Empirical Evidence of Cross-country Monetary Transmissions: A Panel Factor-augmented Vector Autoregression Analysis

Hiroshi Morita[1]

The City University of New York

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Abstract

This paper explores an econometric model of cross-country monetary transmission mechanism. We particularly examine the effect of the U.S. monetary policy shock on the other Group of Seven (G-7) countries. In this paper, we develop a panel version of the factor-augmented vector autoregressive (FAVAR) model, allowing both common and country-specific unobservable factors. We also allow interdependency among the country-specific factors to produce the comovement of business cycles in the G-7 countries. The findings are that: (1) the existence of such interdependency is statistically significant. (2) This interdependency reduces the magnitude of U.S. business cycle. (3) After the contractionary U.S. monetary shock, Japan's output is affected most, following by Anglo-Saxon countries the UK and Canada. The continental European countries, Germany, France, and Italy, are least affected among them. (4) The results are consistent with international consumption risk sharing. (5) Our 2-country DSGE model shows the evidence of such risk sharing, and also suggests that the comovement of business cycles might be caused as a result that foreign central bank reacts to the change in home monetary policy and the exchange rate.

JEL classification: C11; C5; E5; F3; F4

Keywords: cross-country monetary transmission; factor-augmented vector regression; business cycle comovement; international consumption risk sharing

[1]Department of Economics, The City University of New York, 365 Fifth Avenue, New York, NY 10016-4309, U.S.A. Email: hmorita@gc.cuny.edu, Tel: +1-212-817-8255, Fax: +1-212-817-1514

1 Introduction

The conventional view among policy makers is that, at least in the short run, monetary policy can influence the real economy. In fact, recent empirical evidence supports the seminal work by Friedman and Schwartz (1963) that the effect of monetary policy on the real economy may last for at least two years (See Bernanke and Mikov, 1998). However, there is no consensus over how much monetary policy exerts its influence. The main reason for disagreement is the differences in model specifications adopted in these analyses. Since Bernanke and Blinder (1992) and Sims (1992), many researchers employed standard vector autoregression (VAR) to identify and measure the effects of monetary policy innovation on macroeconomic variables. However, a different identification strategy leads to quite different inference in the analysis.

A more critical problem with the existing studies is that they are highly constrained by the dimension of the VAR system and, therefore, can consider only a limited number of variables affecting monetary policy decisions. Thus, the set of limited control variables considerably varies from one study to another. It is a well-known fact that central banks consider a large set of variables to assess and forecast economic conditions before implementing their monetary policies. Moreover, monetary policy transmission occurs through various channels, such as the interest rate channel, the exchange rate channel, and the credit channel. Thus, the ”omitted variable bias” might lead to biased inference, such as the ”price puzzle” (Sims, 1992) that a contractionary monetary policy shock is followed by a rise in price level instead of a fall. For this reason, the factor-augmented vector autoregressive (FAVAR) has been gaining its popularity in analyzing monetary transmission since Stock and Watson (2005a) and Bernanke, Boivin, and Eliasz (2005). Their works reduce over a hundred variables into a couple of unobservable factors to deal with a curse of dimensionality. However, most of literature concentrates on a single country or regional analysis.

In this paper, we explore a cross-country monetary transmission mechanism. We particularly examine whether the interdependency among the G-7 economic variables plays a significant role in the cross-country monetary transmission of a monetary policy shock originating in the United States. Following the terminology of Forbes and Rigobon (2002), contagion is due to significant changes in shocks, such as financial crises, while interdependency simply refers to correlations among countries' macroeconomic variables. This paper focuses the latter.

This paper has four contributions. First, we develop a general class of panel-FAVAR model. This is a new methodology that is more suitable for analyzing cross-country effects among the G-7 countries than a standard VAR or a non-panel FAVAR, since it allows modeling a large number of variables and cross-country variations simultaneously. When examining the monetary transmission mechanism in an open economy context, leaving out the interdependency as majority of literature does, could distort results severely. Second, we systematically examine the interdependency among the country-specific unobservable factors, which produces the comovement of business cycles among the G-7 countries widely documented in literature. We test whether the existence of such interdependency is statistically significant. Third, we examine the robustness of this model, using Bayesian marginal density of data, to check whether the number of factors is optimized. This is a point that has not been paid attention in literature, yet can crucially affect the results. Last, we extend a 2-country of dynamic stochastic general equilibrium (DSGE) of Fererro, Gertler, and Svensson (2008), and compare it with our empirical results. We show that the results of our DSGE model are consistent with our panel-FAVAR analysis in terms of business cycle comovement.

Our findings are: (1) the existence of interdependency among country-specific factors is statistically significant. This interdependency plays an important role in our cross-country monetary transmission mechanism. (2) Such interdependency reduces the magnitude of U.S. business cycle. This suggests that most of VAR or FAVAR literature of the U.S. monetary transmission mechanism without interdependency across countries, might overstate the effect on the U.S. real economy. This view is consistent with the argument of Sims and Zha (2006) among many others that the effect of monetary policy on the real economy might be very little. (3) As a result of interdependency, the effects on the other G-7 countries' real economy are similar. However, there are differences in magnitude. For example, after the contractionary U.S. monetary shock, Japan's output is affected most, following by Anglo-Saxon countries the UK and Canada. The continental European countries, Germany, France, and Italy, are least affected among them. (4) The consumption in each country falls less than its own output. This result is consistent with international consumption risk sharing documented in many existing literature. (5) Our 2-country DSGE model shows the evidence of such risk sharing, and also suggests that the comovement of business cycles might be caused as a result that the foreign central bank reacts to the change in the home-country monetary policy and the exchange rate.

We organize this paper as follows: Section 2 discusses literature review, Section 3 explains methodology, Section 4 describes data, Section 5 provides posterior simulation, Section 6 reports statistical inference, Section 7 develops a 2-country DSGE model, and Section 8 concludes.

2 Literature Review

Since the works by Bernanke and Blinder (1992) and Sims (1992), the VAR model has been the workhorse of monetary transmission literature among researchers and policy makers. However, recently, the limitations of a typical VAR system came to the front, in that it is severely restricted by relatively the small number of variables in order to conserve degrees of freedom. Central banks follow a large number of indicators to monitor the monetary and financial pressures in the markets when they implement the monetary policy. Moreover, the monetary transmission may occur through various channels: the interest-rate channel (Sims, 1992, Leeper, Sims, and Zha, 1996, Sims and Zha, 1998, 2005); the exchange rate channel (Eichenbaum and Evans, 1995, Kim and Roubini, 2000, Kim, 2001, Canova, 2005, Scholl and Uhlig, 2006); and the non-neoclassical credit channel (Bernanke, Gertler, and Gilchrist, 1999). Thus, a limited-dimension VAR creates an "omitted variable bias", which might lead to biased inference, creating undesirable puzzles, e.g. the "price puzzle" where a contractionary monetary policy is followed by a rise in price level instead of a fall (Sims, 1992). Leeper, Sims, and Zha (1996) increased the number of variables by applying Bayesian inference, but their VAR system still contains less than 20 variables.

In response to this limitation, the FAVAR model has been gaining in popularity especially in analyzing monetary transmission in a single country framework since Stock and Watson (2005a) and Bernanke, Boivin, and Eliasz (2005). In their works, just a couple of unobservable factors represent over one hundred U.S. macroeconomic variables. The FAVAR models can significantly reduce the degree of the omitted-variable bias that would be present in the standard VAR model. Thus, the FAVAR model has much richer information than the standard VAR model. Furthermore, many macroeconomic variables, such as gross domestic product and price index, contain measurement errors. Since the unobservable factors in the FAVAR model might be able to capture these errors, this model also represents a solution to the measurement-error bias issue (Bernanke, Boivin, and Eliasz, 2005). With a panel FAVAR model, Boivin, Giannoni, and Mojon (2008) examine the effect of the German monetary policy shocks on the other euro member countries in the context of a cross-country monetary transmission analysis. Their main finding is that the introduction of the euro currency did change the monetary transmission mechanism within the euro area and reduced the magnitude of the business cycles. However, their study employs euro common unobservable factors only and does not consider country-specific unobservable factors.

The FAVAR approach has also been adopted in the business cycles literature. Kose, Otrok, and Whiteman (2008) utilize one common and one country-specific unobservable factors for each country. In their model, each country-specific factor depends on its own lags. We extend this model by allowing multiple common factors and country-specific factors. Later, we formally test to find out the optimized number of factors. We also let the country-specific factors in our model to depend on the lags of the other country's country-specific factors as well as their own lags, and therefore create the interdependency among the G-7 economies. This interdependency plays an important role in our cross-country monetary transmission analysis and distinguishes it from all the other studies examining the same question. Since interdependency refers to correlations among countries' macroeconomic variables, it can generate business cycles comovement across countries. We also test whether the existence of such interdependency is statistically significant.

The conventional view in international macroeconomics states that the degree of interdependency increases with high volumes of international trade and capital flows. For example, international trade linkages lead to spillovers across countries both in demand and supply sides. Through these spillover effects, stronger international trade linkages can lead to a higher correlation in business cycles across countries. Financial linkages can also lead to a similar result in the degree of interdependency. For example, after a positive productivity shock in home country, foreign investors would benefit from capital gains in this country, leading to the wealth effect in the foreign country as well. However, stronger trade linkages may also affect interdependency negatively. If stronger trade linkages are associated with increased inter-industry specialization across countries, and industry-specific shocks are dominant, then the degree of interdependency is expected to decrease (Frankel and Rose, 1998).

Paralleling with the theoretical arguments, empirical evidence is also ambiguous as to global linkages affecting the international comovement of business cycles. Kalemli-Ozcan, Sørensen, and Yosha (2003) show that trade linkages mitigate cross-country business cycle comovements as they stimulate specialization of production. However several studies support a positive relation between global linkages and international business cycles. Imbs (2004a, 2004b) finds that the extent of financial linkages, sectoral similarity, and the volume of intra-industry trade all have a positive impact on business cycle correlations. However, Baxter and Kouparitsas (2005) and Otto, Voss, and Willard (2003) document that international trade is the most important transmission channel in business cycles. The results by Kose, Prasad, and Terrones (2007) provide evidence showing that both trade and financial linkages have a positive impact on cross-country output correlations.

Empirical literature is unable to provide an unequivocal support for increased global interdependency. Heathcote and Perri (2004) and Stock and Watson (2005b) find that the correlation of business cycles between the United States and the other G-7 countries has fallen in the second half of the last century. However, using a much longer sample of annual data (1880-2001), Bordo and Helbling (2003) document that the degree of interdependency across industrialized countries has increased over time. Using the FAVAR model, Kose, Otrok, and Whiteman (2008) find that the degree of comovement of business cycles across the G-7 countries has increased after 1986. Using data of 106 countries, Kose, Otrok, and Prasad (2012) find that business cycles have converged over time within industrial countries and within emerging economies, but decoupled between these two groups of countries. They argue that one of reasons might be much higher levels of financial integration in industrial countries than emerging economies.

A major component of output, and therefore possible contributor to international business cycles, is the correlation of consumption across countries. Research has uncovered what is now known as the "consumption correlation puzzle", which suggests that consumption growth is less correlated than output growth, whereas it should be highly correlated if business cycles are highly correlated. A promising explanation for this puzzle might be international consumption risk sharing models developed by Backus, Kehoe and Kydland (1992) and Obstfeld and Rogoff (1996). The models show that, within a given country, consumption growth should be less volatile than output growth, as risk sharing should allow to smooth consumption in the face of output shocks. Initial empirical works testing of this theory (e.g., Backus, Kehoe and Kydland, 1995, Lewis, 1996) find the degree of risk sharing to be very low or even negative. Various extensions of this empirical work by Ambler et al. (2003), Obstfeld (1994), Sørensen and Yosha (1998), and Stockman and Tesar (1995), have largely confirmed the limited degree of risk sharing.

More recent literature has documented an increase of risk sharing since the 1990s. Artis and Hoffmann (2006) and Sørensen et al. (2007) estimate that risk sharing among industrial countries has increased steadily throughout the 1990s and early 2000s. A similar increase in risk sharing is found in Kose et al. (2007), but they also show that this result does not hold for emerging market economies. Giannone and Reichlin (2006) find an increase in risk sharing since the early 1990s within the euro area. Sørensen et al. (2007) attribute it mainly to the growing internationalization of portfolios due to a decline in home bias, in particular on equity holdings. Fratzscher and Imbs (2007) confirm this finding also for bilateral portfolio holdings.

3 Methodology

3.1 Class of Model

We allow our model to have multiple country-specific factors as well as multiple world-common factors. For the ith country $(i = 1, 2, \cdots, N)$:

$$X_{i,t} \quad = \quad \lambda^{(f)} f_t + \lambda_i^{(fi)} f_{i,t} + \lambda^{(y)} y_t + \lambda_i^{(yi)} y_{i,t} + e_{i,t}$$

$X_{i,t}$ is observable variables, f_t is world common unobservable factors at a time t, $f_{i,t}$ is the ith country-specific unobservable factors at a time t, y_t is common observable variables at a time t, $y_{i,t}$ is the ith country-specific observable policy variables at a time t. $\lambda^{(f)}$, $\lambda^{(fi)}$, $\lambda^{(y)}$, and $\lambda^{(yi)}$ are the loading matrices. In a matrix form, we rewrite the preceding equation as:

$$\begin{bmatrix} X_{1,t} \\ \vdots \\ X_{N,t} \end{bmatrix} = \left[\begin{array}{c|ccc} \lambda^{(f)} & \lambda_1^{(fi)} & & O \\ & & \ddots & \\ & O & & \lambda_N^{(fi)} \end{array} \right] \begin{bmatrix} f_t \\ \hline f_{1,t} \\ \vdots \\ f_{N,t} \end{bmatrix} + \left[\begin{array}{c|ccc} \lambda^{(y)} & \lambda_1^{(yi)} & & O \\ & & \ddots & \\ & O & & \lambda_N^{(yi)} \end{array} \right] \begin{bmatrix} y_t \\ \hline y_{1,t} \\ \vdots \\ y_{N,t} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ \vdots \\ e_{N,t} \end{bmatrix} \quad (1)$$

The unobservable factors evolves in the following VAR model:

$$\begin{bmatrix} f_t \\ f_{1,t} \\ \vdots \\ f_{N,t} \end{bmatrix} = \sum_{l=1}^{p} \phi_l \begin{bmatrix} f_{t-l} \\ f_{1,t-l} \\ \vdots \\ f_{N,t-l} \end{bmatrix} + u_{i,t} \quad (2)$$

We have to stress that there are no restrictions the parameters ϕ_l to produce interdependency among the world-common and country-specific unobservable factors. Now, we turn to derive the first-order representation of Equation (1) and (2). Define $X_t \equiv (X'_{1,t},\ X'_{2,t},\ \cdots, X'_{N,t})'$, $F_t \equiv \{f'_t,\ f'_{1,t},\ f_{2,t},\ \cdots, f'_{N,t}\}'$, $Y_t \equiv \{y'_t,\ y'_{1,t},\ y_{2,t},\ \cdots, y'_{N,t}\}'$, $\lambda_f \equiv \{\lambda^{(f)},\ \lambda_1^{(fi)},\ \lambda_2^{(fi)},\ \cdots,\ \lambda_N^{(fi)}\}$, $\lambda_y \equiv \{\lambda^{(y)},\ \lambda_1^{(yi)},\ \lambda_2^{(yi)},\ \cdots,\ \lambda_N^{(yi)}\}$, $E_t \equiv \{e'_{1,t},\ e'_{2,t},\ \cdots, e'_{N,t}\}'$, and $U_t \equiv \{u'_{1,t},\ u'_{2,t},\ \cdots, u'_{N,t}\}'$. We can rewrite Equation (1) and (2) as:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \lambda_f & \lambda_y \\ O_{N_y \times N_f} & I_{N_f} \end{bmatrix} \begin{bmatrix} F_t \\ Y_t \end{bmatrix} + \begin{bmatrix} E_t \\ O \end{bmatrix}$$
$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \sum_{l=1}^{p} \phi_l \begin{bmatrix} F_{t-l} \\ Y_{i,t-l} \end{bmatrix} + \begin{bmatrix} U_t \\ O \end{bmatrix}$$

