

Asymmetric Volatility Spillover Between European Equity and Foreign Exchange Markets: Evidence From The Frequency Domain

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Abstract

In this study, we explore the nature of volatility spillovers between European equity and foreign exchange markets. In contrast to the traditional empirical methods, we employ a novel approach using high-frequency constructed realized volatility within a frequency domain causal analysis framework. We test for the presence of volatility spillover amongst the CAC 40, DAX, and FTSE 100 and their major underlying exchange rates over the 2009 to 2016 period. We find that volatility spillover is bidirectional and asymmetric across the frequency domain. Daily volatility spillover from equity to foreign exchange markets is significant at high, mid-range, and low frequencies, whereas foreign exchange to equity market spillover is significant at mostly lower frequencies beyond ten days. Weekly analysis reinforces these results; however, weekly volatility spillover from equity to foreign exchange markets is no longer significant at low frequencies and insignificant in two cases. Our findings have implications for portfolio risk management.

Key Words: Volatility spillover; Frequency domain causality; Realized volatility; Equity prices; Exchange rates

JEL Classifications: C32, F31, G15

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1 Introduction

Europe has experienced great economic and socio-political turmoil in the years following the Global Financial Crisis. Over 2010 to 2013, much of Europe suffered from sovereign debt crises. More recently, there are mounting concerns with regards to a fragile Italian banking sector plagued by non-performing loans and regional socio-political upheaval (e.g., Brexit). All of these factors, as well as many more, influence the behavior of European equity and foreign exchange markets which in turn affect each other. Such circumstances highlight the need for a better understanding of the transmission of volatility across these markets. In particular, both individual and institutional investors stand to gain from such an analysis by improving the assessment and management of risk. We approach the problem of untangling the nature of volatility spillovers between European equity and foreign exchange markets from a new perspective within the context of the frequency domain, revealing further complexities that persist in volatility spillover channels.

What does existence of volatility spillover between financial markets imply? This particular question has been widely addressed in the existing literature. The transmission of volatility is closely linked to the flow of information (Ross, 1989). More specifically, volatility spillover is the market response to the flow of information. Engle et al. (1990) attribute the delay between the arrival and processing of information as one of the main contributing factors to the spread of volatility across markets. In addition, there may be financial market friction related delays. For example, trading costs and insufficient asset liquidity may prohibit rapid actionable responses to news shocks. Furthermore, not all investors hold the same belief structure and as a result may interpret and respond to market developments differently (Shalen, 1993). Ultimately, volatility spillover across markets is a complex process with many factors affecting initiation (e.g., rapid or delayed), strength, and duration. As such, focusing on volatility spillover in terms of causality over varying frequencies is particularly

useful.

We focus on the three largest equity markets in Europe, proxied by the CAC 40, DAX, and FTSE 100, and their respective exchange rates (USD/EUR, GBP/EUR, and USD/GBP). We adopt a novel approach to analyze the nature of volatility spillovers between these markets. First, we use high frequency intra-day data to generate realized volatility measures at both the daily and weekly observational frequency. Utilizing realized volatility provides a “model free” means of capturing the integrated, or latent, volatility processes (Andersen et al., 2001, 2002; Barndorff-Nielsen and Shephard, 2001). The most widely used alternative approaches are to estimate either GARCH or stochastic volatility models; however, these models can be quite restrictive in their assumptions, provide a potentially over-smoothed approximation of volatility, and carry with them the potential for misspecification. Second, we use the Breitung and Candelon (2006) approach to test for causality across the frequency domain. Frequency domain analysis can capture both directional and frequency-specific asymmetries, allowing for substantially richer analysis. To the best of our knowledge, this is the first work to analyze volatility spillovers across financial markets using realized volatility and frequency domain causal analysis.

Our findings show that volatility spillover between European equity and foreign exchange markets is asymmetric across the frequency domain and bidirectional in nearly all cases. Daily volatility spillover from equity prices to exchange rates is significant at high, mid-range, and low frequencies for all six equity index and exchange rate pairs. Evidence of daily volatility spillover originating from foreign exchange markets is generally weaker but significant at low frequencies. Our weekly analysis re-affirms these results for volatility spillover from exchange rates to equity prices. Some minor differences are seen for the equity to exchange rate direction when using weekly volatilities; however, the overall results are still consistent with the daily analysis. The result of bidirectional spillover is in contrast to most empirical studies that consider the same variables and similar time periods. Furthermore,

our analysis highlights the need to consider a wide range of frequencies when testing for the presence of volatility spillover across financial markets (e.g., extremely low frequencies).

The rest of the paper is as follows. Section 2 provides an overview of the relevant literature on volatility spillovers between equity and foreign exchange markets. Section 3 derives and discusses the frequency domain causality methodology. Section 4 presents the data. Section 5 discusses all empirical findings. Finally, we provide concluding remarks in Section 6.

2 Literature Review

2.1 Theoretical Foundations

There are two main frameworks typically used to explain the transmission of volatility between foreign exchange and equity markets. The first, directly links a firm's value to currency fluctuations (Jorion, 1990; Sercu, 1992). Consider the case of a domestic exporter and assume the firm's stock price is the simple presented discounted value of future cash flows. An increase (decrease) in exchange rate volatility increases (decreases) the level of uncertainty surrounding their real income and future cash flows. Such a development should be reflected in the stock price's behavior. A similar argument can be made for domestic importers. Here, exchange volatility affects expectations of future import costs and the production costs associated with using imported intermediate goods. The aggregate level impact of this causal channel is somewhat unclear, dependent on the degree of trade openness and relative mix of importers and exporters.

The second framework focuses on the investor's role in shaping the relationship between markets. Hau and Rey (2006) propose an international portfolio diversification model where investors cannot completely hedge exchange rate risk due to the presence of incomplete financial markets. Portfolio re-balancing is conducted in response to both equity price and exchange rate fluctuations. Suppose that a portfolio's foreign equities rapidly appreciate in

value. In the context of this model, re-balancing implies that the investor will sell some of the foreign assets and buy domestic ones. Any foreign profits, assuming that there are for this example, must be converted in to the domestic currency in order to purchase domestic equities; as such, the increase in the demand for the domestic currency affects the exchange rate. Suppose instead there was an exchange rate shock such as an unanticipated depreciation. The investor will buy and sell domestic and international equities to offset any changes to their portfolio's risk profile. Therefore, the initial exchange rate shock is reflected to some extent in the behavior of the respective equity price movements.

