# Correlation Aversion and Insurance Demand

**Abstract**

This study deals with decision problems under two-dimensional risk. This can be interpreted as risk on income and health. Hence, we have presented a basic theoretical insurance model. An index of absolute correlation aversion allows us to conveniently classify bivariate preferences with respect to attitudes towards such risk. Based on the comparative statics of a one-period model for changes in correlation aversion, we derive very simple testable implications about the specification of utility functions. Using evidence from the 2007 Survey of Consumer Finance on the demand of disability insurance, we find the type of preferences that best fits observed behaviors. Finally, we suggest an improvement to the measurement of the intensity of absolute correlation aversion, and an extension to multi-period models.

*Key words* : Correlation Aversion, Insurance

# 1  Introduction

The notion of risk aversion proposed by Arrow (1965) and Pratt (1964) has played a key role in the research on the economics of risk. There are a lot of frameworks or behavioral consequences to define risk aversion. A risk-averse decision maker could be simply described, within an expected-utility framework, as an individual with a concave utility function. In a more general setting, as proposed by Rothschild and Stiglitz (1970), a risk-averse decision maker is someone who is averse to mean-preserving spreads. Furthermore, the definition could be based on the propensity to purchase full insurance at an actuarially fair price. This behavioral consequence as definition allows the concept of risk aversion to be used to a broader array of applications while it also helps the deeper understanding of the properties of risk aversion within the expected-utility framework.

The concept of correlation aversion is considered as a generalization of risk aversion. It was first defined by Richard (1975) who used the term multivariate risk aversion. Richard (1975) described correlation aversion by using two simple binary lotteries. Since the correlation of discrete variables with only two outcomes is 1 or -1, the proposed framework makes easier the introduction of the concept without loss of generality. A correlation averse agent would prefer a lottery with two perfectly negatively correlated attributes than a lottery with two perfectly positively correlated attributes. The utility function of such an agent is characterized by a negative second cross derivative. In the literature, the second cross derivative functions have been used for a lot of different definitions, within a utility function framework, such as the characterizations of two commodities as ALEP(Ausitz-Lieben-Edgeworth-Pareto) complimentary. Epstein and Tanny (1980) change the name of the notion of multivariate risk aversion to correlation aversion and introduce the concept of elementary correlation increasing transformation. The correlation increasing transformation could be considered as an extension of the notion of a mean-preserving risk increase from one to two variables.

Richard (1975) mentions that a simple measure of absolute correlation aversion such as the measure suggested by Arrow (1965) and Pratt (1964) for risk aversion, could not be easily introduced.

The notion of correlation aversion could be useful to various decisions making problems. Examples of such are definitely portfolio diversification and hedging. Furthermore, the same notion has been applied to the study of inter-temporal models when the attributes of a utility function represent consumption at different points in time. In an inter-temporal model, correlation seeking or averse preferences could affect the choices of a decision maker regarding the optimal consumption in various periods. While Richard (1975) had introduced the concept of correlation premium and proved sufficient conditions for more correlation aversion, an index had not been suggested. Quite remarkably, despite the significant amount of literature on the concept, the first attempt for the development of an index for the correlation aversion was made by Bommier (2007) who proposes a measure of intertemporal correlation aversion and shows that the difference between the inverse of the intertemporal elasticity of substitution and the coefficient of relative risk aversion is related to this measure. Bommier (2007) discusses the notion of correlation aversion in the context of an inter-temporally non-separable utility function where all the attributes represent consumption at different moments in time. The proposed index is defined with respect to two arguments. In the same direction, a generalized definition of correlation aversion for an intertemporal model with multiattribute utility functions was suggested by Lichtendahl et al. (2012). In their model, the attributes represent a finite period lifetime consumption stream. Lichtendahl et al. (2012) show that a correlation averse decision maker is a risk-averse in one dimension and study whether habit forming utility functions could be considered as correlation seeking or not.