The error term E_t and U_t follows *i.i.d.* normal distribution $\mathcal{N}(0, Q)$ and $\mathcal{N}(0, R)$, respectively. We define $\mathbb{X}_t \equiv (X_t', Y_t')'$, $\lambda \equiv (\lambda_f, \lambda_y)$, $\mathbb{E}_t \equiv (e_t', O_{N_y \times 1}')'$, $\mathbf{f}_t \equiv (F_t', Y_t')'$, $\phi \equiv (\phi_1, \phi_2, \cdots, \phi_K)'$, a $K \times PK$ matrix, we can further rewrite preceding two equations as:

$$\begin{aligned} \mathbb{X}_t &= \lambda \mathbf{f}_t + \mathbb{E}_t \\ \mathbf{f}_t &= \sum_{s=1}^{p} \phi_s \mathbf{f}_{t-s} + u_t \end{aligned}$$

We define $\mathbb{F}_t \equiv (\mathbf{f}_t, \mathbf{f}_{t-1}, \cdots, \mathbf{f}_{t-p+1})$, and $\mathbb{U}_t \equiv (u_t, 0, 0, \cdots, 0)$. Furthermore, we define $pK \times pK$ companion matrix as:

$$\Phi \equiv \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_p \\ I_{(p-1)K} & O_{(p-1)K \times 1} & \cdots & O_{(p-1)K \times 1} \end{bmatrix}$$

and we also define:

$$\begin{aligned} \mathbb{Q} &\equiv \begin{bmatrix} Q & O_{(p-1)K \times (p-1)K} \\ O_{(p-1)K \times K} & O_{(p-1)K \times (p-1)K} \end{bmatrix} \\ \mathbb{R} &\equiv \begin{bmatrix} R & O_{(p-1)K \times (p-1)K} \\ O_{(p-1)K \times K} & O_{(p-1)K \times (p-1)K} \end{bmatrix} \end{aligned}$$

and $\Lambda \equiv (\lambda, 0, \cdots, 0)'$, $N \times pK$ matrix. Finally, we obtain the first-order representation of FAVAR as follows:

$$\mathbb{X}_t = \Lambda \mathbb{F}_t + \mathbb{E}_t, \quad \mathbb{E}_t \sim \mathcal{N}(0, \mathbb{Q}) \tag{3}$$

$$\mathbb{F}_t = \Phi \mathbb{F}_{t-1} + \mathbb{U}_t, \quad \mathbb{U}_t \sim \mathcal{N}(0, \mathbb{R}) \tag{4}$$

We estimate the parameters $\theta \equiv \{\Lambda, R, \phi, Q\}$ where $\phi = \{\phi_1, \phi_2, \cdots, \phi_p\}$ along with the unobservable factors in $\mathbb{F}^T \equiv \{\mathbb{F}_1, \cdots, \mathbb{F}_T\}$.

3.2 Prior and Posterior Distributions

We follow most of FAVAR literature in specifying prior distribution, such as Bernanke, Boivin, and Eliasz (2005), and Boivin, Giannoni, and Mojon (2008) as follow:

$$Q_u \sim \mathcal{IW}(S_0, \nu_0) \tag{5}$$

$$vec(\phi)|\, Q_u \sim \mathcal{N}(\phi_0, Q_u \otimes N_0^{-1}) \tag{6}$$

In the nth equation, the prior is:

$$R_{nn} \sim \mathcal{IG}(\delta_0/2, \eta_0/2) \tag{7}$$

$$\Lambda_n|\, R_{nn} \sim \mathcal{N}(\Lambda_{n,0}, R_{nn} M_{n,0}^{-1}) \tag{8}$$

Using prior above, we derive the posterior distributions. See more details in Appedix B.

3.3 Identification Strategy

This paper employs the sign restriction of Uhlig (2005). For FAVAR model, Ahmad and Uhlig (2009) examines the sign restrictions, compared with a standard Cholesky decomposition. They found that their sign restriction approach outperforms the competing Cholesky identification, and they argued that it is robust across different subsamples and monetary regimes avoiding anomalies present in Cholesky identification, such as price puzzle described in Sction 2. In this paper's analysis, since it is almost impossible to rank the country-specific factors among the G-7 countries, it is not reasonable to use Cholesky in Equation (4). There is simply no reasonable explanation which country-specific factors are not contemporaneously affected by others. Thus, we choose the sign restrictions in our identification strategy.

4 Data

The sample period is 1985Q1:2010Q4. We show the description of data in Appendix A. In order to ensure stationary, the data transformations, such as level, difference, logarithmic, percentage change, or difference in percentage change, are made in the exact same fashion as Stock and Watson (2005a), and Bernanke, Boivin, and Eliasz (2005). See more details in Appendix A. All data are obtained from IMF International Financial Statistics. Both nominal and real exchange rates are effective (which are weighted by trade volumes), and an increase in the exchange rate represents a home currency appreciation. We use oil price for a common observable variable (y_t), and Federal funds rate and overnight call rate for each country's policy variable ($y_{i,t}$).

5 Posterior Simulation

We estimate the posterior moments of the parameters in the model in (1), using the Gibbs sampler as a posterior simulation described in Section 3.3. In this simulation, each draw is iterated 24,000 times and the first 4,000 iterations are burned in. Then, we choose a thinning parameter of 2 so that only every second draw was kept after the burn-in in order to reduce potential autocorrelation of the sequence in the Gibbs sampler. For the parameters in the prior distributions, we set $S_0 = O$, $\nu_0 = 0$, $\phi_0 = 0$, $N_0 = O$, $\delta_0 = 6$, $\eta_0 = 10^{-3}$, $\Lambda_n = O$, and $M_{n,0} = I$. Note that the choice of S_0 and ν_0 as they are canceled out in the posterior.

6 Statistical Inference

In this section, we present the impulse response functions of our panel FAVAR model with interdependency among the G-7 countries. We show the estimated common unobservable factors and country-specific ones in Figure 1 and 2. We choose one common factor and three country-specific factors for each country. Later, we show that these numbers of factors provides the best fit to our model. We present the impulse response functions in Figure 3 to 9. We normalize the contractionary US monetary shock to a 10-basis-point innovation in the Federal funds rate.

6.1 Impulse Response Functions

We begin with the responses of US macroeconomic variables in Figure 3. After the contractionary U.S. monetary policy shock, both output and consumption fall by 0.13% and 0.22%, respectively. The immediate effect in investment is ambiguous. Both nominal and real effective exchange rates initially rise (a home currency appreciation), and then fall (a home currency depreciation) after one to two quarter. This is in the line of the overshooting of Dornbusch (1976) although he refers to bi-lateral exchange rates. Export falls more than import does. Overall, net export improves by 0.30% one quarter after the shock.

According to Figure 4 to 9, the contractionary monetary policy in the U.S. are transmitted to falls in outputs of the other G-7 countries: a fall by 0.20% for Japan, 0.09% for Germany, 0.14% for U.K., 0.10% for France, 0.09% for Italy, and 0.17% for Canada, although the immediate effect on Germany's output is not statistically significant. These results show that Japan's real economy is most affected by the U.S. monetary policy. Then, it is following by Canada and UK that are Anglo-Saxon countries. The similarity in the movement of macroeconomic variables among the U.S., the UK, and Canada are reported in some literature, and called the Anglo-Saxon effect. Our model's results are consistent with this effect. The continental Europe's real economy is also affected by this policy shock. Yet, its magnitudes are the least among the G-7 countries.

It is similar story in the responses of their consumptions. There are also falls in these countries' consumptions: 0.17% for Japan, 0.09% for the UK, 0.09% for France, 0.07% for Italy, while ambiguous for Germany. An important thing we would like to emphasize is that, for these countries excluding the U.S., their consumptions falls less than their output do. This result is consistent with international risk sharing documented in many existing literature.

6.2 Robustness Check

In this section, we check the robustness of the model. We test whether the numbers of the common and country-specific unobservable factors are optimized in terms of the marginal density of data (MDD). Using MDD is a common practice to find an optimized number of unobservable states in Markov switching literature such as Sims and Zha (2006). However, to our best knowledge, none of Bayesian FAVAR literature concerning monetary transmission mechanism uses MDD to optimize the number of unobservable factors.

Since there exists no analytical solution of MDD in this model, we apply the approximation method of Chib (1997) to our analysis. See more details in Appendix B. We reports MDDs in Table 1. The results show that the model with one common factor and three country-specific factors has the highest MDD -100129, indicating that it is the best model. Although their model is non-panel, Bernanke, Boivin, and Eliasz (2005) choose four factors for the U.S. This paper officially tests the number of factors, and obtains one for the common factor and three for the country-specific ones, thus four for the total factors for each G-7 country.

6.3 Role of Interdependency

In this section, we examine whether interdependency among the country-specific factors in our model are statistically significant. As we did in the preceding section, we use MDD in this test. We conduct a test to impose the restriction on parameter Φ in Equation (4). More specifically, to test the restriction of no interdependency, we set zeros in Φ corresponding to the coefficients of the lags of the other country's country-specific factors. Thus, our model reduces to the model of Kose, Otrok, and Whiteman (2008). In this test, we use the optimized number of factors found in the preceding section, *i.e.*, one common factor and three country-specific factors. We report MDDs in Table 2. The results show that the model with interdependency has the MDD of -100129 while the one without interdependency has -343210. This indicates that the model with interdependency is better model than the one without it. Thus, we conclude that interdependency among country-specific factors plays a significant role in our cross-country monetary analysis mechanism among the G-7 economies.

Using the model without interdependency, we also present the responses of U.S. macroeconomic variables in Figure 10. Compared with the model with interdependency in Figure 3, we can see some clear distinctions between them. The initial shock to U.S. output and consumption are less by .07 and .06 percentage point respectively in the model with interdependency than in without it. This result implies that the existence of our model's interdependency mitigates the effect of U.S. monetary policy shock on U.S. real economy. Thus, this evidence suggests that most of VAR or FAVAR literature concerning U.S. monetary transmission mechanism without using other countries' variables, might overstate the effect on the real economy. This view is consistent with the argument of Sims and Zha (2006) among many others that the effect of monetary policy on the real economy might be very little.

6.4 Euro Effect

In this section, we briefly examine whether the introduction of euro currency after 1999 changes our results. We present the impulse responses of G-7 macroeconomic variables using the subsample beginning 1999Q1 in Figure 11 to 17. Although this sample period might be relatively short for this analysis, we see no clear difference between our full sample beginning 1985Q1 and this euro period. However, there is a marginal difference in response of outputs and investments in the euro area, Germany France, and Italy. This might suggest that the introduction of common currency in this area contributes to the reduction in their business cycles due to the U.S. monetary shock as some literature suggest, e.g., Boivin, Giannoni, and Mojon (2008). However, our results show the differences are marginal, and we leave further researches to the outside of this paper. Our main point here is that the introduction of currency euro does not critically affect this paper's results.

7 2-country Dynamic Stochastic General Equilibrium Model

In this section, we examine a 2-country dynamic stochastic general equilibrium (DSGE) model compared with our panel-FAVAR model. We extend the model of Ferrero, Gertler, and Svensson (2008) (FGS08 henceforth) by allowing the habit formation in consumption dynamics so that its responses to shocks shape smoother as in many of DSGE studies. Unlike their model, we assume that the foreign country's monetary policy also responds to the changes in the exchange rate and the home country's interest rate, while home country's (U.S.) monetary policy respond only to its own inflation and output as in Clarida, Gali, and Gertler (1998). This monetary policy would be reasonable for non-U.S. G-7 countries because those countries' central bank might react to a change in the U.S. monetary policy as well as a change in the exchange rate.

7.1 Model

(1) Households:

Aggregate consumption (C_t) consists of tradable-good consumption ($C_{T,t}$) and non-tradable-good consumption ($C_{N,t}$) at a time t, defined as:

$$C_t \equiv \frac{C_{T,t}^{\gamma} C_{N,t}^{1-\gamma}}{\gamma(1-\gamma)} \tag{9}$$

Furthermore, tradable-good consumption consists of home-made-good consumption ($C_{H,t}$) and foreign-made-good consumption ($C_{F,t}$) at a time t, defined as:

$$C_{T,t} \equiv \left[\alpha^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{10}$$

Households maximize the following utility function:

$$E_t \left\{ \sum_{s=0}^{\infty} \theta_{t+s-1} \left[\frac{(C_{t+s} - \omega C_{t+s-1})^{1-\sigma}}{1-\sigma} - \left(\int_0^{\gamma} \frac{L_{H,t+s}(f)^{1+\varphi}}{1+\varphi} df + \int_{\gamma}^{1} \frac{L_{N,t+s}(f)^{1+\varphi}}{1+\varphi} df \right) \right] \right\} \tag{11}$$

where

$$\theta_t = \beta_t \theta_{t-1} \tag{12}$$

$$\beta_t = \frac{e^{\varsigma_t}}{1+\psi(\log \overline{C}_t - \vartheta)} \tag{13}$$

We allow the discount factor θ_t to be time-varying, and its tmie-varying coefficient β_t depends on the preference shock (ς_t) and the long-run level of consumption ($\overline{C}_t$). The parameter ω is the degree of internal habit persistence, $1/\sigma$ corresponds to the elasticity of intertemporal substitution in the absence of habit formation, and $1/\varphi$ is the Frisch elasticity of labor supply. Cost minimization problem yields the following price index:

$$P_t = \frac{P_{T,t}^{\gamma} P_{N,t}^{1-\gamma}}{\gamma(1-\gamma)} \tag{14}$$

The price index of tradable goods is:

$$P_{T,t} = \left[\alpha P_{H,t}^{1-\eta} + (1-\alpha) P_{F,t}^{1-\eta}\right]^{\frac{\eta}{\eta-1}} \tag{15}$$

Also we obtain the following demand functions:

$$C_{T,t} = \gamma \left(\frac{P_{T,t}}{P_t}\right)^{-1} C_t \tag{16}$$

$$C_{N,t} = (1-\gamma) \left(\frac{P_{N,t}}{P_t}\right)^{-1} C_t \tag{17}$$

$$C_{H,t} = \alpha \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \tag{18}$$

$$C_{F,t} = (1-\alpha) \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \tag{19}$$