While Hau and Rey (2006) assume the investor is already a market participant, Karoui (2006) also considers the case of investors who have yet to enter the market. Exchange rate fluctuations affect the relative value of equities which may entice more investors to enter or leave the market and impact equity prices vis-à-vis their demand. Similarly, investors may enter or leave the market in light of equity price fluctuations (e.g., momentum investing and algorithm high-frequency trading). If a domestic investor wishes to enter a foreign equity market then they must purchase the foreign equities in their local currency. In doing so, the demand for the foreign currency increases and affects its value.

2.2 Characteristics of Volatility Spillovers

The nature of volatility spillovers between equity and foreign exchange markets is complex with empirical findings varying based on factors such as the sample period, level of financial development, and economic state to name a few. One question addressed in the literature is whether or not volatility spillovers are asymmetric; asymmetric spillover implies that equity and foreign exchange markets process positive and negative cross-market news shocks in the same manner. Kanas (2000) and Kanas (2002) find that volatility spillovers are positive and symmetric. More recent evidence suggests the prevalence of positive but asymmetric volatility spillover (Aloui, 2007; Morales, 2008; Walid et al., 2011). Multivariate (MV)

EGARCH models are typically estimated to capture asymmetric behavior, combining the cross-placement of volatilities amongst variance equations with leveraged effects.

In addition to asymmetry in terms of positive versus negative shocks there is also evidence of asymmetry in terms of spillover direction. Equity market volatility tends to more strongly influence foreign exchange market volatility than the other way around (Kanas, 2000, 2002; Apergis and Reztis, 2001; Aloui, 2007; Morales, 2008; Choi et al., 2009). Dominance of the equity market to foreign exchange rate path is noted for both developed and developing markets, as well as for commodity and non-commodity currencies.¹

However, recent studies by Aloui (2007), Choi et al. (2009), and Walid et al. (2011) suggest a time varying component to the volatility spillover relationship. Focusing on New Zealand equity markets and major NZD denominated exchange rates, Choi et al. (2009) find that volatility spillover is weaker following 1997-1998 Asian Financial Crisis. Aloui (2007) examines European equity and foreign exchange markets pre- and post-Euro adoption. Volatility spillover from equity to foreign exchange markets is stronger in both the pre- and post-Euro periods, exhibiting significant asymmetric effects. Spillover persistence is substantially higher in the pre-Euro period for both directions of transmission. As opposed to performing sub-sample analysis, Walid et al. (2011) directly model time variation using a Markov-Switching (MS) EGARCH framework. They classify two regimes to capture general economic states: “calm” (high mean, low variance) and “turbulent” (low mean, high variance). Interestingly, they find a significant difference in the spillover sign across regimes. Volatility spillover is positive during calm regimes and negative during turbulent regimes. News signals tend to be noisier during turbulent regimes, which may partially explain the sign change.

While the aforementioned empirical approaches to testing for volatility spillover can re-

¹The behavior of commodity currencies are closely tied to the price fluctuations of goods such as precious metals, crude oil, natural gas, industrial metals, and ores. Several major commodity currencies are the AUD, CAD, NZD, and RUB (floating period).

veal direction, sign, and magnitude, they are limited as a consequence of defining the relationship as contemporaneous. Volatility, in particular equity market volatility, tends to be highly persistent (Corsi, 2009). Introducing the necessary additional lags to capture long-memory volatility processes in a MV-EGARCH or BEKK-GARCH model greatly increases the parameter space and further complicates estimation. As such, casual analysis provides an attractive alternative empirical approach.

2.3 Causal Linkages

Empirical research focusing on causal linkages between equity and foreign exchange market volatility typically employs econometric methods centered around either univariate (Cheung and Ng, 1996; Hong, 2001) or BEKK-GARCH type models (Caporale et al., 2002, 2006). For the former, cross-correlation function (CCF) tests are performed at various lags using the standardized residuals from a first-stage GARCH model. The BEKK-GARCH causality-invariance (CIV) approach entails estimating a bivariate GARCH system and then separately performing upper and lower diagonal Wald tests on the BEKK variance structure.

Using the CIV approach, Caporale et al. (2002) study the causal relationship between equity and foreign exchange market volatility for several East Asian countries before and after the Asian Financial Crisis. Results depend on the level of financial development and sub-period with evidence in favor of unidirectional CIV from equity to foreign exchange markets. Similarly, Aloui (2007) find evidence in support of unidirectional CIV for French and German equity markets. Using the CCF methodology, Koseoglu and Cevik (2013) also report unidirectional causality in the equity to exchange rate direction for a collection of Central and Eastern European countries.

There are limitations associated with the aforementioned econometric methodologies. While the CCF approach can test for causality at various lags and leads, it is hindered by its reliance on first-stage univariate GARCH estimates. As such, there is potential for model

misspecification. Comparing the properties of CCF and CIV tests, Hafner and Herwartz (2008) find that the CCF test exhibits lower power and is more sensitive to misspecification than CIV. That said, the CIV test suffers from size and power distortions in the presence of extreme volatility clustering such as around financial crises (Javed and Mantalos, 2015). Furthermore, the CIV test methodology can only recover spillover sign and causal direction.

Taking an alternative approach, Leung et al. (2017) estimate a two-stage regression model. First, they estimate univariate GARCH models. Next, regressions are performed on the recovered conditional volatilities. Control variables are included to account for financial stress, contagion, and crisis. Consistent with past studies, volatility spillover is significant and positive. Volatility spillover strength between equity and foreign exchange markets is lower during financial crisis, reflecting the state-dependent nature explored by Walid et al. (2011). That said, there are two serious econometric concerns which call in to question parameter estimate reliability. Leung et al. (2017) use a highly restrictive GARCH model with no conditional mean structure (e.g., ARMA) and assume that disturbances are normally distributed. Financial returns often exhibit highly non-normal behavior with fat-tails and some degree of skewness. GARCH model misspecification brings in to question the conditional volatility estimates. Finally, their second-stage regression analysis implicitly assumes directional exogeneity from equity to foreign exchange markets. While instruments are used to control for potential control variable endogeneity, no such effort is done to account for endogeneity in volatilities.