Crainich et al. (2014) develop an index for the absolute correlation aversion. The proposed measurement of intertemporal correlation aversion by Bommier (2007) is based on a

different setting than the discrete framework with binary lotteries used by Richard (1975) and Crainich et al (2014). Furthermore, without loss of generality, Crainich et al.(2014) assume that the first attribute could be the subject to a monetary payment while the second attribute is nonpecuniary. This is a key assumption that simplifies the calculations for the resulted index which plays the role of a measurement of the intensity of correlation aversion. Actually, the index is related to a monetary payment that is not similar to the concept of correlation premium as introduced by Richard (1975). Although the assumption of a nonpecuniary variable is restrictive, the index of absolute correlation aversion could be used for a framework with a bivariate utility function where both attributes are monetary. The index proposed by Bommier (2007) is similar to the index proposed by Crainich et al. (2014) under the assumption that the second attribute is nonpecuniary.

The introduction of such an index of absolute correlation aversion can shed more light not only on the study of insurance models and applications but also a wide range of multidimensional risk problems. In his seminal work on the theory of insurance demand, Mossin (1968) shows the important role of the index of absolute risk aversion and prove that that insurance is an inferior good if the decision maker has a utility function with one attribute that exhibits decreasing absolute risk aversion (DARA). However, decisions often have multidimensional consequences, and a good example is health insurance. Individuals choose an insurance program and pay the related premium. Thus, the insurance program affects the wealth of the policyholder but furthermore affects her health through the offered basic medical treatment in case of illness. Usually, the wealth and the health status are positively correlated. For example, a reduction of health status, including a degree of disability, affects the wealth and income level as well. Therefore a measurement of the intensity of correlation aversion that is a generalization of risk aversion could help the study of insurance models. Eeckhoudt and Kimball (1992) introduced a background risk in the framework of insurance theory. Utilizing an attribute, they show that the demand for insurance is not independent

of the background risk, even when the two risks are independent. On the other hand, Rey (2003) uses a utility function with two attributes that could represent wealth and health for the study of a nonpecuniary background risk. His results on the theory of the optimal insurance show that the sign of the second cross derivative plays an important role. A lot of studies emphasize the role of the sign of the second cross derivative as Rey and Rochet (2004), Eeckhoudt et al. (2007) and Lee (2005). Lee (2005) studies how a change in initial wealth modifies the level of prevention against a health loss while Rey and Rochet (2004) use a static model for decisions on health insurance, disability insurance and flu shots in order to derive testable implications about the specification of individual preferences. The implications of the static model are not tested in their study based on observed behaviors. On the other hand Scott and Finke (2013) explore factors that affect demand for disability insurance by using the 2007 Survey of Consumer Finances.

The purpose of this paper is to improve on the index of correlation aversion and find possible applications to insurance theory and multidimensional risk problems. The paper is organized as follows. In the first section, we study the existing measures of correlation aversion and possible applications to insurance demand. We present a basic theoretical model of insurance demand in a one-period expected-utility setting. In the framework of this study, the indemnity payment is considered as one of the simplest cases in which the insurer pays a fixed proportion of the loss. The comparative statics of a one-period model for changes in correlation aversion allows us to derive simple testable implications about the specification of utility functions. Based on evidence from the 2007 Survey of Consumer Finance, we find the preferences on health and wealth that best fit observed behaviors. In the second section, we propose a new index for measuring the absolute correlation aversion and compare the result to the index proposed by Crainich et al.(2014). We prove that the new index for the measurement of the intensity of correlation aversion improves on the index by Crainich et al. (2014) in capturing differences in the degree of correlation aversion for a

wide variety of utility functions that are not captured by the index proposed by Crainich et al. (2014). The assumption that the first attribute is monetary remains to our approach as well. In the last section, we generalize the proposed index of absolute correlation aversion for utility functions with more than two attributes. This approach will help us to find possible applications to dynamic problems.

## 2 Insurance Demand

### 2.1 Index of Absolute Correlation Aversion

By using the notation proposed by Crainich er al., we consider a decision maker with utility function u(x,y) who faces two binary lotteries A and B as shown in Figure 1. Both attributes are subject to a loss l and k respectively, where l and k are positive numbers. Lottery A gives the decision maker a 0.5 probability of receiving (x-l, y-k) and a 0.5 probability of receiving (x, y). On the other hand, lottery B offers to the decision maker a chance of receiving (x-l, y) with probability 0.5 or a chance of receiving (x, y-k) with the same probability. Thus for each attribute, there are two levels: high and low. The low-level values for the two attributes are x-l and y-k while the high-level values are x and y respectively.