Household's budget constraint is:

$$P_t C_t + B_t = I_{t-1} B_{t-1} + \int_0^{\gamma} W_{H,t}(f) L_{H,t}(f) df + \int_{\gamma}^{1} W_{N,t}(f) L_{N,t}(f) df + \Upsilon_t \tag{20}$$

where B_t is a government bond holding at the end of a time t, I_t is nominal interest rate at the end of a time t, $L_{k,t}(f)$ is labor supply function in firm f in the tradable good sector ($k = H$) and in the non-tradable good sector ($k = N$), and $W_{k,t}(f)$ is nominal wage in firm f for $k = H$, N. Υ_t is dividends. Within the household, a fraction γ of workers work in the tradable goods sector, while a fraction $1 - \gamma$ work in the nontradable goods sector. Maximizing the utility function subject to budget constraint yields the following Euler equation:

$$E_t \Lambda_t P_t = E_t \left[(C_t - \omega C_{t-1})^{-\sigma} - \omega \beta_t (C_{t+1} - \omega C_t)^{-\sigma} \right] \tag{21}$$

where Λ_t is the household's marginal utility of additional nominal income at a time t. Also, we obtain labor supply function:

$$\frac{W_{k,t}(f)}{P_t} \frac{1}{C_t} = L_{k,t}(f)^{\varphi} \tag{22}$$

for $k = H, N$. We defines nominal exchange rate as $e_t \equiv P_{k,t}/P^*_{k,t}$ for $k = H$, N. The superscript $*$ denotes its foreign counterpart. As in FGS08, we assume that the foreign country bond is not traded internationally. Thus, while home citizens trade only in domestic bonds, foreign citizens may hold either domestic or foreign country bonds. Finally, we obtain the following uncovered parity condition:

$$E_t \left\{ I_t \frac{e_t P^*_t}{e_{t+1} P^*_{t+1}} \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-1} \right\} = E_t \left\{ I^*_t \frac{P^*_t}{P^*_{t+1}} \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-1} \right\} \tag{23}$$

(2) Firms

(2-1) Final Goods Firms

Final good producers convert intermediate goods into final goods in competitive market.

$$Y_{H,t} = \left[\gamma^{-1/\theta} \int_0^{\gamma} Y_{H,t}(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \tag{24}$$

$$Y_{N,t} = \left[\gamma^{-1/\theta} \int_{\gamma}^{1} Y_{N,t}(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \tag{25}$$

where the parameter θ is the elasticity of substitution among intermediate goods. From cost minimization, we obtain:

$$Y_{H,t}(f) = \gamma^{-1}\left[\frac{P_{H,t}(f)}{P_{Ht}}\right]^{-\theta} Y_{H,t} \tag{26}$$

$$Y_{N,t}(f) = (1-\gamma)^{-1}\left[\frac{P_{N,t}(f)}{P_{Nt}}\right]^{-\theta} Y_{N,t} \tag{27}$$

The price indices are:

$$P_{H,t}(f) = \left[\gamma^{-1}\int_0^{\gamma} P_{H,t}(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{28}$$

$$P_{N,t}(f) = \left[(1-\gamma)^{-1}\int_{\gamma}^{1} P_{N,t}(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{29}$$

(2-2) Intermediate Goods Firms

For $k = H,\ N$, monopolistic competitive firm f produces intermediate goods by the follwoing production function:

$$Y_{k,t}(f) = A_t L_{k,t}(f) \tag{30}$$

$$A_t = Z_t e^{a_t} \tag{31}$$

$$\frac{Z_t}{Z_{t-1}} = 1+g \tag{32}$$

where A_t is the total factor productivity, a_t is the technology shock, and Z_t is the long-run trend of total factor productivity which grows at the rate of g. For each firm f, the marginal cost $MC_{k,t}(f)$ (for $k = H,\ N$) can be expressed as follows:

$$MC_{k,t}(f) = \frac{W_{k,t}(f)}{A_t} \tag{33}$$

Intermediate good firms set price in the manner of Calvo-type sticky price in which only a fraction of $1-\xi$ can choose a new price in each period:

$$E_t \sum_{s=0}^{\infty} \xi^s \Lambda_{t,t+s}[P_{k,t}(f) - MC_{k,t+s}(f)]Y_{k,t+s} \tag{34}$$

where $\Lambda_{t,t+s} = \beta_{t+s}(C_{t+s}/C_t)^{-1}(P_t/P_{t+s})$. The first order condition for the optimal reset price $P_{k,t}$ is given by:

$$E_t \left\{ \sum_{s=0}^{\infty} \xi^s \Lambda_{t,t+s}[P^o_{kt}(f) - (1+\mu)MC_{k,t+s}(f)]Y_{k,t+s} \right\} = 0 \tag{35}$$

where $\mu \equiv (\sigma-1)^{-1}$. Finally, the price index in each sector evolves according to:

$$P_{kt} = [\xi P^{1-\sigma}_{k,t-1} + (1-\xi)(P^o_{kt})^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{36}$$

(2-3) Import retailers

As is FGS08, we add imperfect pass-through following Monacelli (2005). We introduce monopolistically competitive retailers who import foreign tradables and sell them to domestic residents. The law of one price may not hold because local retailers set the price of imported goods on a staggered fashion. Each period, a fraction $\tilde{\xi}$ of retailers hold their price constant while the remaining fraction $1-\tilde{\xi}$ solve an optimal problem. In particular, these retailers who change their price $P_{F,t}$ to maximize:

$$E_t \left\{ \sum_{s=0}^{\infty} \tilde{\xi}^s \Lambda_{t,t+s}(P_{F,t} - e_{t+s}P^*_{F,t+s})C_{F,t+s} \right\} \tag{37}$$

subject to demand function. The first order condition is:

$$E_t \left\{ \sum_{s=0}^{\infty} \tilde{\xi}^s \Lambda_{t,t+s}[P_{F,t} - (1+\tilde{\mu})e_{t+s}P^*_{F,t+s}]C_{F,t+s} \right\} = 0 \tag{38}$$

where $\tilde{\mu} \equiv (\eta - 1)^{-1}$. The law of large numbers implies that the price index for imported goods becomes:

$$P_{F,t} = \tilde{\xi} P_{F,t-1} + (1 - \tilde{\xi}) P_{F,t}^{o} \tag{39}$$

We define gap from the law of one price as:

$$\Psi_{F,t} \equiv \frac{e_t P_{F,t-1}^{*}}{P_{F,t}} \tag{40}$$

In case of perfect pass-through, $\Psi_{F,t} = 1$.

(3) Current Account Dynamics

Total nominal domestic bond holdings, B_t , evolve according to:

$$\frac{B_t}{P_t} = \frac{I_{t-1} B_{t-1}}{P_t} + NX_t \tag{41}$$

where NX_t is the real net exports, defined as:

$$NX_t \equiv \frac{P_{H,t} Y_{H,t}}{P_t} - \frac{P_{T,t} Y_{T,t}}{P_t} \tag{42}$$

Real exchange rate (Q_t) is defined as:

$$Q_t \equiv \frac{e_t P_t^{*}}{P_t} \tag{43}$$

(4) Monetary Policy

We assume that the home national central banks sets its nominal interest rate according to the following monetary policy rule:

$$i_t = \rho i_{t-1} + (1 - \rho)(\phi_\pi E_t \pi_{t+1} + \phi_y y_t) + \epsilon_t \tag{44}$$

where $i_t \equiv \log(I_t/I)$ corresponds to the deviations of the interest rate from its steady-state value (I), $\pi_t \equiv \log(P_t/P_{t-1})$ denotes deviations of CPI inflation around the steady state (assumed to be zero), y_t represents percent deviations of output from trend, $\Delta e_t = \log(\mathcal{E}_t/\mathcal{E}_{t+1})$ denotes percent nominal depreciation of the home currency, and the *iid* shock ϵ_t measures unexpected interest-rate disturbances. The foreign central bank follows a similar rule, but also responds to a change in the exchange rate and a change in home country's nominal interest rate as follows:

$$i_t^* = \rho^* i_{t-1}^* + (1-\rho^*)(\phi_\pi^* E_t \pi_{t+1}^* + \phi_y^* y_t^* + \phi_i^* i_t + \phi_e^* \Delta e_t) + \epsilon_t^* \quad (45)$$

(5) Resource Constraint

For both home and foreign tradables, production must equal demand:

$$\begin{aligned} Y_{H,t} &= C_{H,t} + C_{H,t}^* \\ Y_{F,t} &= C_{F,t} + C_{F,t}^* \\ Y_{N,t} &= C_{N,t} \\ Y_{N,t}^* &= C_{N,t}^* \end{aligned}$$

International financial markets must clear:

$$B_t + B_t^* = 0 \quad (46)$$

where B_t^* represents the nominal holdings of the domestic bond by the foreign household.

7.2 Calibrations

We follow FGS08 for most of calibrations. The steady-state growth rate of the economy g is set to 0.5%, so that annual growth is 2%. The steady-state discount factor β is set to 0.99. The parameters describing the evolution of the discount factor $\vartheta = -1000$ and $\psi = 7.2361 \times 10^{-6}$. The Frisch elasticity of labor supply is $\varphi^{-1} = 0.5$. The elasticity of substitution among intermediate

$\theta = 11$ results in a steady-state markup of 10% in the tradable and nontradable sectors. We set the probability that intermediate-goods firms and importing retailers do not re-optimize their price to $\xi = \tilde{\xi} = 0.66$, corresponding to a mean duration between price re-optimizations of 3 quarters. For the parameters that determine the openness of the economies, we set the share of tradables in the consumption basket to $\gamma = 0.7$, the preference share for home tradables $\alpha = 0.5$, and the elasticity of substitution between home and foreign tradables $\eta = 2$. Depart from FGS08, we calibrate $\omega = 0.60$ assuming that there are some degree of habit formation in consumption. For the parameters in monetary policy rule, $\rho = \rho^* = 0.75$, $\phi_\pi = \phi_\pi^* = 2.00$, , $\phi_y = \phi_y^* = 0.30$, $\phi_i^* = 0.40$, and $\phi_e^* = 0.40$.

7.3 Results

We present the responses of key variables to the home country's monetary shock in Figure 18. The solid lines are for home country variables and the break lines are for foreign country ones. There are a couple of notable results compared with our panel FAVAR model. First, the consumption falls less than own output in both countries. Although the difference between output and consumption are very small, this simple 2-country DSGE model also shows the evidence of international consumption risk sharing. Notice that there is no explicit capital in this model, and consumption includes investment. Thus, even including investment, a fall in consumption is still slightly less than a fall in output. This suggests that there is a possibility of international risk sharing. Second, the home currency initially appreciates (overshooting), and then depreciates with the presence of imperfect pass through. Note that this exchange rate in the 2-country DSGE is a bilateral one, instead of an effective one. Third, net export improves in home country.

Next, we change the monetary policy rule in the foreign country to see whether it changes the effect of contractionary monetary shock in the home country. Especially, the foreign central bank react in the same way as home central bank does. In other words, it doesn't react to the change in home monetary policy nor the exchange, *i.e.*, $\phi_i^* = 0$ and $\phi_e^* = 0$. We represent the impulse response functions of key variables in Figure 19. This result shows very different from the previous result in Figure 18. In foreign country, both output and consumption are not affected significantly compared with the previous result. Although this model has only 2 countries, the result suggests that the comovement of outputs in both home and foreign countries might be generated as a result that foreign central bank react to the change in home monetary policy and the exchange rate.

8 Conclusion

In this paper, we explored a cross-country monetary transmission mechanism. Particularly, we examined the effect of the U.S. monetary policy shock on the G-7 economies. To facilitate our analysis, we developed the panel FAVAR model. Unlike existing literature, we allow the country-specific factors in our model to have interdependency among them in order to produce the comovement of business cycles in the G-7 countries. Such comovement is documented elsewhere in international macroeconomic literature. Our findings are that: (1) The existence of interdependency among country-specific factors is statistically significant. This interdependency plays an important role in our cross-country monetary transmission mechanism. (2) Such international dependency reduces the magnitude of the U.S. business cycle. This suggests that most of VAR or FAVAR literature of the U.S. monetary transmission without using other countries' variables, overstate the effect on the U.S. real economy. This view is consistent with the argument of Sims and Zha (2006) among many others that the effect of monetary policy on real economy might be very little. (3) As a result of interdependency, the effect on other G-7 countries' real economy are similar. However, there are differences in magnitude. For example, after the U.S. contractionary monetary shock, Japan's output is affected most, following by Anglo-Saxon countries the UK and Canada, and then followed by the continental European countries, Germany, France, and Italy. (4) The consumption in each country falls less than its output. This is evidence of international consumption risk sharing documented in many literature. (5) Our 2-country DSGE model shows the evidence of such risk sharing, and also suggests that the comovement of business cycles among countries might be caused as a result that foreign central bank react to the change in home monetary policy and the exchange rate.

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Table 1: Optimal Numbers of Unobservable Factors

The Number of World-common Factors	The Number of Country-specific Factors	Marginal Density of Data
1	1	-265415
1	2	-124740
1	3	-100129*
1	4	-257598
2	1	-236633
2	2	-130664
2	3	-105437
2	4	-260074
3	1	-176426
3	2	-121821
3	3	-101571
3	4	-314325

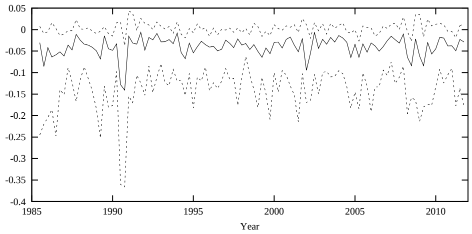
* denotes a best model.

Table 2: Interdependency Test

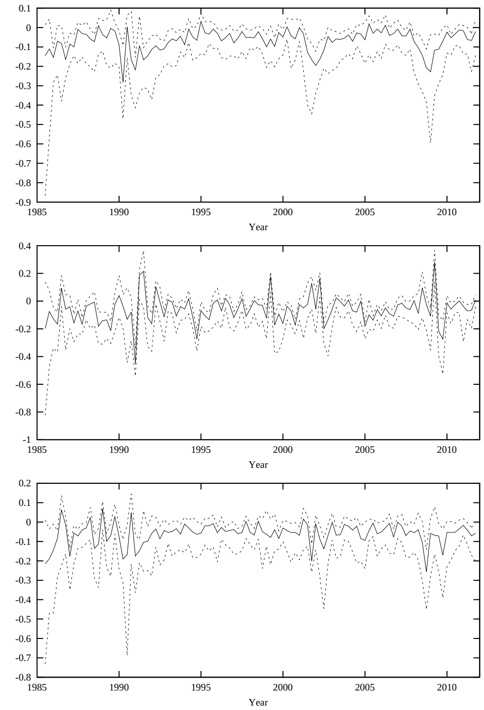
Restriction	Marginal Density of Data
Model with Interdependency	-100129*
Model without Interdependency	-343210

* denotes a best model.