This study adopts a more flexible approach, providing richer causal analysis. Taking advantage of the information conveyed by high frequency data, intra-day returns are used to generate more accurate “model free” realized volatility measures. In doing so, we bypass the typical first-stage GARCH estimations and lessen the chance for misspecification. Focusing on the frequency domain, we address not only the existence of causality but can pinpoint the significant frequencies in each direction. Consider two daily asset returns A

and B. Suppose there exists bidirectional causality in the volatilities of A and B. Further assume that causality is significant from A to B at high frequencies (< 5 days) but only at low frequencies (> 30 days) from B to A. Such a relationship can easily be recovered using frequency domain analysis.

3 Testing For Causality In The Frequency Domain

Developed by Breitung and Candelon (2006), we test for volatility spillovers between equity and foreign exchange markets using the frequency domain causality framework. Focusing on the frequency domain by means of spectral densities facilitates the testing of whether or not a series causes another at a given frequency; this approach can more easily reveal both directional *and* frequency-specific asymmetries.

Let $z_t = [x_t, y_t]'$ be a two-dimensional vector for $t = \{1, \dots, T\}$. Assume z_t has a finite-order VAR functional form given by

$$\Theta(L)z_t = \epsilon_t. \tag{1}$$

$\Theta(L) = I - \Theta_1 L - \Theta_2 L^2 - \dots - \Theta_p L^p$ is a 2 x 2 polynomial lag operator such that $L^k z_t = z_{t-k}$. Further assume that ϵ_t is a white noise process defined by $\epsilon_t \stackrel{i.i.d}{\sim} (0, \Sigma)$ where Σ is positive definite. For simplicity, the following derivations abstract from the inclusion of deterministic terms such as a constant, trend, or dummy variables.

Let G be the lower triangular matrix of the Cholesky decomposition $G'G = \Sigma^{-1}$ such that $E[\eta_t \eta_t'] = I$ and $\eta_t = G\epsilon_t$. If the system is stationary, then the moving average representation

takes the form

$$z_t = \Phi(L)\epsilon_t = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} \quad (2)$$

$$= \Psi(L)\eta_t = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}, \quad (3)$$

where $\Phi(L) = \Theta(L)^{-1}$ and $\Psi(L) = \Phi(L)G^{-1}$. From Eq.(3), the spectral density of x_t can be expressed as

$$f_x(\omega) = \frac{1}{2\pi} \{ |\Psi_{11}(e^{-i\omega})|^2 + |\Psi_{12}(e^{-i\omega})|^2 \}, \quad (4)$$

where ω defines the angular frequency.² Based on Geweke (1982), the measure of causality for a given ω is

$$M_{y \rightarrow x}(\omega) = \log \left[\frac{2\pi f_x(\omega)}{|\Psi_{11}(e^{-i\omega})|^2} \right] \quad (5)$$

$$= \log \left[1 + \frac{|\Psi_{12}(e^{-i\omega})|^2}{|\Psi_{11}(e^{-i\omega})|^2} \right]. \quad (6)$$

Note that $M_{y \rightarrow x}(\omega) = 0$ only if $|\Psi_{12}(e^{-i\omega})| = 0$. If this is true, then we say that y does not cause x at frequency ω .

Breitung and Candelon (2006) propose testing for the null hypothesis of no causality by using

$$\Omega_{12}(L) = -\frac{g^{22}\Theta_{12}(L)}{|\Theta(L)|}, \quad (7)$$

where g^{22} is the lower diagonal element of G^{-1} and $|\Theta(L)|$ is the determinant of $\Theta(L)$. It

²Let τ be the observational frequency of the underlying data. The relationship between the observational and angular frequencies takes the form $\tau = 2\pi/\omega$.

follows that y does not cause x at frequency ω if

$$|\Theta_{12}(e^{-i\omega})| = \left| \sum_{k=1}^P \theta_{12,k} \cos(k\omega) - \sum_{k=1}^P \theta_{12,k} \sin(k\omega) \right| = 0. \quad (8)$$

$\Theta_{11,j}$ and $\Theta_{12,j}$ are the coefficients of the lag polynomials $\Theta_{11}(L)$ and $\Theta_{12}(L)$. We can write the reduce form equation for x_t as

$$x_t = \sum_{j=1}^p \Theta_{11,j} x_{t-j} + \sum_{j=1}^p \Theta_{12,j} y_{t-j} + \epsilon_{1,t}. \quad (9)$$

Necessary and sufficient conditions for no causality at frequency ω are

$$\begin{aligned} \sum_{j=1}^p \Theta_{12,j} \cos(j\omega) &= 0 \\ \sum_{j=1}^p \Theta_{12,j} \sin(j\omega) &= 0 \end{aligned} \quad (10)$$

The set of linear restrictions in Eq.(10) are tested using a simple F test where $F \sim F_{2,T-2p}$ for $\omega \in (0, \pi)$.

4 Data

4.1 Realized Volatility Construction and Behavior

The data consists of five minute intra-day spot prices for the CAC 40, DAX, and FTSE 100 equity indices, as well as the USD/EUR, USD/GBP, and GBP/EUR exchange rates over January 2nd, 2009 through December 30th, 2016.³ Equity indices are denominated in local currencies. We only consider five minute observations that occur during normal trading hours. British, French, and German equity markets are each open from 9:00AM to

³Source: Bloomberg.

5:30PM Central European Time (CET), leading to a total of 103 intra-day observations on complete trading days.⁴ Intra-day returns are defined as $r_{i,t} = 100 \times [\ln(P_{i,t}) - \ln(P_{i-1,t})]$ for $i = \{1, \dots, N\}$ returns and $t = \{1, \dots, T\}$ days.

Daily realized variance for each series calculated using the scaled sum of squared intra-day returns,

$$RV_t^d = \hat{c} \sum_{i=1}^N r_{i,t}^2, \quad (11)$$

where $\hat{c} = \sum_{t=1}^T (r_t - \bar{r})^2 / \sum_{t=1}^T RV_t$. r_t is the daily close-to-close return and $\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t$. Hansen and Lunde (2005) show that \hat{c} is a consistent estimator of $c \equiv E[IV_t] / E[RV_t]$, where IV_t is the full-day integrated variance.⁵ We scale by \hat{c} to account for the fact that realized variance is calculated only using data when equity markets are open but foreign exchange markets trade approximately 24-hours each day. Scaled daily realized volatility is then defined as $RVOL_t^d = \sqrt{RV_t^d}$.