Figure 1: Binary Lotteries

Richard (1975) showed that a decision maker who prefers B to A, has a utility function u(x,y) where $u_{12}(x,y) = \frac{\partial u(x,y)}{\partial x \partial y} < 0$. The derivative $u_{12}(x,y)$ has been used for the condition of characterization of two commodities as ALEP complementary. Assume $u(x_1, x_2, x_3, x_4, ...)$

is a utility function over a set of commodities $\{x_1, x_2, x_3, x_4, ...\}$, that represents the preference of a consumer. Two commodities considered as ALEP complementary at $(x_i, x_j)$ if $\frac{\partial u(x,y)}{\partial x_i \partial x_j} < 0$. This condition has been criticized as not invariant under monotone (increasing) transformations. Since $\frac{\partial u(x,y)}{\partial x \partial y}$ is not invariant under linear transformations, it could not play the role of an index. The neutrality of an index of correlation aversion to linear transformations of the utility function is required.

By assuming that only the first variable is monetary, Crainich et al.(2014) create an index of correlation aversion by answering the question: "What is the amount of money (m) to be paid to the decision maker who initially faces lottery A so that he becomes indifferent between A and B?". This amount of money (m) is being paid only if the outcome of the lottery is the ordered pair that involves the high level of the attributes, (x,y), as indicated clearly in figure 2.



Figure 2: payment (m) that makes the decision maker indifferent between the lotteries

In a sense m measures the willingness to accept a perfectly positive correlation when initially the decision maker faces a perfectly negative one. By following Pratt (1964), the authors prove that the payment is given by:

$$m = -\frac{lku_{12}(x, y)}{2u_1(x, y)}. \tag{1}$$

Thus, the proposed index is the fraction $C(x, y) = \frac{-u_{12}(x,y)}{2u_1(x,y)}$ that does not depend on the losses l and k. The index above, which is invariant under linear transformations, could be considered as a measurement of the absolute correlation aversion. The assumption that the

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first variable is monetary is convenient for the development of a basic insurance problem where the first attribute is interpreted as wealth and the second attribute as health.

Richard(1975) had introduced the concept of correlation premium as an ordered pair $(m_x, m_y)$ such that if we reduce x and y in lottery B to $x - m_x$ and $y - m_y$ then the decision maker is indifferent between A and B. Richard (1975) was not able to provide a simple measure of correlation aversion, however, based on the correlation premium, he proved the following theorem that could be considered as the first step in the literature for the introduction of more correlation aversion.

**Theorem 1.** *Suppose u(x,y) and v(x,y) be two bivariate utility functions. Let $u_1 > 0$, $u_2 > 0$, $u_{12} < 0$ and $v_1, v_2 > 0$. Then the following are equivalent*

(i) $\frac{-u_{11}(x,y)}{u_1(x,y)} \geq \frac{-v_{11}(x,y)}{v_1(x,y)}$

(ii) $\frac{-u_{12}(x,y)}{u_1(x,y)} \geq \frac{-v_{12}(x,y)}{v_1(x,y)}$

(iii) $\frac{-u_{12}(x,y)}{u_2(x,y)} \geq \frac{-v_{12}(x,y)}{v_2(x,y)}$

(iv) $\frac{-u_{12}(x,y)}{u_1(x,y)} \geq \frac{-v_{12}(x,y)}{v_1(x,y)}$

(v) $\frac{-u_{12}(x,y)}{u_2(x,y)} \geq \frac{-v_{12}(x,y)}{v_2(x,y)}$

(vi) *If $(m_x, m_y)$ is the correlation premium for u then $v(x - l, y - k) + v(x,y) < v(x - m_x, y - k) + v(x - l, y - m_y)$*

Although the payment in the framework of Crainich et al. (2014) is different from the concept of correlation premium as suggested by Richard (1975), the second condition of Theorem 1 is the index C(x,y). Thus, for two decision makers with twice differentiable utility functions u(x,y) and v(x,y), we could find who is more correlation averse by comparing the index of absolute correlation aversion C(x,y).