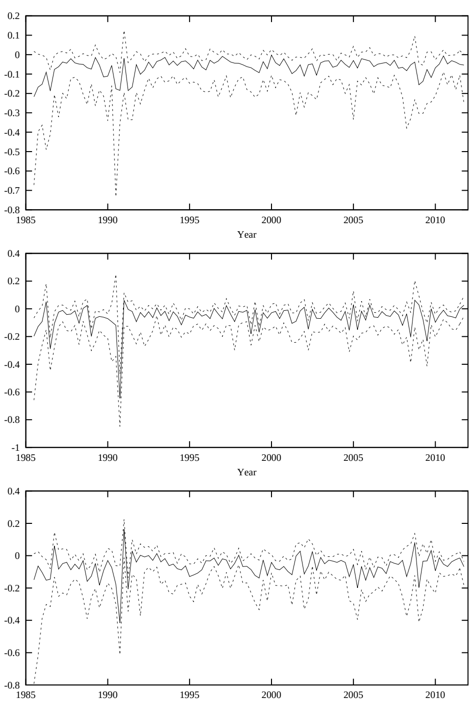
Figure 1: Unobservable Factors (1)



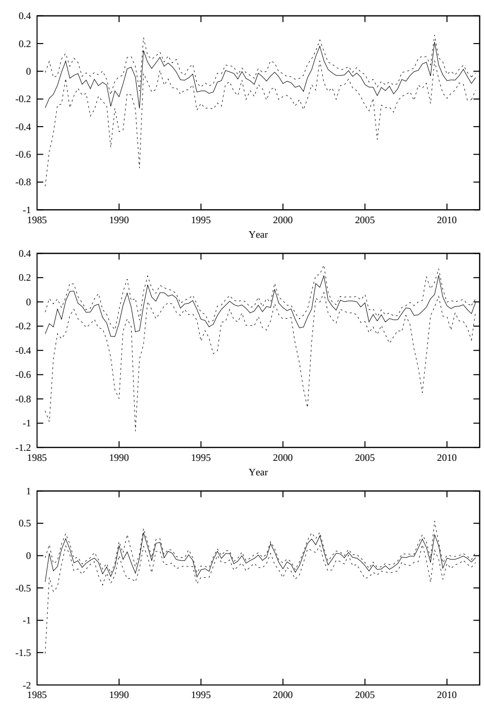
(a) Common Factor



(b) U.S.-specific Factors

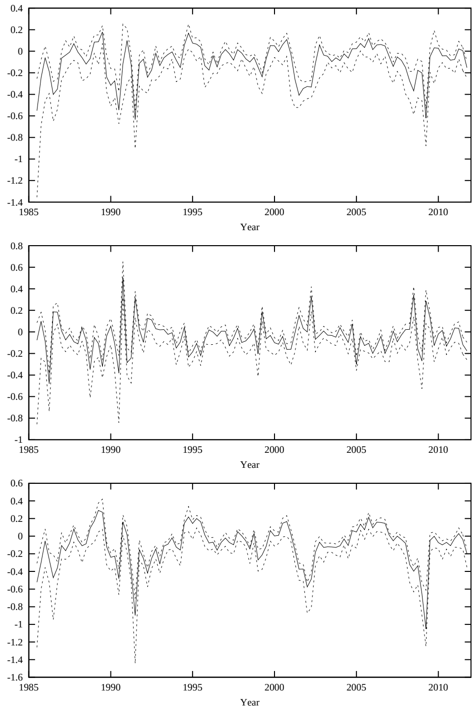


(c) Japan-specific Factors

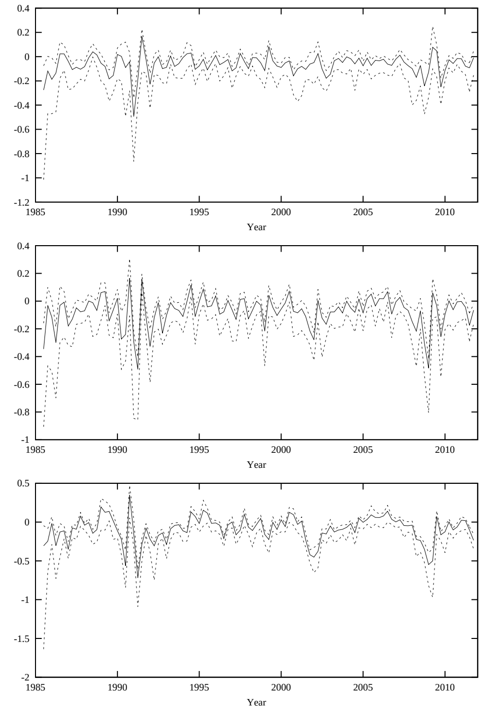


(d) Germany-specific Factors

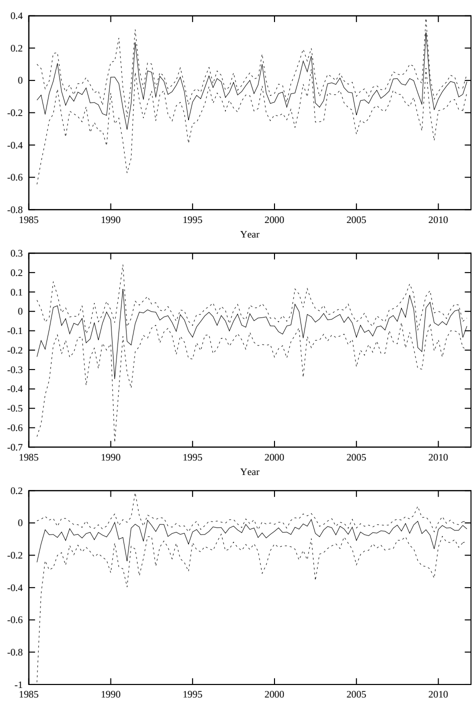
Figure 2: Unobservable Factors (2)



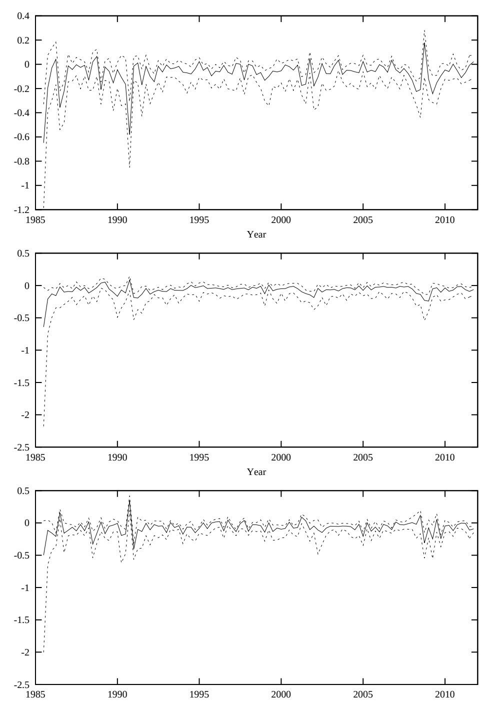
(a) UK-specific Factor



(b) France-specific Factors

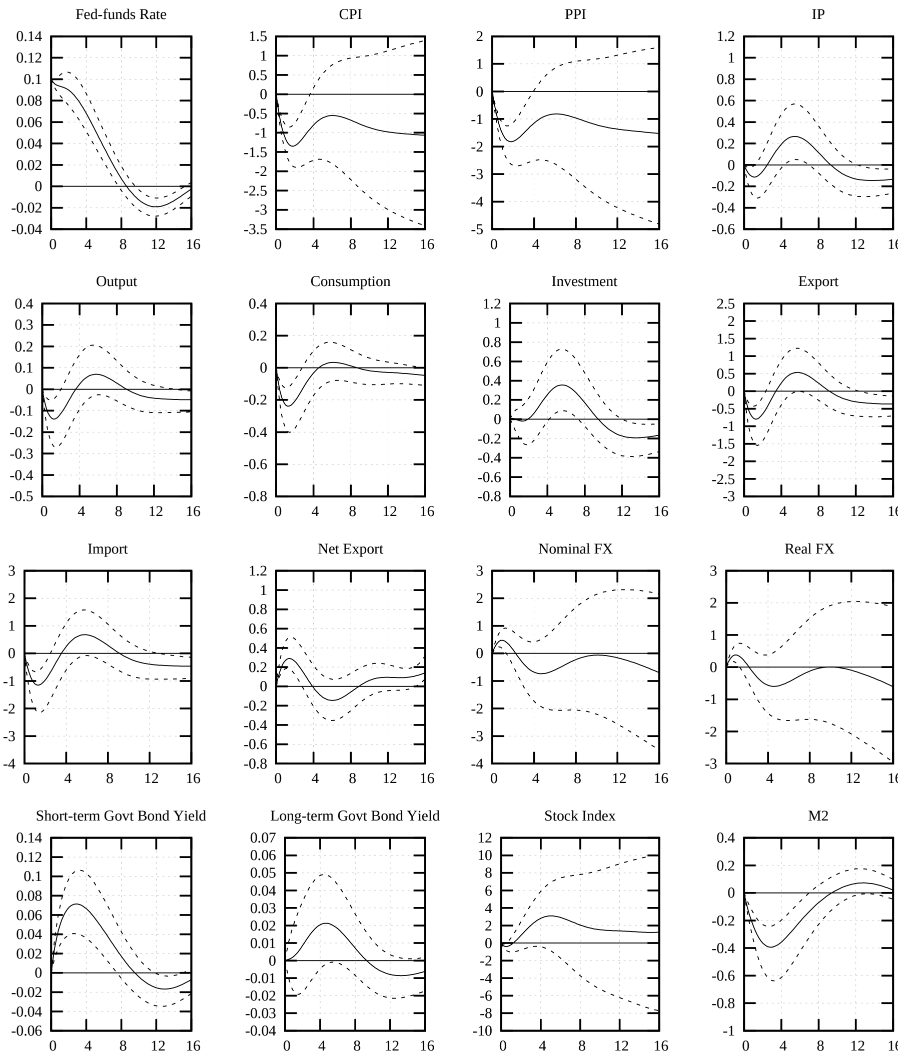


(c) Italy-specific Factors



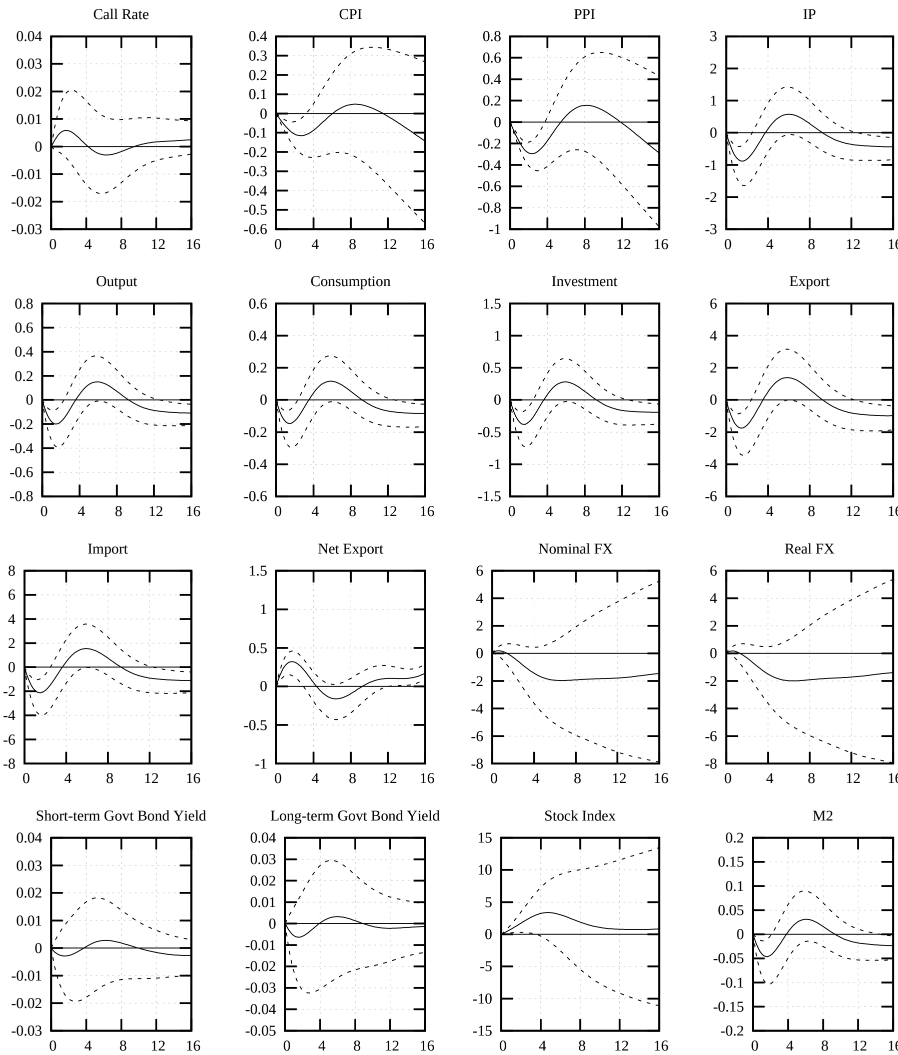
(d) Canada-specific Factors

Figure 3: Responses of US Macroeconomic Variables



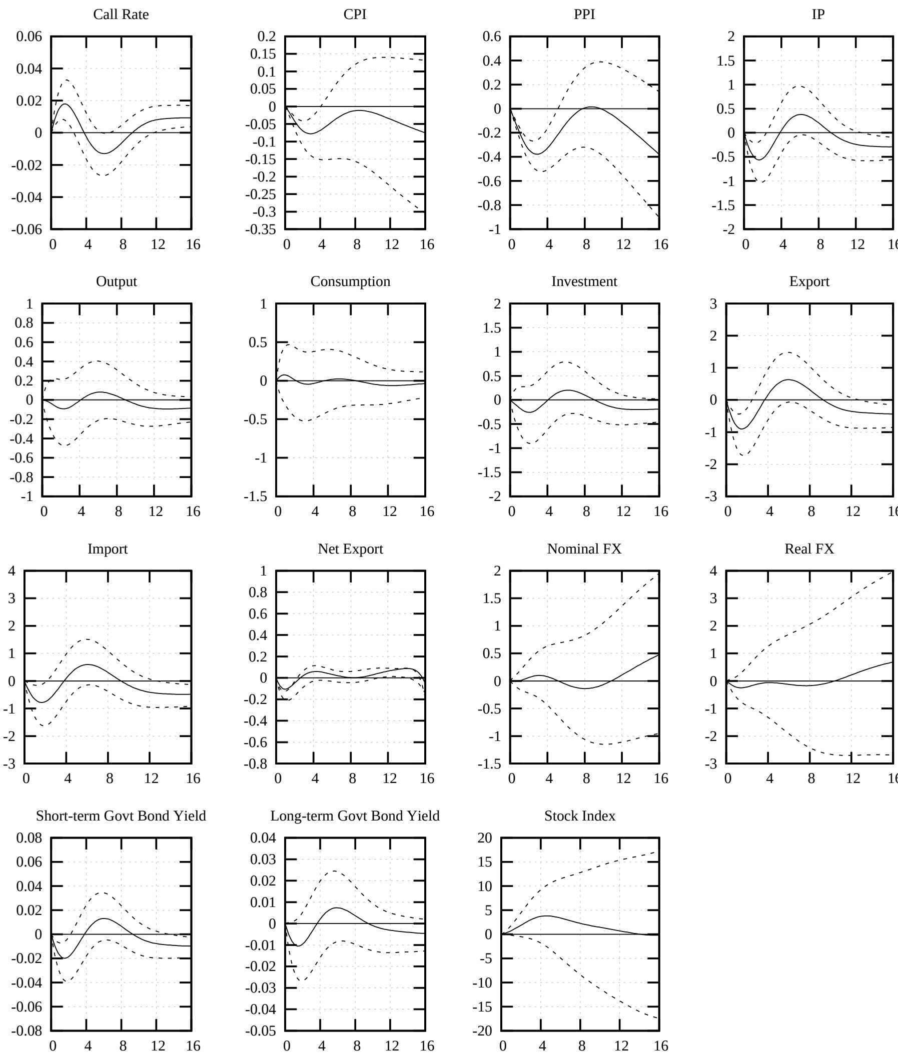
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 4: Responses of Japan's Macroeconomic Variables