We proceed using the log of realized volatility, $\log(RVOL_t^d)$, due to more desirable distributional properties and to avoid non-negativity issues. Exploring differences that may arise at varying observational frequencies, we define weekly realized volatility as

$$\log(RVOL_s^w) = \frac{1}{K_s} \sum_{k=1}^{K_s} \log(RVOL_{k,s}). \quad (12)$$

K_s is the number of daily observations in a given week $s = \{1, \dots, S\}$. Thus, log weekly realized volatility is the intra-week average of log daily realized volatility. Our approach here is similar to the traditional heterogeneous autoregressive (HAR) empirical methodology, which uses a five day trailing average instead of intra-week average.⁶

Table 1 presents descriptive statistics each series. Consistent with Corsi et al. (2008), each

⁴Incomplete trading days are a consequence of holiday related early closings. On such days, French and German equity markets close at 2:00PM CET while the British equity market closes at 1:30PM CET.

⁵See Martens (2002), Hansen and Lunde (2005), and Koopman et al. (2005) for further details on similar adjustments approaches.

⁶See Corsi et al. (2012) for further details.

series exhibits non-normal distributional behavior, significant ARCH effects, and volatility persistence. Bodart and Candelon (2009) show that non-normality and/or the presence of GARCH type effects do not significantly impact the power or size of the frequency domain causality test; however, the presence of outliers in the data is problematic. Meaningful differences between the traditional skewness and the outlier robust (MC-Skew) estimates suggests that we should be concerned about outliers for most series.⁷

For illustrative purposes we plot $RVOL_t^d$ in Figure 1. Several outliers are clearly visible. Notable spikes in equity and exchange rate volatility are present towards the end of 2011 (peak Greek sovereign debt crisis), mid-2015 (negative oil shock and Chinese stock market crash), and June 2016 (Brexit). Bodart and Candelon (2009) note that even a small number of outliers can affect the power of the frequency domain causality test, ultimately leading to over-rejection of the null hypothesis of no causality for a given ω . As such, we must account control for outliers before proceeding with any causal analysis.

4.2 Outlier Adjustments

We adopt the *adjusted outlier* (AO) methodology developed by Hubert and van der Veen (2008) to isolate outliers in the data; this particular approach is attractive as it accounts for outliers in a manner robust to skewness and does not assume any particular underlying distribution. Let X be a $(T \times 1)$ vector of data. Define AO_t such that

$$AO_t = AO(x_t, X) = \begin{cases} \frac{x_t - med(X)}{w_2 - med(X)}, & \text{if } x_t > med(X) \\ \frac{med(X) - x_t}{med(X) - w_1}, & \text{if } x_t < med(X) \end{cases} \quad (13)$$

⁷Introduced by Brys et al. (2004), we employ the medcouple approach to calculating outlier robust skewness estimates.

where $AO_t = 0$ only if $x_t = med(X)$ where $med(\cdot)$ is the median function and

$$w_1 = \begin{cases} Q_{0.25} - 1.5e^{-4MC}IQR, & \text{if } MC > 0 \\ Q_{0.25} - 1.5e^{-3MC}IQR, & \text{if } MC < 0 \end{cases} \quad (14)$$

$$w_2 = \begin{cases} Q_{0.75} + 1.5e^{3MC}IQR, & \text{if } MC > 0 \\ Q_{0.75} + 1.5e^{4MC}IQR, & \text{if } MC < 0 \end{cases} \quad (15)$$

MC is the outlier robust skewness measure of X , Q is the quantile function, and $IQR = Q_{0.75} - Q_{0.25}$. Note that if $MC = 0$ then w_1 and w_2 collapse to standard Tukey box plot whiskers.

AO_t is calculated for all $t \in T$ yielding a $(T \times 1)$ vector AO . Define the cutoff statistic, k , calculated from the distribution of AO as

$$k = Q_3(AO) + 1.5e^{3MC(AO)}IQR(AO). \quad (16)$$

If a given $AO_t > k$ then the underlying x_t is marked as an outlier. Table 2 displays the corresponding percentage of observations marked as outliers for each $log(RVOL)$ series. There are a small but non-zero percentage of outliers for each of the daily series. As expected, outliers are less problematic at the weekly frequency; in constructing the weekly estimates we smooth over large intra-week volatility spikes.

We replace any outliers with their m -observation centered average,

$$x_{m,j} = \left(\frac{1}{m-1} \right) \sum_{k=j-\frac{m-2}{2}}^{j+\frac{m-2}{2}} x_k, \quad (17)$$

where $j = \{t, s\}$ for daily or weekly frequencies. The range of observations (m) is set equal to 10 if the underlying data is daily and 6 if weekly. A 10-day centered average is consistent

with Bodart and Candelon (2009) for daily data. Furthermore, a wide range is necessary to minimize the potential issue of clustered outliers due to volatility persistence.

5 Empirical Results

Each daily VAR model is estimated including a constant term and set of day of the week dummies, $D = \{D_{TU}, D_W, D_{TH}, D_F\}$. Daily dummy variables are included to control for intra-week seasonality typically exhibited by international equity (Berument and Kiymaz, 2001; Kiymaz and Berument, 2003; Scharth and Medeiros, 2009) and to a lesser extent foreign exchange markets (Ke et al., 2007; Popović and Durović, 2014). Each weekly VAR model only includes a constant term. Tables 3 presents the optimal VAR lag lengths by group and frequency.⁸ The large number of lags for each daily model reflects volatility persistence in both equity and foreign exchange markets.

All frequency domain causality test results are presented by plotting the p-values corresponding to each F test for Eq.(10) over the range of $\omega \in (0, \pi)$. P-values below 0.10 imply rejection of the null hypothesis of no causality at a given ω . Evidence of causality for ω near π indicates short-run (high frequency) volatility spillover, whereas causality at ω near 0 are reflective of co-movement in the long-run (low frequency) or permanent components of market volatility (Bodart and Candelon, 2009).