## 2.2 Basic Insurance Model

The introduction of such an index of absolute correlation aversion allows us to classify preferences of two attributes contingent on the properties of function C(x,y). For example, we could denote preferences that exhibit decreasing first variable absolute correlation aversion (DFACA) if C(x,y) is decreasing in variable x. In the same way, we could denote preferences that exhibit decreasing second variable absolute correlation aversion (DSACA) if C(x,y) is decreasing in variable y.

We develop an insurance model where the insurer has a twice differentiable utility function u(x,y) where x and y are interpreted as income and health. Therefore, we classify preferences as follows.

(i) Preferences are said to exhibit decreasing health absolute correlation aversion (DHACA) if C(x,y) is decreasing in health level y.

(ii) Preferences exhibit increasing health absolute correlation aversion (IHACA) if C(x,y) is decreasing in health level y

(iii) Preferences exhibit constant health absolute correlation aversion (CHACA) level y when if C(x,y) is constant in health level y

Let the random variable $\tilde{y}$ to denote the decrease of the health index while $\tilde{w}$ denote the loss of income. Variables $\tilde{w},\tilde{y}$ are perfectly correlated. Variable $\tilde{w}$ follows the distribution F with support [0,X]. For this example, we assume the simplest type of indemnity in which the insurer pays a fixed proportion of the loss. The indemnity payment is given by the function P(w)=aw, where a is the fixed proportion paid by the insurer. Of course $P(w) < w$. Also, the premium is given as

$$P(a) = a(1 + \lambda)E\tilde{w} \tag{2}$$

where $\lambda$ is the loading factor asked by the insurer.

The final Wealth and Health are given as $X(a) = X - a(1+\lambda)E\tilde{w} - \tilde{w} + a\tilde{w}$ and $Y = y - \tilde{y} = y - h(\tilde{w})$, where $h()$ is a monotonically increasing function. The notation for the optimal insurance is given as $a^*$. A really good illustration of the above setting is disability insurance. A loss in the Health Index of an individual who became disabled will cause a loss of her income as well.

**Proposition 1.** *For a perfectly correlated risk in Health and Wealth, an increase in Health Index Y will decrease (increase/not affect) the optimal level of insurance under DHACA (IHACA/ CHACA) preferences*

*Proof.* Assume DHACA

$$\left.\frac{\partial^2 Eu(x,y)}{\partial a \partial y}\right|_{a^*} = \int_0^X u_{12}\left(X(a),Y\right)\left(X - (1+\lambda)E\tilde{w}\right)dF \qquad (3)$$

Thus, by setting $w_0 = (1+\lambda)E\tilde{w}$ and $x_0 = x - a(1+\lambda)E\tilde{w} - w_0 + aw_0$ and $y_0 = y - \tilde{y} = y - h(\tilde{w}_0)$ then (3) could be written as

$$-\int_0^{x_0} C\left(X(a),Y\right)u_1\left(X(a),Y\right)\left(X - (1+\lambda)E\tilde{w}\right)dF$$

$$-\int_{x_0}^X C\left(X(a),Y\right)u_1\left(X(a),Y\right)\left(X - (1+\lambda)E\tilde{w}\right)dF <$$

$$-C(x_0,y_0)\int_0^X u_{12}\left(X(a^*),Y\right)\left(X - (1+\lambda)E\tilde{w}\right)dF = 0$$

Thus, an increase in Health index Y will decrease the $a^*$. The statement for IHACA and CHACA preferences can be proved in a similar manner.

The proposition generalizes the result of Eeckhoudt and Schlesinger (2013) for multi-

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variate preferences. Eeckhoudt and Schlesinger (2013) conclude that the optimal level of insurance would need to be increased when the financial loss is accompanied by a loss in health status. Furthermore, the above example is an extension of the basic theoretical model of insurance demand as presented from Schlesinger (2013) to multi-attribute preferences.