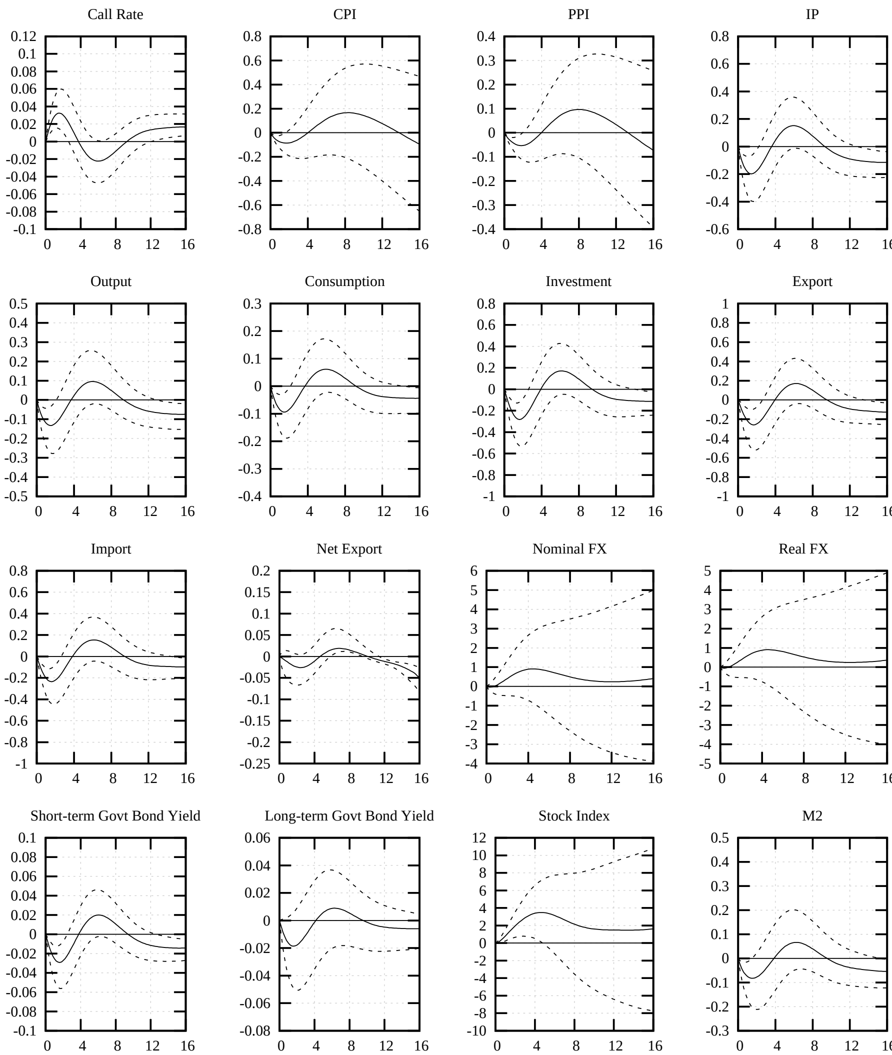
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 5: Responses of Germany's Macroeconomic Variables



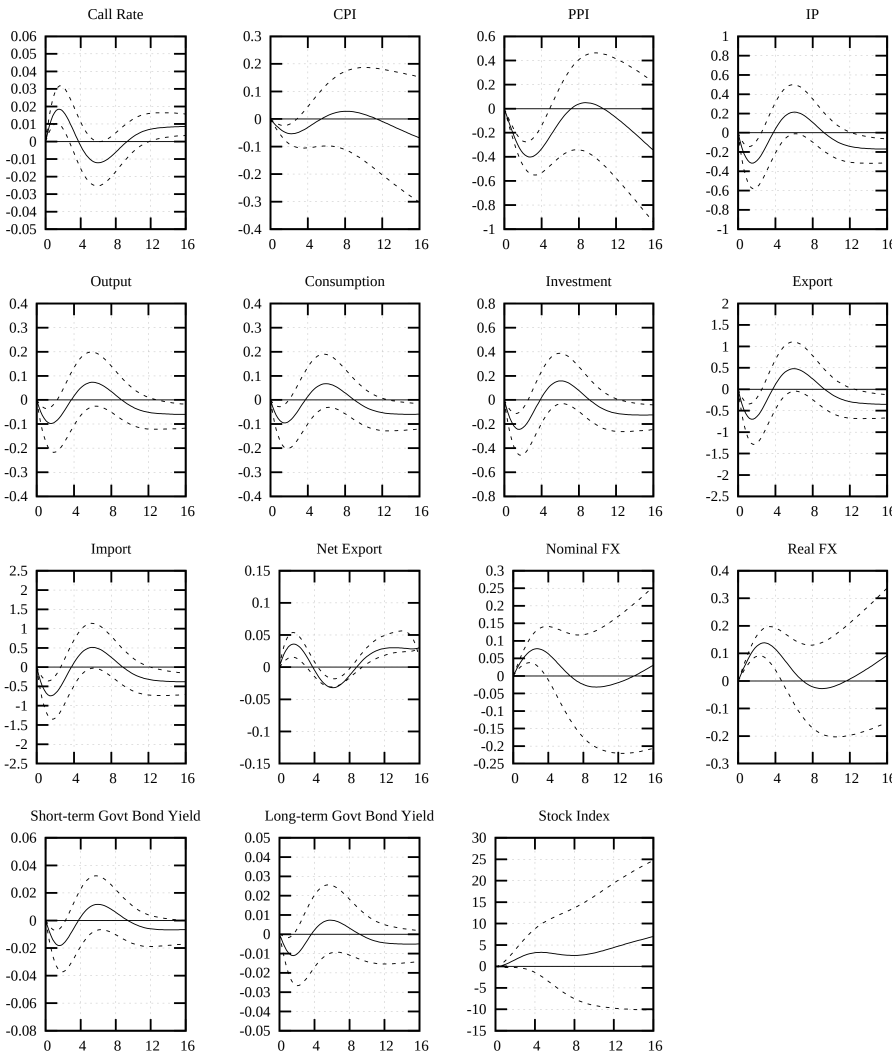
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 6: Responses of UK Macroeconomic Variables



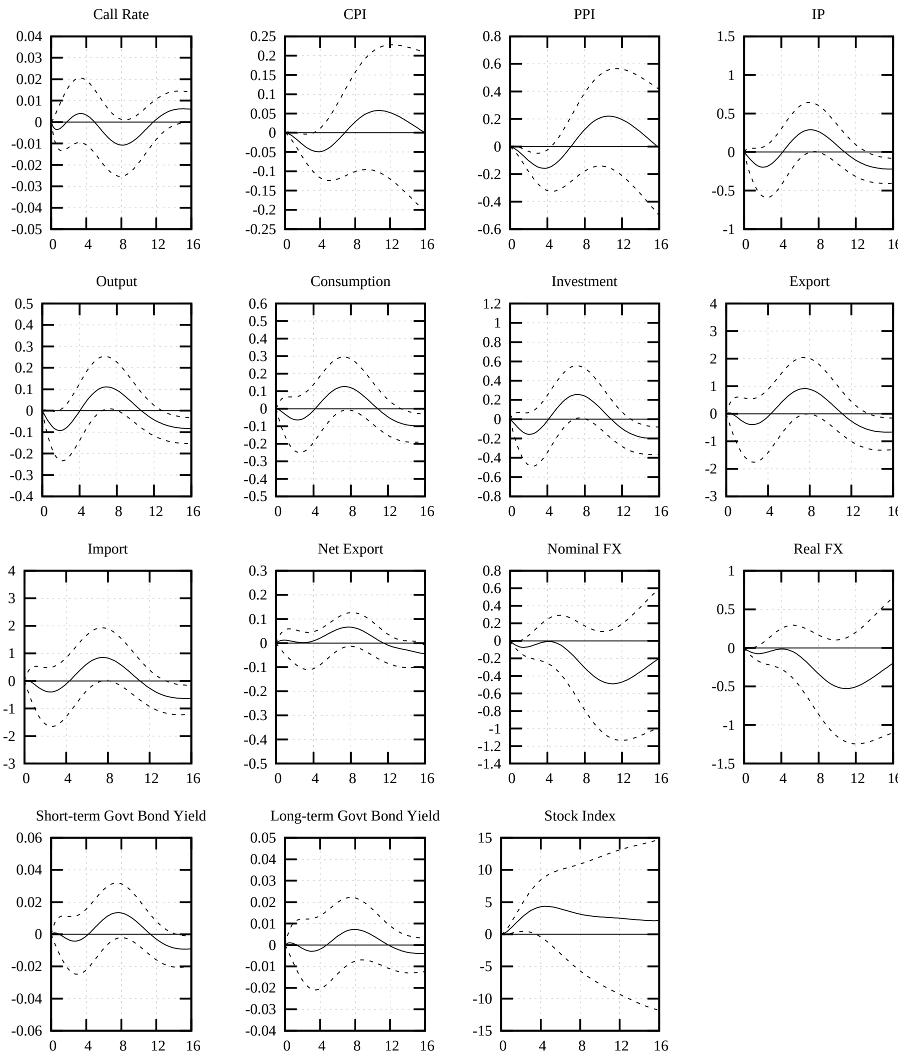
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 7: Responses of France's Macroeconomic Variables



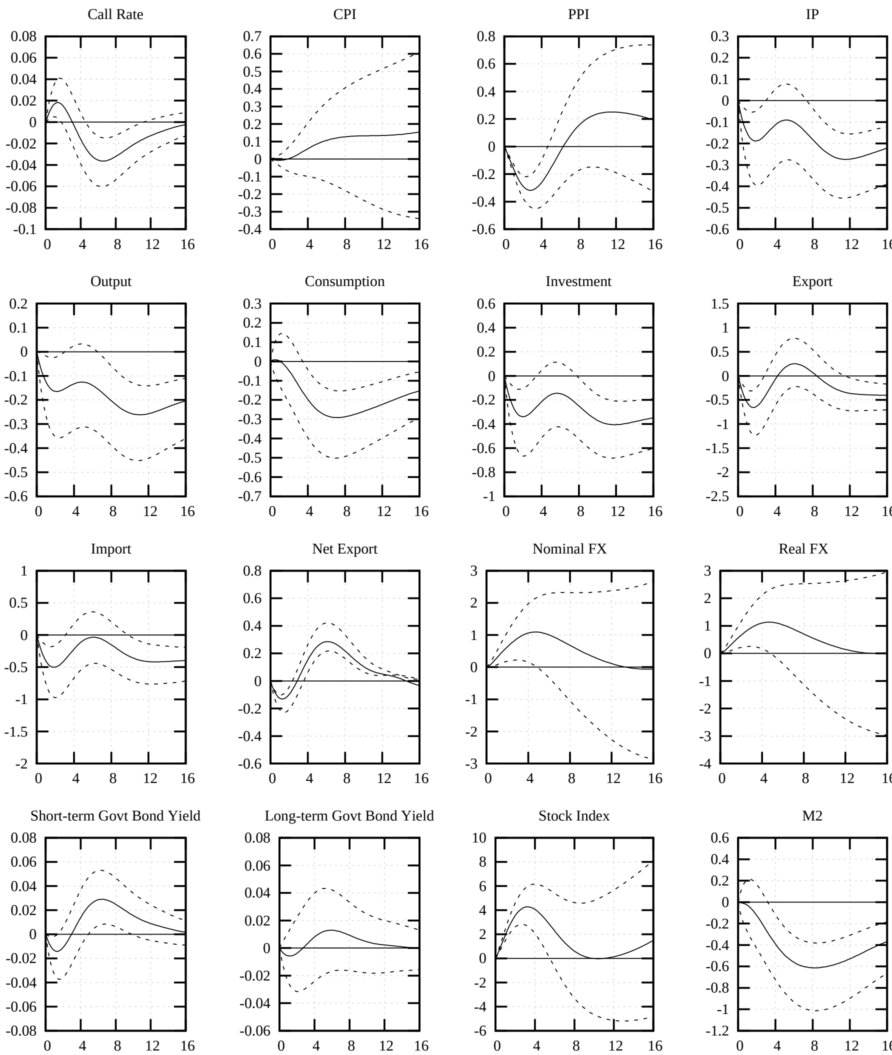
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 8: Responses of Italy's Macroeconomic Variables



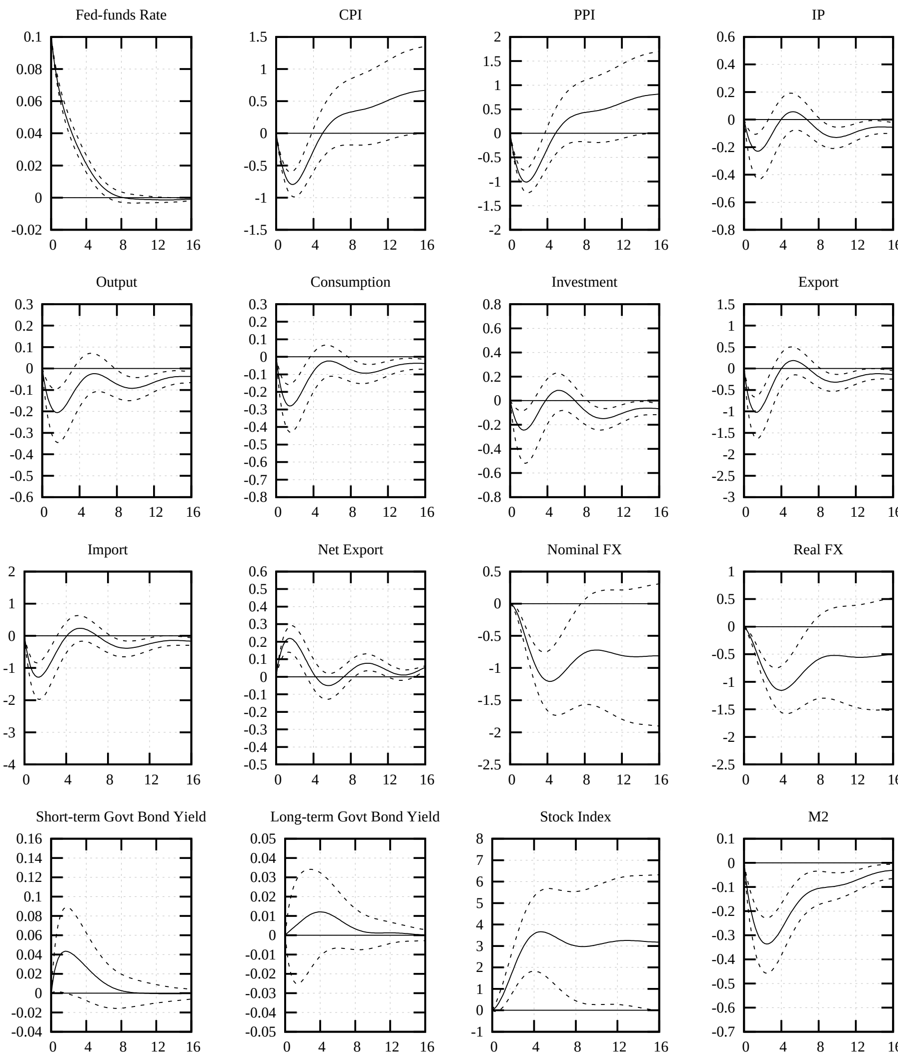
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 9: Responses of Canada's Macroeconomic Variables