5.1 Daily Volatility Spillover

Figure 2 presents the frequency domain causality test results at the daily level. For the CAC 40, volatility spillover is stronger in the equity to exchange rate direction than vice versa for both exchange rate pairs. More specifically, CAC 40 volatility spillover is significant across most tested frequencies (high, mid-range, and low). In contrast, USD/EUR and GBP/EUR volatility only cause CAC 40 volatility at mid-to-low frequencies. For the USD/EUR, we see

⁸Full results are available upon request.

marginal rejection for $\omega \in [1.908, 2.042]$ but strong evidence of causality for $\omega \in [0.686, 1.218]$ and $\omega \leq 0.360$. Disregarding the former, this translates to significant volatility spillover between approximately 5 to 9 days and beyond 17 days. Similarly, GBP/EUR volatility only causes CAC 40 volatility for low frequencies $\omega \leq 0.578$ (≥ 11 days). These findings are contrary to the evidence presented by Aloui (2007) in the post-Euro period, who only finds evidence of causality using the CCF test from the CAC 40 to the USD/EUR. Similarly, Kanas (2000) and Yang et al. (2004) only find significant volatility spillover from the CAC 40 to USD/EUR; however, their analysis pre-dates the adoption of the Euro.

As with the CAC 40, volatility spillover from the DAX to the USD/EUR and GBP/EUR exchange rates is significant at high, mid-range, and low frequencies. Volatility spillover from the USD/EUR and GBP/EUR to the DAX is primarily significant at low frequencies. For the USD/EUR, strong evidence of spillover is present for $\omega \in [0.252, 0.630]$ (10 to 25 days). Focusing on the GBP/EUR, we reject the null of no causality for $\omega \in [0.195, 0.549]$. Here, this translates to roughly 11 to 32 days. As before, our finding of bidirectional causality between the DAX and USD/EUR deviates from Aloui (2007) post-Euro analysis. In his study, he only finds significant spillover from the USD/EUR to the DAX.

Volatility spillover is again stronger in the equity to foreign exchange market direction for the FTSE 100 against the GBP/EUR and USD/GBP. However, as compared to the CAC 40 and DAX analysis, we see weaker evidence of causality at mid-range frequencies between 3 and 10 days. This behavior is more prominent for volatility spillover from the FTSE 100 to the USD/GBP exchange rate. GBP/EUR volatility significantly causes FTSE 100 volatility for $\omega \leq 1.260$ (≥ 5 days), whereas USD/GBP volatility spillover is weakly significant for $\omega \in [0.943, 1.220]$ and strongly significant for $\omega \leq 0.609$ (≥ 10 days). Leung et al. (2017) also find evidence of significant volatility spillover from the USD/GBP and GBP/EUR to the FTSE 100. However, their results suggests a weaker relationship when considering the GBP/EUR than we find here. In their analysis, they do not consider spillover

in the equity to exchange rate direction. Focusing on the pre-Euro period, Kanas (2000) and Kanas (2002) only find significant volatility spillover from the broader FT All Share index to the USD/GBP.

Overall, these results suggest bidirectional volatility spillover that is asymmetric across the frequency domain. Moreover, our findings are consistent with the portfolio balance framework. There are substantial differences in the significant causal frequencies by direction, an important quality of the relationship between equity and foreign exchange markets overlooked by past empirical work. Equity market volatility causes exchange rate volatility at high, mid-range, and low frequencies for all six equity index and exchange rate pairs. Volatility spillover in the other direction tends to be weaker but still significant. Exchange rate volatility primarily causes exchange market volatility at lower frequencies; here, volatility spillover is highly persistent except for in the (DAX, USD/EUR) and (DAX, GBP/EUR) cases. That said, volatility spillover from the USD/EUR and GBP/EUR is still significant up to approximately 25 and 32 days, respectively.

5.2 Weekly Volatility Spillover

We now focus our attention to volatility spillover occurring at a weekly observational frequency to see if market behavior deviates from what was observed at the daily frequency. In part, this approach is motivated by Corsi (2009), who remarks that traders react differently to volatility measured over various horizon lengths. More specifically, long-term volatility may matter more to short-term traders than short-term volatility. At the same time, the processing speed of information conveyed over long versus short periods of time may vary. Portfolio managers, or individual and institutional investors, may react differently to slower moving volatility signals.

Figure 3 presents the weekly volatility spillover test results for each equity and exchange rate pair. We find evidence of bidirectional causality for four of the six equity market and

exchange rate pairs, where volatility spillover is significant in the exchange rate to equity direction for each pair. Moreover, we observe the same pattern in this direction as compared to the daily analysis. On average, exchange rate volatility causes equity price volatility beyond four weeks at the high end (FTSE 100, GBP/EUR) and seven weeks on the low end (DAX, USD/EUR).

The results somewhat differ when focusing on the equity to exchange rate direction. We fail to reject the null hypothesis of no causality at any $\omega \in (0, \pi)$ for the CAC 40 and DAX to the USD/EUR exchange rate, implying that volatility spillover is insignificant. However, there is evidence of significant spillover for the CAC 40 and DAX towards the GBP/EUR exchange rate. For the former this occurs between $\omega \in [0.411, 1.491]$ (~ 4 to 15 weeks) and the latter over $\omega \in [0.669, \pi]$ (2 to ~ 9 weeks). Volatility spillover from the FTSE 100 to the GBP/EUR and USD/GBP exhibits a similar behavior as the DAX and GBP/EUR where spillover is significant up to ~ 7 and ~ 11 weeks, respectively.

These findings reinforce our daily analysis, suggesting bidirectional volatility spillover between equity and foreign exchange markets which is asymmetric in the frequency domain. The only exceptions being the unidirectional results for the (CAC 40, USD/EUR) and (DAX, USD/EUR) pairs. While the weekly volatility spillover from equity to foreign exchange markets do not exhibit the same degree of persistence as in the daily analysis, causality remains significant for up to two months on average.

6 Concluding Remarks

In this study, we analyze the nature of volatility spillovers between the CAC 40, DAX, and FTSE 100, and the USD/EUR, GBP/EUR, and USD/GBP exchange rates using a novel approach over the 2009 to 2016 period. Starting from high-frequency intra-day data, we construct realized volatility measures for each series to capture their integrated volatility

and avoid the issues that typically plague GARCH and stochastic volatility models. The main contribution of this paper is to focus on volatility spillover between these markets in the context of the frequency domain, exploring the non-linear nature of the relationship. We test for causality in the frequency domain using the Breitung and Candelon (2006) framework.