Since a disability affects health status and income, we could find implications of Proposition 1 to the demand of disability insurance. Proposition 1 generalizes the result of Rey and Rochet (2004) for the demand of disability insurance as well. They show that the individual wants to take full insurance, less than full insurance or more than full insurance if the occurrence of the illness does not affect, decreases, or increases its marginal utility of wealth respectively. Hence their result is related to the sign of the second cross derivative.

The specification of preferences plays an important role in the literature in health economics. Bleichordt and Quiggin (1999) suggest life-cycle preferences where both health status and consumption at different time periods are considered as arguments. At each time period the proposed utility function is given as multiplicative separable between consumption and health $u(x, y) = v(x)w(y)$, where $v(x)$ and $w(y)$ are considered as utilities associated with consumption and health status respectively. On the other hand, Ma and Riordan (1997) propose a utility function that is additive separable between consumption and health $u(x, y) = v(x) + w(y)$.

The empirical model of Scott and Finke (2013) for the demand for disability insurance uses variables that are related to financial characteristics, attitude, and demographics of individuals from the 2007 Survey of Consumer Finances. The study finds evidence that the demand for disability insurance increases as the health index increases as well. Based on Proposition 1, we could conclude preferences exhibit IHACA. Hence, the assumption of an additive separable utility function should be rejected. Under specific conditions, a utility function $u(x, y) = v(x)w(y)$ does not contradict to the empirical evidence from Scott and Finke (2013). An example of such preferences is presented in the next section.

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# 3  New Index of Correlation Aversion

In this section, we suggest an alternative index for correlation aversion. First of all, the index proposed by Crainich et al. (2014) includes only the second cross derivative while the partial derivatives with respect to the second argument do not play any role. Furthermore, in a case of a multiplicative separable utility function $u(x,y) = v(x)w(y)$, the index of absolute correlation aversion is given as $C(x,y) = \frac{w_1(y)}{w(y)}$. Thus, C(x,y) is not a function of the first variable x and cannot capture differences in the degree of correlation aversion for this type of utility functions.

We propose an index for the intensity of correlation aversion based on a payment (m) that is the maximum amount of money to be paid by the decision maker to avoid a perfectly positively correlated loss and face a perfectly negatively correlated loss, as shown in Figure 3.



Figure 3: payment by the decision maker

By expanding the terms up to the second order, it can be proved that m is given by the following formula. The proof is given in the appendix.

$$m = -\frac{l^2 u_{11}(x,y) + k^2 u_{22}(x,y) + 2lk u_{12}(x,y)}{4 u_1(x,y)} \qquad (4)$$

Furthermore, by following Pratt (1964), we assume small losses $\epsilon$ around (x,y) Hence m could be written as

$$m = \frac{\epsilon^2}{4} \frac{-u_{11}(x,y) - u_{22}(x,y) - 2u_{12}(x,y)}{u_1(x,y)} \qquad (5)$$

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The new proposed index for the absolute correlation aversion is given by the following expression

$$I(x, y) = \frac{-u_{11}(x, y) - u_{22}(x, y) - 2u_{12}(x, y)}{4u_1(x, y)} \tag{6}$$

Thus, we observe that the index I(x) could be expressed as a sum of the two terms. The first term is the proposed index by Crainich et al. C(x,y), as indicated by the following equation.

$$I(x, y) = C(x, y) + \frac{-u_{11}(x, y) - u_{22}(x, y)}{4u_1(x, y)} \tag{7}$$

As we have shown above, for multiplicative separable utility functions, as power utilities, the index C(x,y) cannot support our intuition about correlation aversion. On the other hand, the new index I(x) captures differences in the degree of absolute correlation aversion that are not captured by C(x,y). This could be easily illustrated with the following example.