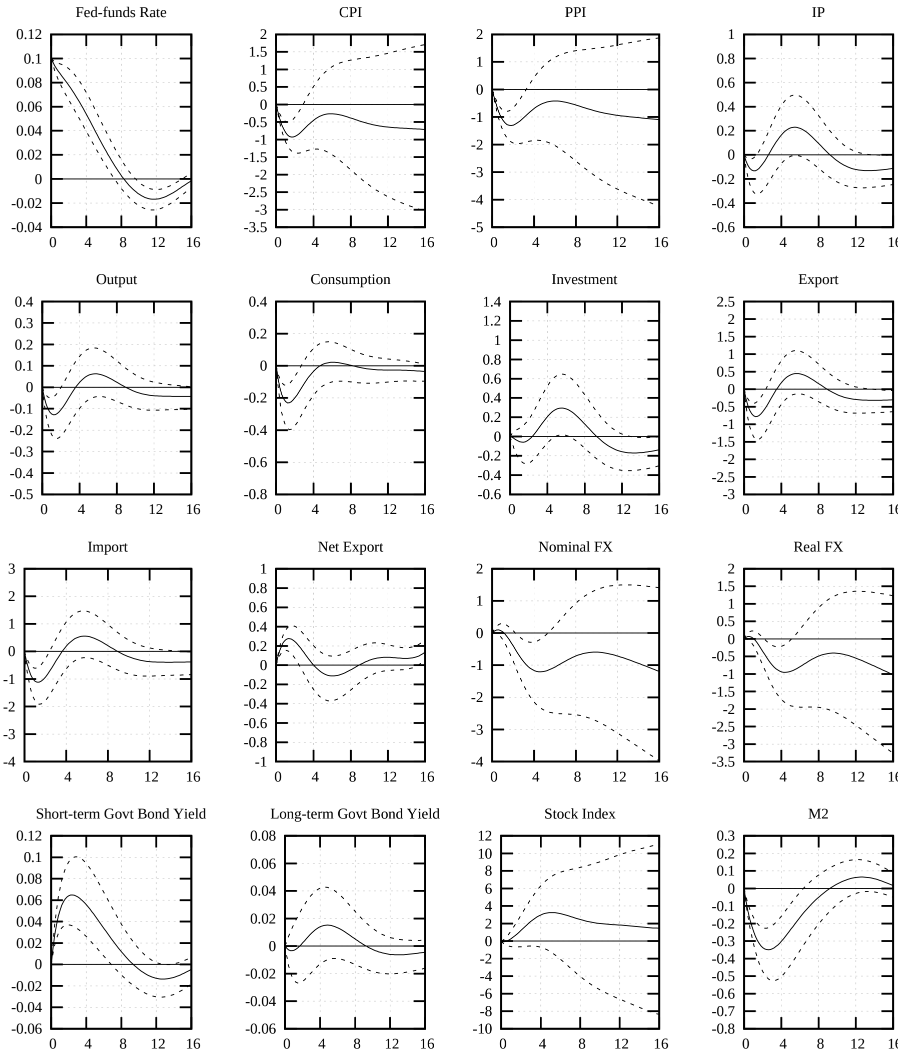
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 10: Responses of US Macroeconomic Variables (without Interdependency)



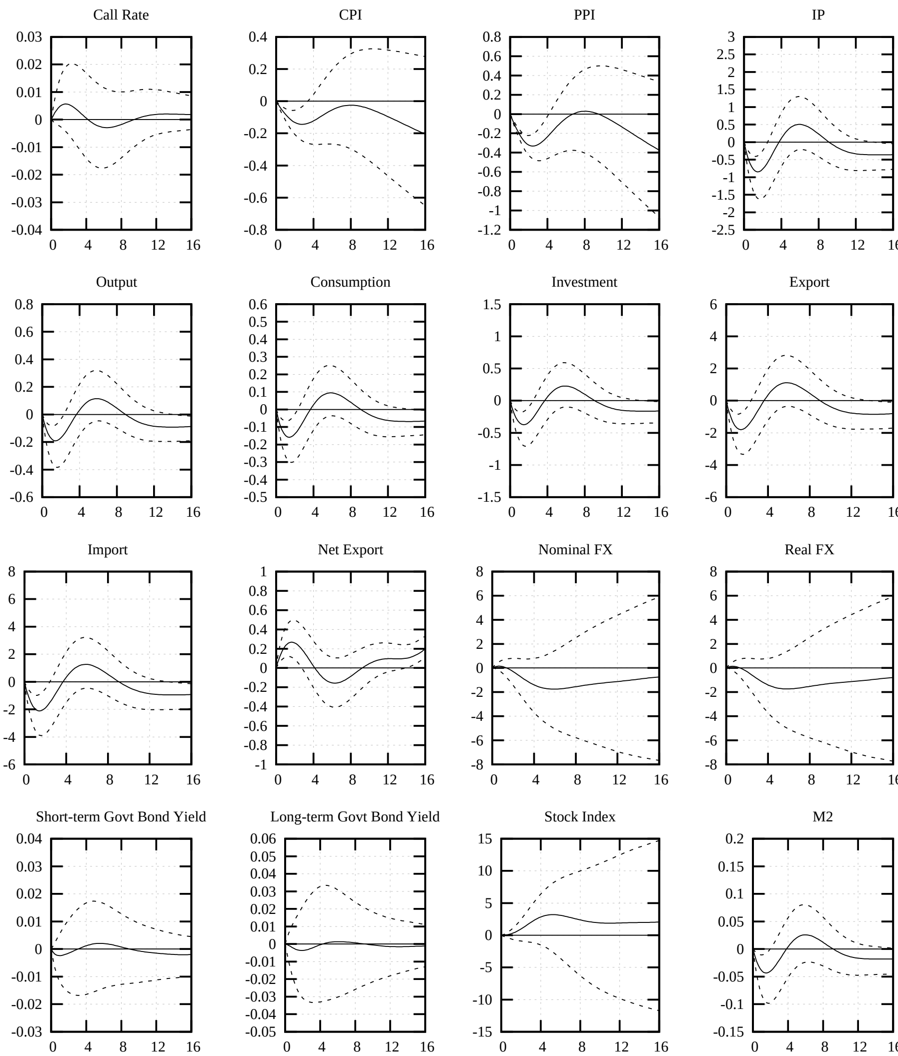
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 11: Responses of US Macroeconomic Variables (Euro Effect)



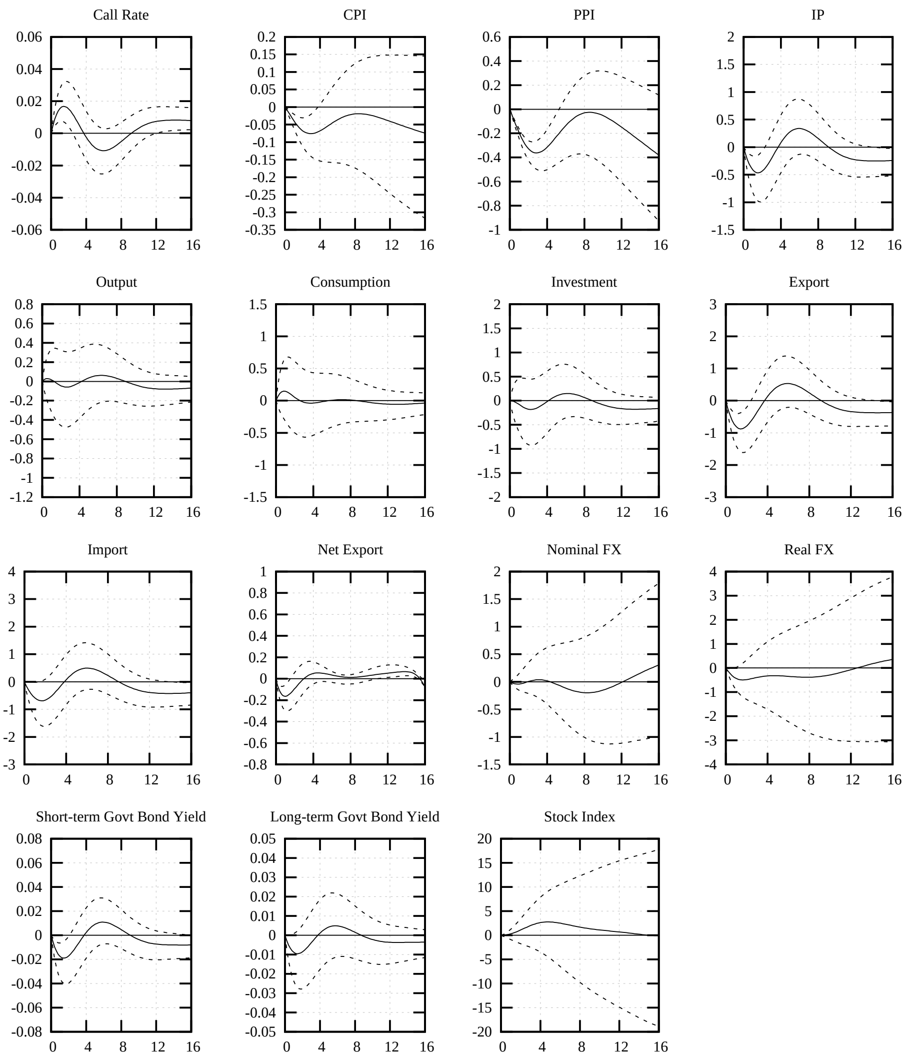
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 12: Responses of Japan's Macroeconomic Variables (Euro Effect)



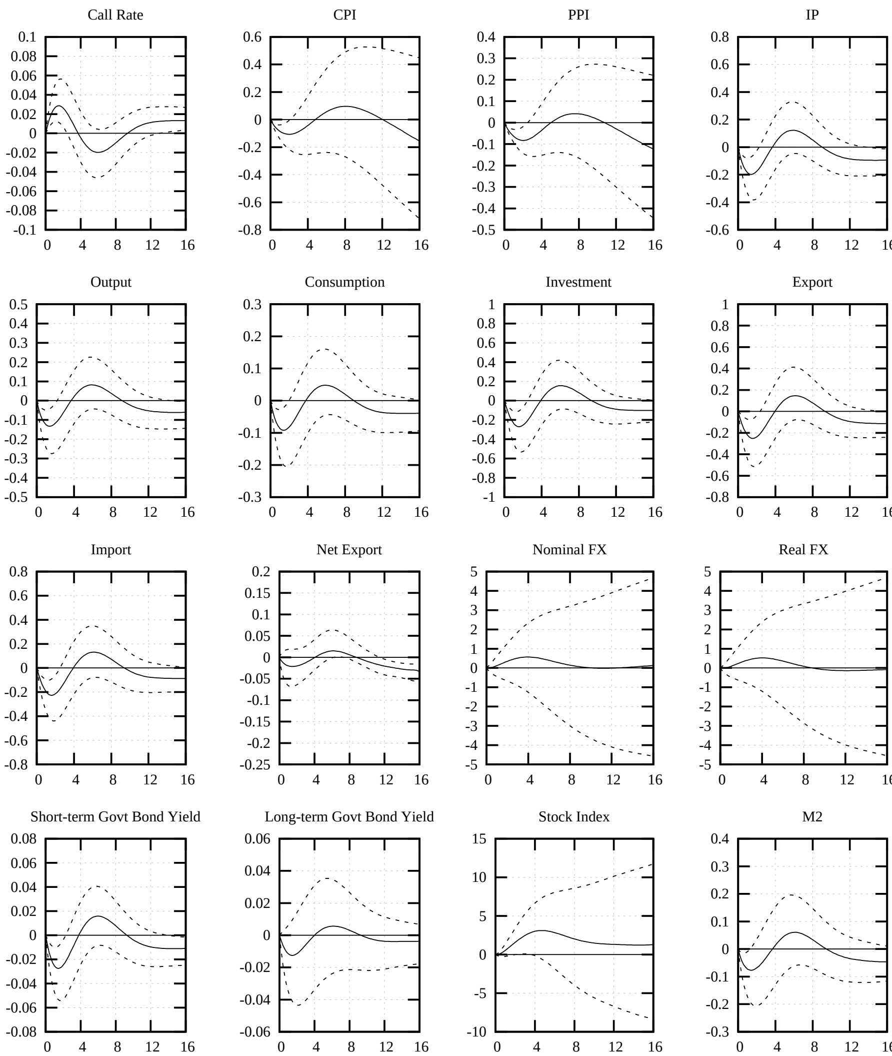
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 13: Responses of Germany's Macroeconomic Variables (Euro Effect)



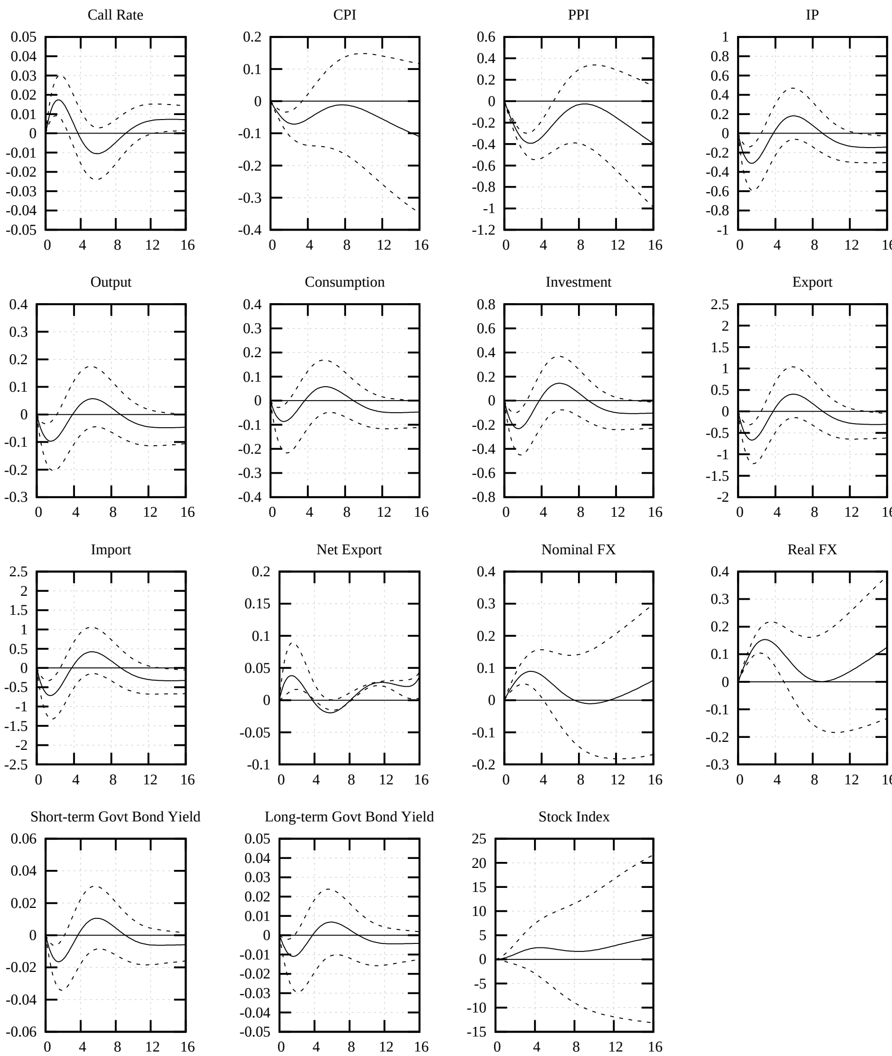
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 14: Responses of UK Macroeconomic Variables (Euro Effect)