Our empirical findings are summarized as follows. Volatility spillovers between European equity and foreign exchange markets are inherently asymmetric across the frequency domain and bidirectional in nature. At the daily frequency, we observe a stronger causal relationship moving from equity to foreign exchange markets than vice versa. Volatility spillover in this direction is significant at high, mid-range, and low frequencies for all six equity index and exchange rate pairs. On the other hand, volatility spillover from the foreign exchange to equity markets is significant only at lower frequencies. Bidirectional causality is consistent with the portfolio balance approach to modeling equity prices and exchange rates. Mid-to-low frequency spillover is suggestive of portfolio re-balancing effects, whereas high-frequency spillover is more likely to be attributable to market entry/exit and contagion-type effects. Overall, our results suggest a stronger relationship in the exchange rate to equity direction than the existing literature, reflecting the importance in considering on a wide range of frequencies (especially extremely low frequencies).

Focusing on a weekly volatility, we observe the same relationship in the foreign exchange to equity market direction. However, volatility spillover is less persistent in the equity to foreign exchange market direction. Only at the weekly level do we find any evidence of unidirectional causality, and this occurs only for the (CAC 40, USD/EUR) and (DAX, USD/EUR) pairs. Overall, the weekly analysis re-affirms the daily volatility spillover findings.

Taken all together, our results reveal that the causal nature of volatility transmission between European equity and foreign exchange markets is asymmetric and non-linear in nature. Ignoring asymmetric behavior across the frequency domain may lead to inaccurate assessments of the strength and degree of persistence of volatility spillovers. As such, con-

sidering volatility spillover across the frequency domain is particularly important for risk management. One of the biggest challenges that a portfolio manager faces is to both quantify their portfolio's current level of risk but also to assess and anticipate future risk. The analysis presented in this study is particularly useful with regards to the latter. Furthermore, the rise of algorithm based high-frequency trading places a greater emphasis on predicting market volatility given that such an investment strategy tends to perform best when markets are volatile and pricing discrepancies arise.

Further analysis may want to explore what differences emerge, if any, when focusing on volatility spillover in the frequency domain amongst emerging equity and foreign exchange markets. Another potential avenue could be to extend the Breitung and Candelon (2006) model to account for HAR effects.