**Example** Assume two decision makers with utility functions $u(x, y) = -x^{-1}y^{-0.1}$ and $v(x, y) = -x^{-0.1}y^{-0.1}$ respectively. The only difference between $u(\cdot)$ and $v(\cdot)$ is that the first function is more sensitive to changes in variable x than in variable y. The decision makers are correlation averse since $\frac{\partial u(x,y)}{\partial x \partial y} < 0$ and $\frac{\partial v(x,y)}{\partial x \partial y} < 0$, while the index of absolute correlation aversion is the same for both utility functions. The decision makers face two lotteries A and B where

(i) Lottery A gives the decision maker a 0.5 probability of receiving (7900, 6000) and a 0.5 probability of receiving (8000, 8000)

(ii) Lottery B gives the decision maker a 0.5 probability of receiving (7900, 8000) and a 0.5 probability of receiving (8000, 6000)

Both indices do not depend on the losses. The proposed losses for the two attributes x and y will help us to illustrate the difference of the expected utility between the lotteries. The utility functions have the same form $U(x,y) = -x^{-a}y^{-b}$, where a and b are positive numbers. Based on the Crainich et al.(2014), in the index C(x,y), the term $x^{-a}$ doesn't play any role in the intensity of correlation aversion. Table 1 shows that the values of C(x,y) doesn't support our intuition contrary to the values of I(x,y).

Table 1: Comparison

| Utility Function | Difference of Expected Utility for lotteries A and B | $C(x,y)\|_{(8000,8000)}$ | $I(x,y)\|_{(8000,8000)}$ |
|---|---|---|---|
| u(x,y) | 2.893E-08 | 3.75 | 5.93 |
| v(x,y) | 89.2618E-06 | 3.75 | 14.65 |

Actually, the differences of the expected utility for lotteries A and B support our intuition that the decision maker with utility function v(x,y) is more correlation averse than a decision maker with utility function u(x,y). As we can observe in the above table, the value of the index C(x,y) is the same for both utility functions while the value of index I(x,y) is higher for the utility function v(x,y) than the utility function u(x,y).

## 4  Index of Correlation Aversion for Utility Functions with n attributes, $n > 2$

In this section, we generalize the index I(x,y) for utility functions with more than two attributes. This index will summarize the information about the correlation among all the attributes. First of all, this generalization will allow us to use the index to insurance models where the utility function is represented by more than two arguments or in a setting that is not static. A good example is an intertemporal model in which preferences depend on the lifetime path of consumption and the decision maker faces various trade-offs between the levels and riskiness of consumption in successive time periods. We will start our analysis with a twice differential utility function u(x,y,z) that has

three attributes. Also, we need to introduce the following operations between vectors. Based on two randomly given vectors $v_0 = (v_{01}, v_{02}, v_{03})$ and $v_1 = (v_{11}, v_{12}, v_{13})$, we denote $v_0 \vee v_1$ as $v_0 \vee v_1 = (max\{v_{01}, v_{11}\}, max\{v_{02}, v_{12}\}, max\{v_{03}, v_{13}\})$ while $v_0 \wedge v_1$ is denoted as $v_0 \wedge v_1 = (min\{v_{01}, v_{11}\}, min\{v_{02}, v_{12}\}, min\{v_{03}, v_{13}\})$. Without loss of generality, consider the following order for the attributes of two given vectors $v_{01} > v_{11}$, $v_{02} < v_{12}$ and $v_{03} > v_{13}$. We create two lotteries where lottery B gives the decision maker a 0.5 probability of receiving $v_0$ and 0.5 probability of receiving $v_1$. On the other hand, lottery A gives the decision maker a 0.5 probability of receiving $v_0 \vee v_1$ and 0.5 probability of receiving $v_0 \wedge v_1$ as shown in figure 4.



Figure 4: $v_0$, $v_1 \in \mathbb{R}^n$ where $n > 2$

These lotteries are similar to lotteries proposed to figure 1. Indeed, lottery B offers a chance to the decision maker on getting the high-level values for all the attributes or all the low-level values with the same probability. On the other hand, lottery A offers two outcomes with some high-level values and some low-level values. We can easily show the following proposition. The proof is given in the appendix

**Proposition 2.** *A decision maker, where all the second cross derivatives are negative, would prefer A than B.*

By following the same process as in the previous section, we could show that the payment (m) that is the maximum amount of money to be paid by the decision maker to avoid a perfectly positively correlated loss is given by the following equation. The proof is given in the appendix

$$m = -\frac{\sum_{i=1}^{3} l_i u_{ii}(v_0 \vee v_1) + \sum_i^3 \sum_j^3 \sum_j^3 2l_i l_j u_{ij}(v_0 \vee v_1)}{4u_1(v_0 \vee v_1)} \qquad (8)$$

where $|l_1| = |v_{01} - v_{11}|$, $|l_2| = |v_0 - y_1|$ and $|l_3| = |z_0 - z_1|$

