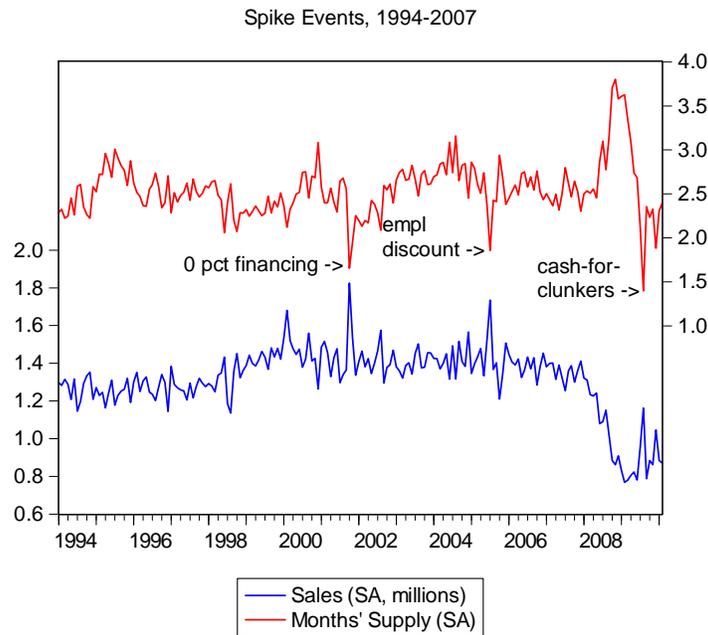
Table 2: Constrained ($\gamma = 5$) Maximum Likelihood Estimates

γ	θ	δ	ω	χ	$g \times 10^2$	$\sigma_\eta \times 10^2$	$\sigma_u \times 10^2$
5.000	0.0112	0.01450	0.3400	0.4334	0.0309	0.0120	0.0100
<i>n.a.</i>	(0.0023)	(0.0003)	(0.0874)	(0.1323)	(0.0015)	(0.0072)	(0.0202)

Likelihood Value: 544.8. We set $\rho = 3$, $\tau = -0.0025$, $\phi = 0.042$.



In light of the above discussion, in future work we plan to explore a more nonlinear approach that puts more weight on specific episodes. Cash-for-clunkers is one such episode, but there are others, even just within the automobile sector. There were several other “spike” events (see figure) even in the 1994-2007 data: in 2001 the automakers offered zero percent financing in the wake of the 9-11 attacks; in 2005 they briefly extended the employee discount plan to all buyers.



There are also strikes, of course, especially in the earlier data. These are obviously not exogenous events, but they are large events that may be more informative about the parameters of the model, especially if responses are nonlinear. Indeed in our earlier paper (Copeland and Kahn, 2011) we found evidence for such nonlinearity by the significance of an included threshold term in the VEC model. We found that when the inventory-sales ratio deviated from the steady state by more than a specified amount, adjustment was faster. We also found evidence of asymmetry: Large negative deviations (very low months' supply) were adjusted more rapidly than large positive deviations.

This work has broader implications that potentially go far beyond narrow policy experiments such as cash-for-clunkers, or the more recent “First-Time Homebuyer Credit.” These findings go to a core issue at the heart of stabilization policy, but generally overlooked: Do consumer expenditures stimulate GDP? There is a large literature testing (and usually rejecting) the simple implication of the permanent income hypothesis with regard to policies such as one-time tax rebates (e.g. Johnson et al, 2006), with the implication that such policies lead to increased spending. But do they actually lead to higher GDP? Or, as with

cash-for-clunkers, is the increased spending largely offset by other factors such as inventory movements? Embedding this model in a fully general equilibrium framework, a straightforward exercise, should provide a reasonable framework for addressing such questions.

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