Figures

Figure 1: Daily Realized Volatility

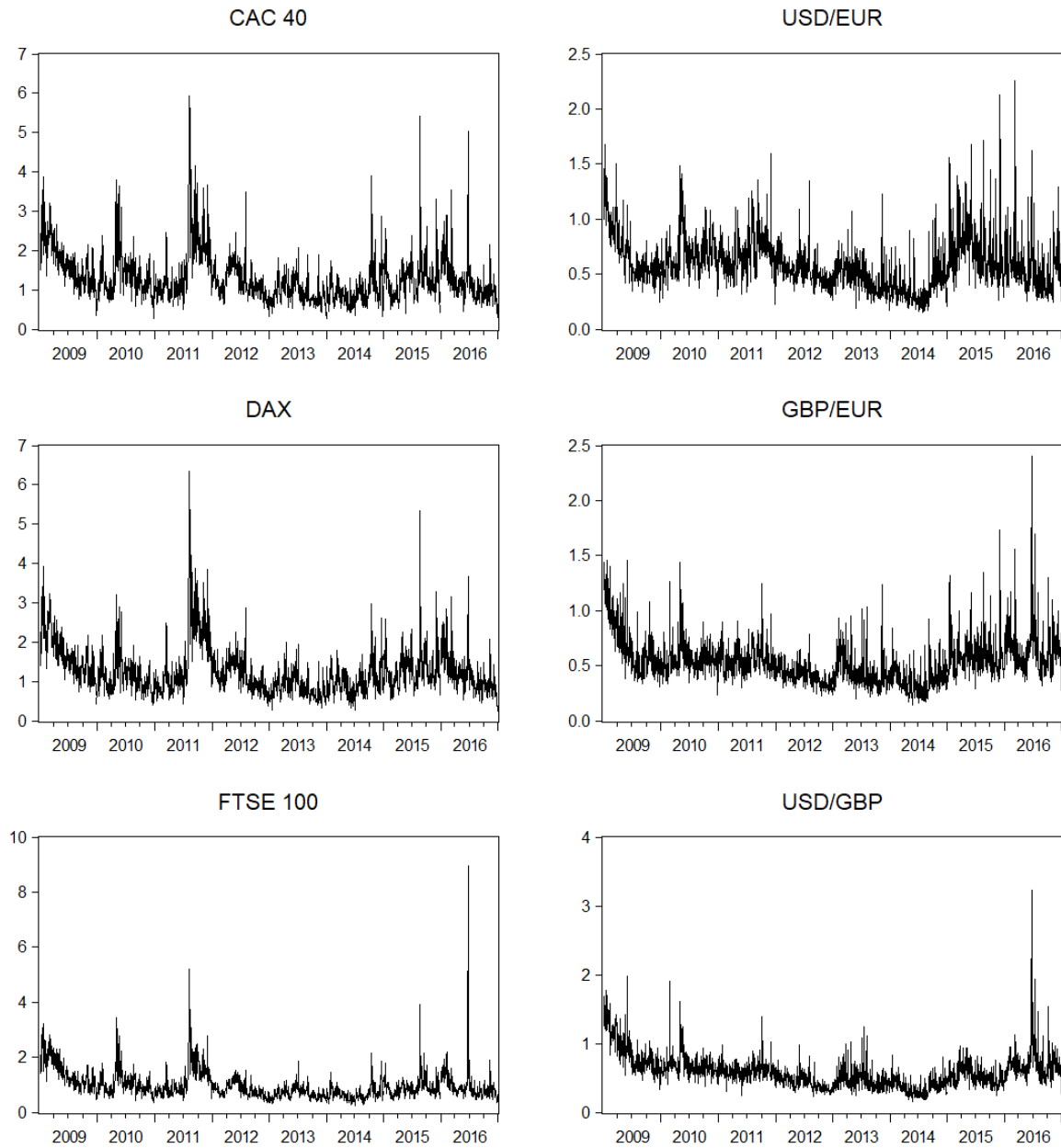


Figure 2: Daily Volatility Spillover

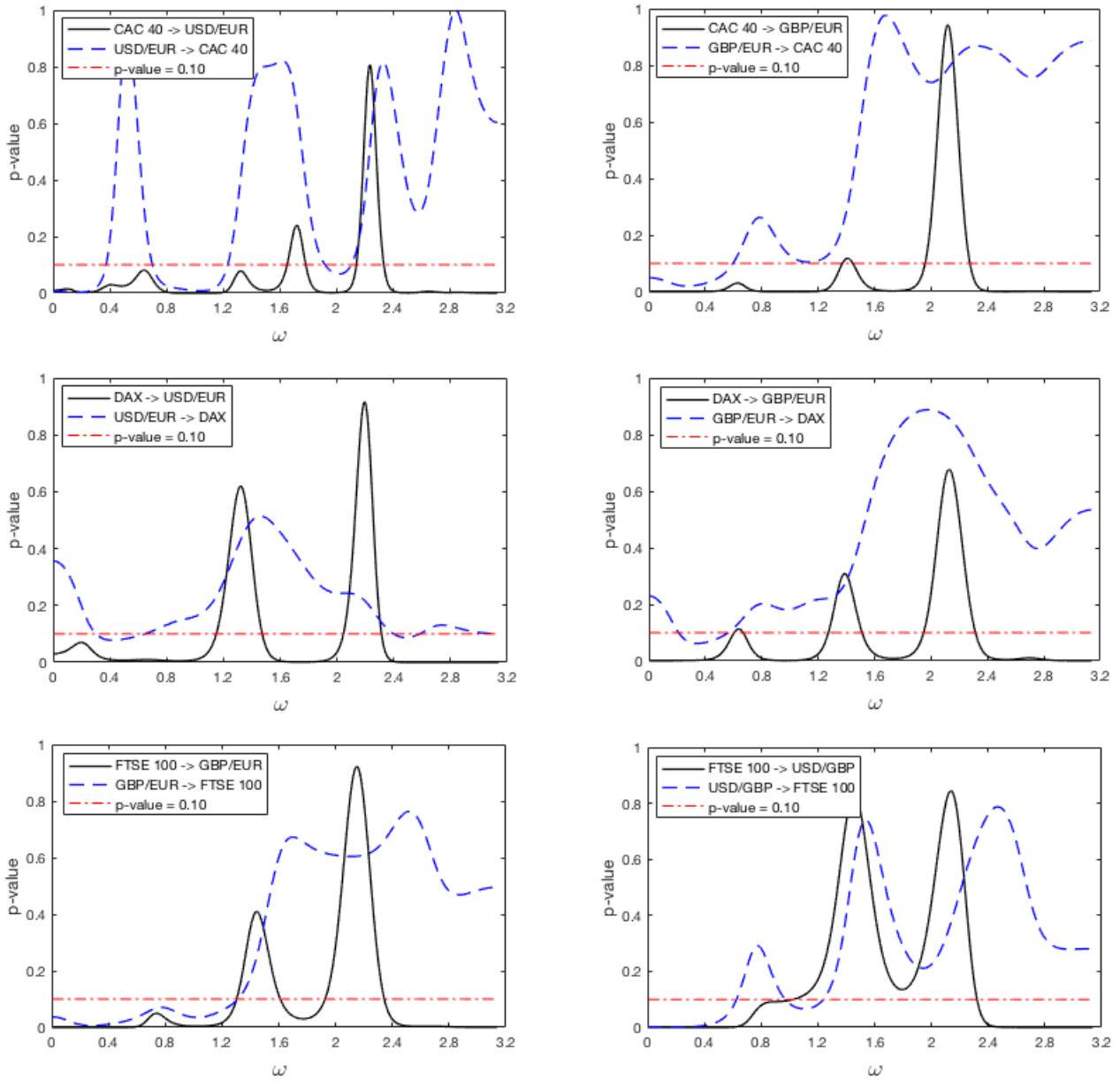
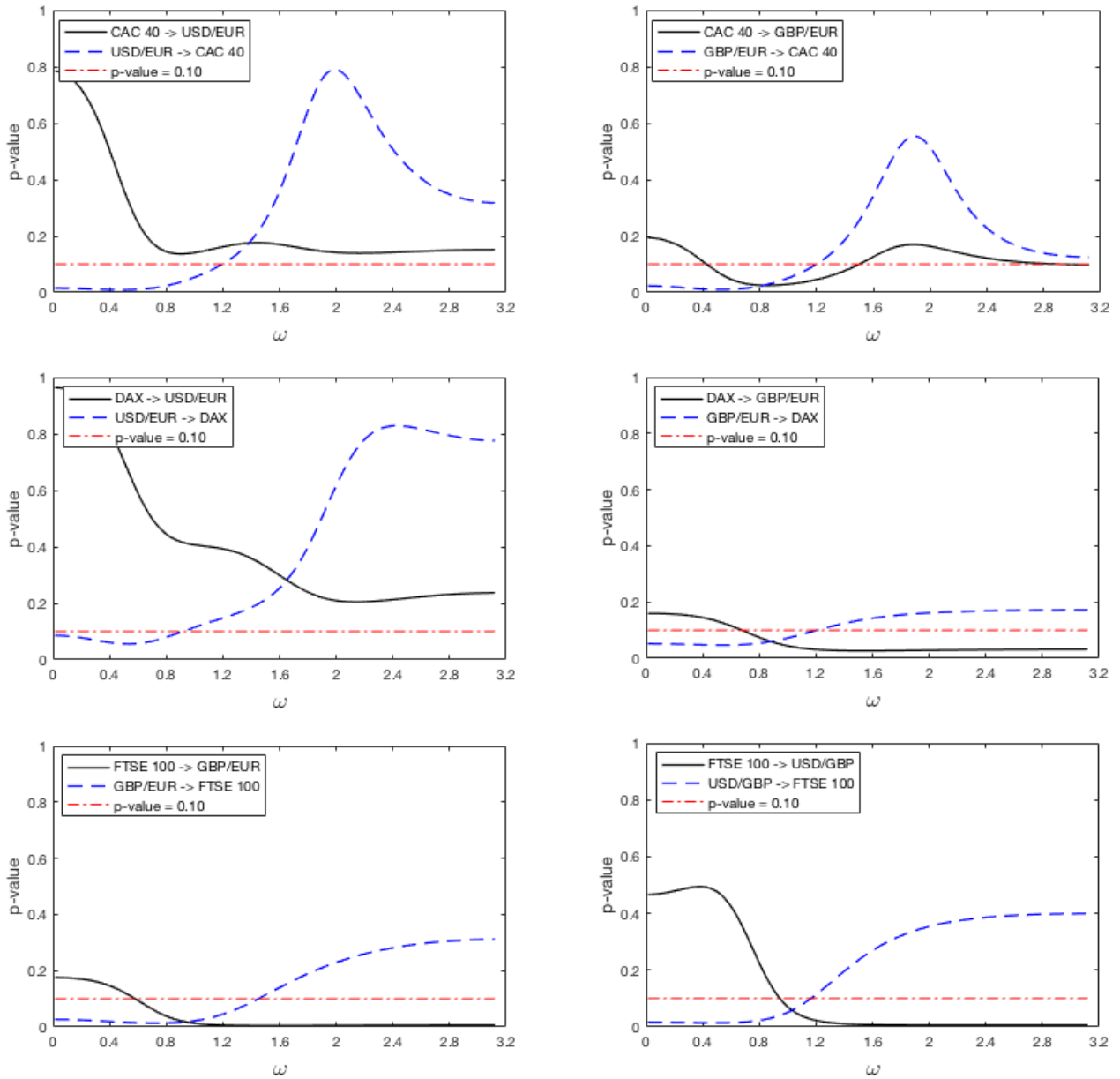