Now assume two vectors $v_0$ and $v_1$ from $\mathbb{R}^n$ where $n \geq 3$ and a utility function with n attributes. Also, we denote the decision maker's utility function as u: $\mathbb{R}^n \to \mathbb{R}$ which is exists and it is twice partially differentiable. Lichtendahl et al. (2012) define correlation aversion in a multiattribute setting as following

**Definition** A decision maker with utility function u, is correlation averse if for all $v_0$ and $v_1$ and any $q_L < q_H$, he prefers lottery $L(q_L)$ to lottery $L(q_H)$, where the lottery L(q) is defined in Figure 4 .
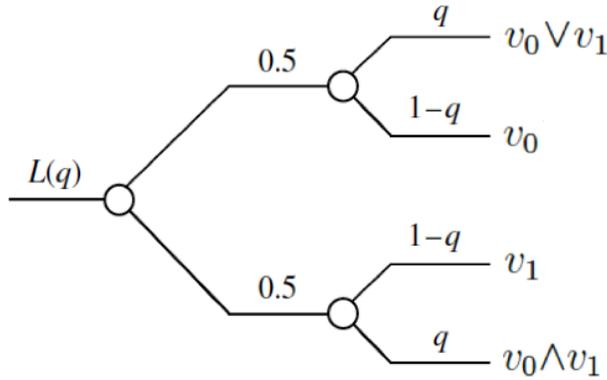


Figure 5: correlation aversion in a multiattribute setting

Lichtendahl et al. (2012) proved that a correlation averse decision maker has a utility function where all the second cross derivatives $u_{ij} < 0$ are negative, where $0 < i, j < n$ and $i \neq j$. The vectors could represent consumptions at different time periods. For example, the vector $v_0 = (v_{01}, v_{02}, v_{03}, ..., v_{0n})$ is interpreted as the n-period lifetime consumption stream

where the decision maker consumes $v_{0i}$ over the period from time i-1 to i. The utility comparison that follows from a preference for lottery $L(q_L)$ to lottery $L(q_H)$ could reduce the definition to the condition $u(v_0 \vee v_1) + u(v_0 \vee v_1) \geq u(v_0) + u(v_1)$. Thus, a correlation averse decision maker, as proposed above, always prefers lottery A than lottery B in figure 4. The payment (m), as denoted in Equation (8) , in the Lichtendahl et al. (2012) setting could be interpreted in a different way. If the lottery realization is either $v_0$ or $v_1$, the decision maker will subtract m units from the first attribute. Thus m is the maximum amount of money to be paid by the decision maker today at branches $v_0$ and $v_1$ to become indifferent about the value of q. By following the same process as in section 3, from the payment m, we can derive the following function that plays the role of an index of correlation aversion for preference with more than two attributes.

$$I = -\frac{\sum_{i=1}^{n} u_{ii}(\cdot) + \sum_{i}^{n}\sum_{j}^{n} 2u_{ij}(\cdot)}{4u_1(\cdot)} \tag{9}$$

Although the payment (m) is related to the first attribute, the above index includes all the second cross-derivatives of the utility functions, $u_{ij}$, where $0 < i, j < n$ and $i \neq j$. Moreover, the index above leads to the index proposed in equation (6) when n=2.

## 5   Conclusion

Despite the significant amount of papers which have been devoted to the study of correlation aversion an index for the measurement of the intensity of absolute correlation aversion had not been developed. The first attempt was made Crainich et al.(2014) for bivariate utility functions. The introduction of such an index could play a major role in the study of multidimensional risk problems. A first implication for one period insurance models under a two dimensional risk that can be interpreted as risk on income and health has been developed in

this study. The comparative statics of a one-period model for changes in correlation aversion are presented. Using evidence from the 2007 Survey of Consumer Finance on the demand of disability insurance, we show that multiplicatively separable preferences between health and wealth best fit observed behaviors. Furthermore, this paper deals with the improvement of the index suggested by Crainich et al. (2014). We compare the two indices for multiplicatively separable preferences and propose an extension to an index of correlation aversion for multi-attribute utility functions. The new index could be used for a framework that is not static and shed more light not only to insurance models and applications but also to a wide range of multidimensional risk problems in different fields.