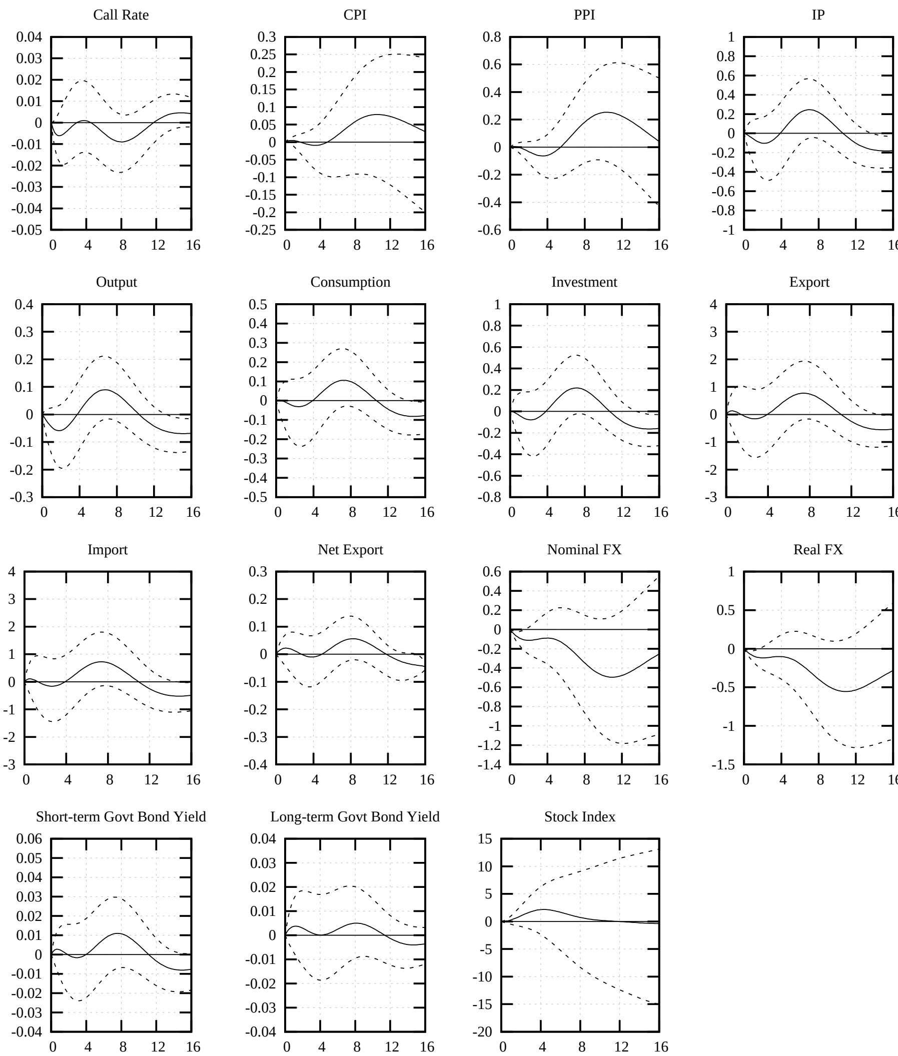
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 15: Responses of France's Macroeconomic Variables (Euro Effect)



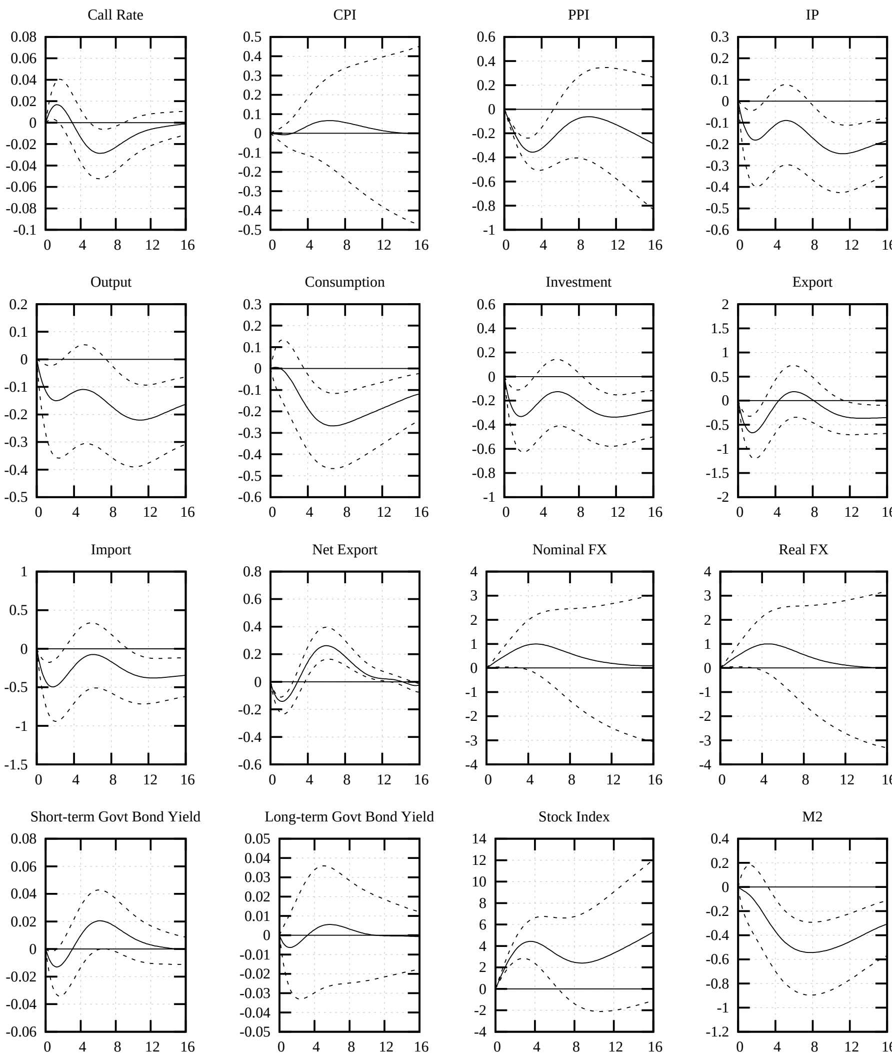
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 16: Responses of Italy's Macroeconomic Variables (Euro Effect)



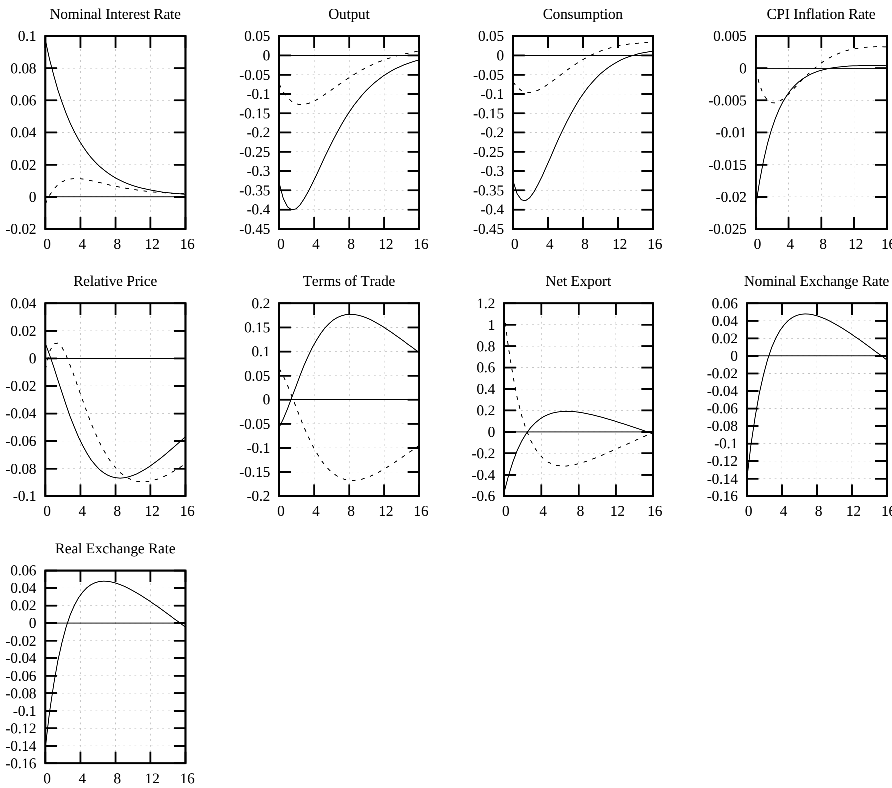
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 17: Responses of Canada's Macroeconomic Variables (Euro Effect)



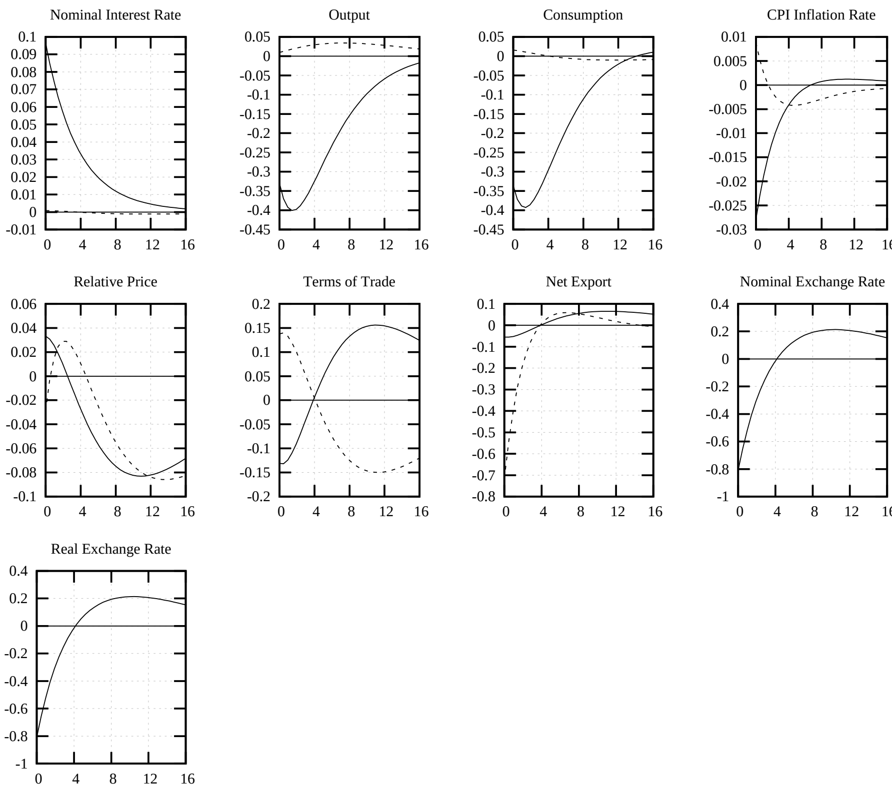
Note: The units are all annualized percentage changes expect for the interest rates. The unit of the interest rates are percentage points.

Figure 18: Impulse Response Functions in 2-country DSGE Model (1)



Note: Solid lines are home country and break lines are foreign country.

Figure 19: Impulse Response Functions in 2-country DSGE Model (2)



Note: Solid lines are home country and break lines are foreign country.

Appendix A: Data

The numbers in parenthesis shows data transformation (1=level, 2=difference, 3=percentage change, 4=change in percentage change).

U.S.
1. Federal funds rate (2)
2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
4. M2 (3)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)
12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
17. Exported good price (4)
18. Imported good price (4)
19. Consumption (3)
20. Investment (3)
21. Output (3)

Japan
1. Overnight call rate (2)
2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
4. M2 (3)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)
12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
17. Exported good price (4)
18. Imported good price (4)
19. Consumption (3)
20. Investment (3)
21. Output (3)

Germany
1. Overnight call rate (2)

2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)
12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
17. Exported good price (4)
18. Imported good price (4)
19. Consumption (3)
20. Investment (3)
21. Output (3)

<u>UK</u>

1. Overnight call rate (2)
2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
4. M0 (3)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)
12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
17. Exported good price (4)
18. Imported good price (4)
19. Consumption (3)
20. Investment (3)
21. Output (3)

<u>France</u>

1. Overnight call rate (2)
2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)

12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
19. Consumption (3)
20. Investment (3)
21. Output (3)

Italy

1. Overnight call rate (2)
2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)
12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
17. Exported good price (4)
18. Imported good price (4)
19. Consumption (3)
20. Investment (3)
21. Output (3)

Canada

1. Overnight call rate (2)
2. Nominal effective exchange rate (4)
3. Real effective exchange rate (4)
4. M2 (3)
5. Short-term government bond rate (2)
6. Long-term government bond rate (2)
7. Deposit rate (2)
8. Lending rate (2)
9. Spread of short-term government bond rate (1)
10. Spread of long-term government bond rate (1)
11. Spread of deposit rate (1)
12. Spread of lending rate (1)
13. Stock index (4)
14. Producer price index (4)
15. Consumer price index (4)
16. Industrial production (3)
17. Exported good price (4)
18. Imported good price (4)
19. Consumption (3)
20. Investment (3)
21. Output (3)

Appendix B: Moving Gibb Sampler Markov Chain Monte Carlo Posterior Simulation

(1) $\mathbb{F}^T|\ \mathbb{X}^T, \theta$

$$\pi(\mathbb{F}^T|\ \mathbb{X}^T, \theta) = \pi(\mathbb{F}_T|\ \mathbb{X}^T, \theta) \prod_{t=1}^{T-1} \pi(\mathbb{F}_t|\ \mathbb{F}_{t+1}, \mathbb{X}^T, \theta) \tag{47}$$

Backward recursion:

$$\begin{cases} \mathbb{F}_T|\ \mathbb{X}_T, \theta \sim N(\mathbb{F}_{T|T}, \mathbb{S}_{T|T}) \\ \mathbb{F}_{t|T}|\ \mathbb{F}_{t+1|T}, \mathbb{X}^T, \theta \sim N(\mathbb{F}_{t|t,F_{t+1|T}}, \mathbb{S}_{t|t,F_{t+1|T}}) \end{cases}$$

Kalman filter:
Set $\mathbb{F}_{1|0} = O_{pK\times 1}$ and $\mathbb{S}_{1|0} = I_{pK}$, then
prediction step

$$\begin{cases} \mathbb{F}_{t|t} = \mathbb{F}_{t|t-1} + \mathbb{S}_{t|t-1}\Lambda' H_{t|t-1}^{-1}\eta_{t|t-1} \\ \mathbb{S}_{t|t} = \mathbb{S}_{t|t-1} - \mathbb{S}_{t|t-1}\Lambda' H_{t|t-1}^{-1}\Lambda\mathbb{S}_{t|t-1} \end{cases}$$

where $\eta_{t|t-1} = \mathbb{X}_t - \Lambda\mathbb{F}_{t|t-1}$ and $H_{t|t-1} = \Lambda\mathbb{S}_{t|t-1}\Lambda' + \mathbb{R}$

update step

$$\begin{cases} \mathbb{F}_{t|t-1} = \Phi\mathbb{F}_{t-1|t-1} \\ \mathbb{S}_{t|t-1} = \Phi\mathbb{S}_{t-1|t-1}\Phi' + \mathbb{Q} \end{cases}$$

Using $\mathbb{F}_{T|T}$ and $\mathbb{S}_{T|T}$ in the last iteration, draw $\mathbb{F}_T|\ \mathbb{X}_T, \theta$.