Figure 3: Weekly Volatility Spillover



Tables

Table 1: Descriptive Statistics For Log Realized Volatility

| | Daily | | | | | Weekly | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|-----------|-----------|
| | CAC40 | DAX | FTSE100 | USD/EUR | GBP/EUR | USD/GBP | CAC40 | DAX | FTSE100 | USD/EUR | GBP/EUR | USD/GBP |
| Mean | 0.147 | 0.112 | -0.139 | -0.601 | -0.693 | -0.608 | 0.142 | 0.105 | -0.142 | -0.604 | -0.695 | -0.611 |
| Median | 0.117 | 0.092 | -0.179 | -0.583 | -0.695 | -0.592 | 0.093 | 0.073 | -0.193 | -0.597 | -0.684 | -0.597 |
| Max | 1.781 | 1.849 | 2.194 | 0.815 | 0.877 | 1.170 | 1.547 | 1.578 | 1.261 | 0.259 | 0.265 | 0.433 |
| Min | -1.382 | -1.454 | -1.396 | -1.911 | -1.919 | -1.848 | -0.901 | -1.210 | -0.915 | -1.543 | -1.486 | -1.420 |
| Std. Dev. | 0.419 | 0.436 | 0.421 | 0.378 | 0.350 | 0.376 | 0.370 | 0.389 | 0.381 | 0.311 | 0.286 | 0.329 |
| Skewness | 0.214 | 0.183 | 0.546 | -0.124 | 0.168 | 0.129 | 0.326 | 0.252 | 0.592 | -0.216 | 0.131 | 0.175 |
| MC-Skew. | 0.089 | 0.050 | 0.092 | -0.065 | -0.006 | -0.086 | 0.174 | 0.103 | 0.142 | -0.007 | -0.072 | -0.085 |
| Kurtosis | 3.407 | 3.284 | 3.676 | 3.342 | 3.641 | 3.510 | 3.094 | 3.390 | 3.243 | 3.186 | 3.500 | 3.465 |
| S.W. | 0.995*** | 0.997*** | 0.983*** | 0.997*** | 0.994*** | 0.995*** | 0.990*** | 0.991** | 0.975*** | 0.994* | 0.991** | 0.987*** |
| LBQ(5) | 4554*** | 4763*** | 5336*** | 3514*** | 3584*** | 5032*** | 794*** | 887*** | 997*** | 1099*** | 1165*** | 1409*** |
| ARCH(5) | 625*** | 858*** | 757*** | 554*** | 548*** | 870*** | 138*** | 164*** | 196*** | 226*** | 228*** | 274*** |
| ADF | -4.295*** | -4.380*** | -5.139*** | -3.459*** | -3.837*** | -3.529*** | -4.337*** | -3.876*** | -4.254*** | -2.951** | -4.380*** | -3.765*** |
| Obs. | 2050 | 2033 | 2021 | 2050 | 2050 | 2050 | 417 | 417 | 417 | 417 | 417 | 417 |

Notes: Daily USD/EUR, GBP/EUR, and USD/GBP statistics are presented for each series paired against the CAC 40 but do not differ substantially for the other sample sizes (available upon request); MC-Skew. is the outlier robust medcouple skewness measure (Bryns et al., 2004); S.W. is the Shapiro-Wilks statistic; Augmented Dickey-Fuller (ADF) unit root tests are performed including a constant term with optimal lag length determined by AIC; * p-val < 0.01, ** p-val < 0.05, *** p-val < 0.01.

Table 2: Percentage of Outliers

| Daily | | | | | |
|---------------|------------|-----------------|----------------|----------------|----------------|
| CAC 40 | DAX | FTSE 100 | USD/EUR | GBP/EUR | USD/GBP |
| 0.59% | 0.25% | 0.10% | 0.10% | 0.05% | 0.78% |
| Weekly | | | | | |
| CAC 40 | DAX | FTSE 100 | USD/EUR | GBP/EUR | USD/GBP |
| 0.00% | 0.24% | 0.00% | 0.00% | 0.00% | 0.96% |

Notes: Daily USD/EUR, GBP/EUR, and USD/GBP outlier percentages correspond to each series paired against the CAC 40 but do differ substantially for the other sample sizes (available upon request).

Table 3: Optimal VAR Lag Structure

| | Series | USD/EUR | GBP/EUR | USD/GBP |
|----------|---------------|----------------|----------------|----------------|
| CAC 40 | Daily | 15 | 10 | |
| | Weekly | 4 | 4 | |
| DAX | Daily | 9 | 10 | |
| | Weekly | 4 | 3 | |
| FTSE 100 | Daily | | 10 | 10 |
| | Weekly | | 3 | 3 |

Notes: Daily VARs are estimated including a constant term and a set of exogenous day of the week dummy variables while the weekly VARs only include a constant term; Optimal lag length determined by AIC.

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