## 6   Appendix.

**Proof of equation (4)**. If the decision maker is indifferent between lottery A and B in Figure 3 then

$$U(x - m - l, y) + U(x - m, y - k) = U(x - l, y - k) + U(x, y)$$

By following Pratt (1964), we find the second order Taylor approximation of the expression on the right-hand side and the first order Taylor approximation on the left hand side.

$$U(x,y) - (m+l)U_1(x,y) + U(x,y) - mU_1(x,y) - kU_2(x,y) = U(x,y) - lU_1(x,y) - kU_2(x,y) +$$
$$\tfrac{1}{2}l^2 U_{11}(x,y) + \tfrac{1}{2}k^2 U_{22}(x,y) + lkU_{12}(x,y) + U(x,y)$$

By simplifying and solving for m, it is easy to show that m is given by the equation (4)

$$m = -\frac{l^2 u_{11}(x,y) + k^2 u_{22}(x,y) + 2lk u_{12}(x,y)}{4u_1(x,y)}$$

where l, k are positive real numbers

**Proof of Proposition 2**. Without loss of generality, assume that the values for the first and the last attribute of $v_0$ are higher than $v_1$. Set $v_0 = (x, y-l_2, z)$ and $v_1 = (x-l_1, y, z-l_3)$, where $l_1$, $l_2$ and $l_3$ are positive real numbers. For a decision maker the difference between expected utility for lottery A and B is given as

$$Eu(A) - Eu(B) = 0.5\big(u(v_0) + u(v_1)\big) - 0.5\big(u(v_0 \vee v_1) + u(v_0 \vee v_1)\big) = 0.5\big(u(x, y-l_2, z) +$$
$$u((x-l_1, y, z-l_3))\big) - 0.5\big(u(x, y, z) + u(x-l_1, y-l_2, z-l_3)\big)$$

By expanding each term up to the second order around u(x,y), we get

$$Eu(A) - Eu(B) = 0.5\big(l_1 l_2 u_{12}(x, y, z) + l_2 l_3 u_{23}(x, y, z) + l_1 l_3 u_{13}(x, y, z)\big)$$

So when all the second cross derivatives are negative the decision maker prefers lotery A.

**Proof of Equation (8)**. Without loss of generality, assume that the values for the first and the last attribute of $v_0$ are higher than $v_1$. Set $v_0 = (x, y-l_2, z)$ and $v_1 = (x-l_1, y, z-l_3)$, where $l_1$, $l_2$ and $l_3$ are positive real numbers. If the decision maker is indifferent between lottery A and B in Figure 4 then

$$u(x-m-l_1, y, z-l_3) + u(x, y-l_2, z) = u(x-m-l_1, y, z-l_3) + u(x, y, z)$$

By following the same process, we find the second order Taylor approximation of the expression on the right-hand side and the first order Taylor approximation on the left hand side.

$u(x, y, z) - (m + l_1)u_1(x, y, z) - l_3u_3(x, y, z) + u(x, y, z) - l_2u_2(x, y, z) = u(x, y, z) - l_1u_1(x, y) - l_2u_2(x, y, z) - l_3u_3(x, y, z) + \frac{1}{2}l_1^2u_{11}(x, y, z) + \frac{1}{2}l_2^2u_{22}(x, y, z) + \frac{1}{2}l_3^2u_{33}(x, y, z) + l_1l_2u_{12}(x, y, z) + l_2l_3u_{23}(x, y, z) + l_1l_3u_{13}(x, y, z) + u(x, y, z)$

By simplifying and solving for m, it is easy to show that m is given by the equation

$$m = -\frac{\sum_{i=1}^{3} l_iu_{ii}(x, y, z) + \sum_{i}^{3}\sum_{j}^{3}\sum_{j}^{3} 2l_il_ju_{ij}(x, y, z)}{4u_1(x, y, z)} \tag{10}$$

where $v_0 \vee v_1 = (x, y, z)$

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