Kalman smoothing

$$\begin{cases} \mathbb{F}^*_{t|t,F_{t+1|T}} = \mathbb{F}_{t|t} + \mathbb{S}_{t|t}\Phi^{*'} J_{t+1|t}^{-1}\xi_{t+1|t} \\ \mathbb{S}^*_{t|t,F_{t+1|T}} = \mathbb{S}_{t|t} - \mathbb{S}_{t|t}\Phi^{*'} J_{t+1|t}^{-1}\Phi^*\mathbb{S}_{t|t} \end{cases}$$

where $\xi_{t+1|t} = \mathbb{F}^*_t - \Phi^*\mathbb{F}_{t|t}$ and $J_{t+1|t} = \Phi^*\mathbb{S}_{t|t}\Phi^{*'} + \mathbb{Q}^*$. $\mathbb{Q}^*$ is an upper $K \times K$ of $\mathbb{Q}$, Φ^* is the first K rows of Φ, and $\mathbb{F}^*_t$ is the first K rows of $\mathbb{F}_t$

(2) $\lambda, \mathbb{R}|\ \mathbb{X}^T, \mathbb{F}^T, \phi, \mathbb{Q}$

Posterior:

$$R_{nn}|\ \mathbb{X}^T, \mathbb{F}^T \sim \mathcal{IG}(\delta_n/2, \eta_n/2) \tag{48}$$

$$\Lambda_n|\ \mathbb{X}^T, \mathbb{F}^T, R_{nn} \sim \mathcal{N}(\bar{\Lambda}_n, R_{nn}\bar{M}_n^{-1}) \tag{49}$$

where

$$\eta_n = \eta_0 + T \tag{50}$$

$$\delta_n = \delta_0 + \hat{e}_n^{c'}\hat{e}_n^c + (\hat{\Lambda}_n - \Lambda_{n,0})'[M_{n,0}^{-1} + (\mathbb{F}^{n'}\mathbb{F}^n)^{-1}]^{-1}(\hat{\Lambda}_n - \Lambda_{n,0}) \tag{51}$$

$$\bar{M}_n = M_{n,0} + (\mathbb{F}^{n'}\mathbb{F}^n) \tag{52}$$

$$\bar{\Lambda}_n = \bar{M}_n^{-1}[M_{n,0}^{-1}\Lambda_{n,0} + (\mathbb{F}^{n'}\mathbb{F}^n)\hat{\Lambda}_n] \tag{53}$$

and $\mathbb{F}^n$ is the regressors of the nth equation, $\hat{\Lambda}_n$ is the equation-by-equation OLS estimator, and $\hat{e}_n^c$ is its residuals.

(3) $vec(\phi), \mathbb{Q}|\, \mathbb{X}^T, \mathbb{F}^T, \lambda, \mathbb{R}$
Posterior:

$$\begin{aligned} Q_u|\, \mathbb{X}^T, \mathbb{F}^T &\sim \mathcal{IW}(S_T, \nu_T) && (54) \\ vec(\phi)|\, \mathbb{X}^T, \mathbb{F}^T, Q_u &\sim \mathcal{N}(\phi_T, Q_u \otimes N_T^{-1}) && (55) \end{aligned}$$

where

$$\begin{aligned} \nu_T &= \nu_0 + T && (56) \\ S_T &= \nu_T^{-1}[\nu_0 + T\hat{Q}_u + (\hat{\phi} - \phi_0)' N_0 N_T^{-1} (\mathbb{F}'_{T-1}\mathbb{F}_{T-1})(\hat{\phi} - \phi_0)] && (57) \\ N_T &= N_0 + (\mathbb{F}'_{T-1}\mathbb{F}_{T-1}) && (58) \\ \phi_T &= N_T^{-1}[N_0\phi_0 + (\mathbb{F}'_{T-1}\mathbb{F}_{T-1})\hat{\phi}] && (59) \end{aligned}$$

Appendix C: Approximation Method of Marginal Likelihood

To compute MDD, we begin with the posterior density function of parameter:

$$p(\tilde{\theta}|\ \mathbb{X}^T) \quad = \quad \frac{p(\mathbb{X}^T\ |\ \tilde{\theta}\ )\ p(\tilde{\theta})}{p(\mathbb{X}^T)}$$

where $p(\cdot)$ is a probability density function, and $\tilde{\theta}$ is any $\theta \in \Theta$ Then, the logarithm of MDD can be written as follow:

$$\log p(\mathbb{X}^T) \quad = \quad \log p(\mathbb{X}^T|\ \theta^*) + \log p(\theta^*) - \log p(\theta^*|\ \mathbb{X}^T) \tag{60}$$

Note that we evaluate the MDD at the posterior mode (θ^*) where it is most efficient. The prior distribution can be applied to $p(\theta^*)$. The likelihood density can be expressed as follows:

$$p(\mathbb{X}^T|\ \theta^*) \quad = \quad \prod_{t=1}^{T} \mathcal{L}(X_t|Y_t, \theta\ ) \tag{61}$$

where

$$\mathcal{L}(X_t|Y_t,\theta\ ) = (2\pi)^{-0.5N_c}|\lambda'\mathbb{S}_{t|t-1}\lambda + R|^{-0.5}\exp\left[-\frac{1}{2}(X_t - \lambda_y Y_t - \lambda_f F^*_{t|t-1})'(\lambda'\mathbb{S}_{t|t-1}\lambda + R)(X_t - \lambda_y Y_t - \lambda_f F^*_{t|t-1})\right]$$

For the posterior density $p(\theta^*|\ \mathbb{X}^T)$, we approximate it using Chib's (1995) method. In this method, we decompose it as follows:

$$\begin{aligned} p(\theta^*|\ \mathbb{X}^T) &= p(\phi^*, Q^*, \lambda^*, R^*|\ \mathbb{X}^T) \\ &= p(\phi^*|\ Q^*, \lambda^*, R^*, \mathbb{X}^T)p(Q^*|\ \lambda^*, R^*, \mathbb{X}^T)p(\lambda^*|\ R^*, \mathbb{X}^T)p(R^*|\ \mathbb{X}^T) \end{aligned} \tag{62}$$

Due to the ergodicity theorem, one can obtain the following almost sure convergence in three conditional density functions in the right hand side of (62):

$$\frac{1}{M}\sum_{m=1}^{M} p(R^*|\ \phi^{(m)},\ Q^{(m)}, \lambda^{(m)}, \mathbb{F}^{T\ (m)}, \mathbb{X}^T) \ \longrightarrow_{a.s.}\ p(R^*|\ \mathbb{X}^T)$$

where M is the number of iterations in the Gibbs sampler.
Since $p(R^*|\ \phi^{(m)},\ Q^{(m)}, \lambda^{(m)}, \mathbb{F}^{T\ (m)}), \mathbb{X}^T)$ is already available in the outputs in the Gibbs sampler, obtaining this approximation requires no additional effort. However, we need to run two additional iterations to get the following three approximations:

$$\frac{1}{M}\sum_{m_1=1}^{M} p(\lambda^*|\ \phi^{(m_1)}, Q^{(m_1)}, R^*, \mathbb{F}^{T\ (m_1)}, \mathbb{X}^T) \ \longrightarrow_{a.s.}\ p(\lambda^*|\ R^*, \mathbb{X}^T)$$

where $\mathbb{F}^{T\ (m_1)} \sim \mathbb{F}^T|\ \phi^{(m_1)}, Q^{(m_1)}, \lambda^*, R^*, \mathbb{X}^T$, $\quad Q^{T\ (m_1)} \sim \mathbb{F}^T|\ \phi^{(m_1)}, \lambda^*, R^*, \mathbb{F}^{T\ (m_1)}, \mathbb{X}^T$, and $\phi^{(m_1)} \sim \mathbb{F}^T|\ Q^{(m_1)}, \lambda^*, R^*, \mathbb{F}^{T\ (m_1)}, \mathbb{X}^T$

$$\frac{1}{M}\sum_{m_2=1}^{M} p(Q^*|\ \phi^{(m_2)}, \lambda^*, R^*, \mathbb{F}^{T\ (m_2)}, \mathbb{X}^T) \ \longrightarrow_{a.s.}\ p(Q^*|\ \lambda^*, R^*, \mathbb{X}^T)$$

where $\mathbb{F}^{T\ (m_2)} \sim \mathbb{F}^T |\ \phi^{(m_2)}, Q^*, \lambda^*, R^*, \mathbb{X}^T$, and $\phi^{(m_2)} \sim \phi |\ Q^*, \lambda^*, R^*, \mathbb{F}^{T\ (m_1)}, \mathbb{X}^T$

$$\frac{1}{M} \sum_{m_3=1}^{M} p(\phi^*, Q^*, \lambda^*, R^* |\ \mathbb{F}^{T\ (m_3)}, \mathbb{X}^T) \ \longrightarrow_{a.s.} \ p(\phi^*, Q^*, \lambda^*, R^* |\ \mathbb{X}^T)$$

where $\mathbb{F}^{T\ (m_3)} \sim \mathbb{F}^T |\ \phi^*, Q^*, \lambda^*, R^*, \mathbb{X}^T$.

Appendix D: Dynamic Stochastic General Equilibrium Model: Log Linear Approximation

$\hat{x}_t$ denotes deviation from its steady state in percent. $\kappa \equiv (1-\xi)(1-\beta\xi)/\xi(1+\varphi\theta)$ and $\tilde{\kappa} \equiv (1-\tilde{\xi})(1-\beta\tilde{\xi})/\tilde{\xi}$

$$\begin{aligned}\hat{\lambda}_t &= \hat{\lambda}_{t+1} + E_t(i_t - \pi_{t+1}) + \hat{\beta}_t \\ \hat{\lambda}^*_t &= \hat{\lambda}^*_{t+1} + E_t(i^*_t - \pi^*_{t+1}) + \hat{\beta}^*_t\end{aligned}$$

$$\begin{aligned}\hat{\lambda}_t &= E_t\left[\frac{-\sigma(1+\beta\omega^2)\hat{c}_t + \sigma\omega\hat{c}_{t-1} + \sigma\beta\omega\hat{c}_{t+1} - \beta\omega(1-\omega)\hat{\beta}_t}{(1-\omega)(1-\beta\omega)}\right] \\ \hat{\lambda}^*_t &= E_t\left[\frac{-\sigma(1+\beta\omega^2)\hat{c}^*_t + \sigma\omega\hat{c}^*_{t-1} + \sigma\beta\omega\hat{c}^*_{t+1} - \beta\omega(1-\omega)\hat{\beta}^*_t}{(1-\omega)(1-\beta\omega)}\right]\end{aligned}$$

$$\begin{aligned}\hat{\beta}_t &= -\psi\beta c_t + \varsigma_t \\ \hat{\beta}^*_t &= -\psi\beta c^*_t + \varsigma_t \\ y_t &= \gamma y_{H,t} + (1-\gamma)y_{N,t} \\ y^*_t &= \gamma y^*_{F,t} + (1-\gamma)y^*_{N,t} \\ y_{H,t} &= (1-\alpha)\alpha\eta(\tau_{H,t} - \tau_{F,t}) + (1-\gamma)[\alpha x_t + (1-\alpha)x^*_t] + \alpha c_t + (1-\alpha)c^*_t \\ y^*_{F,t} &= (1-\alpha)\alpha\eta(\tau_{F,t} - \tau_{H,t}) + (1-\gamma)[\alpha x^*_t + (1-\alpha)x_t] + \alpha c^*_t + (1-\alpha)c_t \\ y_{N,t} &= -\gamma x_t + c_t \\ y^*_{N,t} &= -\gamma x^*_t + c^*_t \\ nx_t &= \gamma\left[y_{H,t} - c_t - (1-\alpha)\tau_{H,t} - (1-\gamma)x_t\right] \\ nx^*_t &= \gamma\left[y^*_{F,t} - c^*_t - (1-\alpha)\tau_{F,t} - (1-\gamma)x^*_t\right]\end{aligned}$$

$$\begin{aligned}\tau_{H,t} &= \tau_{H,t-1} + \pi_{F,t} - \pi_{H,t} \\ \tau_{F,t} &= \tau_{F,t-1} + \pi^*_{H,t} - \pi^*_{F,t} \\ x_t &= x_{t-1} + \pi_{N,t} - \pi_{H,t} - (1-\alpha)(\tau_{H,t} - \tau_{H,t-1}) \\ x^*_t &= x^*_{t-1} + \pi^*_{N,t} - \pi^*_{F,t} - (1-\alpha)(\tau_{F,t} - \tau_{F,t-1})\end{aligned}$$

$$\begin{aligned}E_t(\pi_{H,t} - \delta\pi_{H,t-1}) &= \kappa E_t\left[mc_{H,t} + \beta(\pi_{H,t+1} - \delta\pi_{H,t})\right] \\ E_t(\pi^*_{F,t} - \delta\pi^*_{F,t-1}) &= \kappa E_t\left[mc^*_{F,t} + \beta(\pi^*_{F,t+1} - \delta\pi^*_{F,t})\right] \\ E_t(\pi_{N,t} - \delta\pi_{N,t-1}) &= \kappa E_t\left[mc_{N,t} + \beta(\pi_{N,t+1} - \delta\pi_{N,t})\right] \\ E_t(\pi^*_{N,t} - \delta\pi^*_{N,t-1}) &= \kappa E_t\left[mc^*_{N,t} + \beta(\pi^*_{N,t+1} - \delta\pi^*_{N,t})\right]\end{aligned}$$

$$
\begin{aligned}
mc_{H,t} &= \varphi y_{H,t} - (1+\varphi)a_t - \lambda_t + (1-\alpha)\tau_{H,t} + (1-\gamma)x_t \\
mc^*_{F,t} &= \varphi y^*_{F,t} - (1+\varphi)a^*_t - \lambda^*_t + (1-\alpha)\tau_{F,t} + (1-\gamma)x^*_t \\
mc_{N,t} &= \varphi y_{N,t} - (1+\varphi)a_t - \lambda_t - \gamma x_t \\
mc^*_{N,t} &= \varphi y^*_{N,t} - (1+\varphi)a^*_t - \lambda^*_t - \gamma x^*_t
\end{aligned}
$$

$$
\begin{aligned}
\pi_t &= \gamma\pi_{H,t} + (1-\gamma)\pi_{N,t} + \gamma(1-\alpha)(\tau_{H,t} - \tau_{H,t-1}) \\
\pi^*_t &= \gamma\pi^*_{F,t} + (1-\gamma)\pi^*_{N,t} + \gamma(1-\alpha)(\tau_{F,t} - \tau_{F,t-1}) \\
\pi_{F,t} - \tilde{\delta}\pi_{F,t-1} &= \tilde{\kappa}E_t\left[\hat{\psi}_{F,t} + \beta(\pi_{F,t+1} - \tilde{\delta}\pi_{F,t})\right] \\
\pi^*_{H,t} - \tilde{\delta}\pi^*_{H,t-1} &= \tilde{\kappa}E_t\left[\hat{\psi}^*_{H,t} + \beta(\pi^*_{H,t+1} - \tilde{\delta}\pi^*_{H,t})\right] \\
\hat{\psi}_{F,t} &= \hat{\psi}_{F,t-1} + \Delta e_t + \pi^*_{F,t} - \pi_{F,t} \\
\hat{\psi}^*_{H,t} &= \hat{\psi}^*_{H,t-1} - \Delta e_t + \pi_{H,t} - \pi^*_{H,t}
\end{aligned}
$$

$$
\begin{aligned}
i_t &= \rho i_{t-1} + (1-\rho)(\phi_\pi E_t\pi_{t+1} + \phi_y y_t) + \epsilon_t \\
i^*_t &= \rho^* i^*_{t-1} + (1-\rho^*)(\phi^*_\pi E_t\pi^*_{t+1} + \phi^*_y y^*_t + \phi^*_i i_t + \phi^*_e \Delta e_t) + \epsilon^*_t \\
i_t - i^*_t &= E_t\Delta e_{t+1} + \mu \\
b_t &= \beta^{-1}b_{t-1} + nx_t \\
q_t &= \hat{\psi}_{F,t} + (1-\alpha)\tau_{F,t} + \alpha\tau_{H,t} + (1-\gamma)(x^*_t - x_t)
\end{aligned}
$$