Intangibles, Inequality and Stagnation

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Questions

How does managerial skill (intangibles) accumulate across generations?

How does a small difference in initial endowment lead to a large inequality across people?

How does a negative shock to endowment generate a persistent stagnation and a rise in inequality?

Approach

Overlapping Generations Model with On-the-Job-Training + Financial Friction
Skilled managers can train young workers to become future managers

Training is costly: Investment in intangible capital

Young workers are heterogeneous in initial endowment

Managers offer two options

simple labor: pay wage for labor

career path: apprentice wage and training to be future manager

Without financing constraint, present value of income would be the same between two → no inequality in permanent income
With financing constraint

intangible investment is lower for a given interest rate

rich young workers receive more training, while poor young workers receive no training - working as simple workers for life

skill level becomes diverse across managers

more skilled managers train richer young more intensively →

large inequality

temporary decrease in initial endowment →

intangible investment ↓, aggregate production ↓, inequality ↑ persistently
Model

Overlapping generations: a unit measure of agents are born every period and lives for 2 periods.

When young, each agent is endowed with goods $e$ and initial skill $\kappa$.

$$e \sim G(e) \text{ on } [0, \bar{e}]$$

$$G(e) = 1 - \omega + \omega \frac{e}{\bar{e}}$$

In Basic Model, $\kappa$ is the same across young.

No aggregate nor idiosyncratic uncertainty.

Everyone is endowed with one unit of labor. Can work as a worker or a manager.
When a manager of skill $k$ hires $n$ labor, output is

$$y = Ak^\alpha n^{1-\alpha}$$

Each manager can train $n^m$ number of young workers to become future managers with skill $k'$ by investing $i$ goods

$$n^m k' = \frac{1}{b} \left[ i^\gamma + \psi (\hat{k})^\gamma \right]^{1/\gamma}$$

$$\hat{k} = \left[ \eta k^\rho + \bar{\eta}(n^m\bar{k})^\rho + (1 - \eta - \bar{\eta})(n^m\kappa)^\rho \right]^{1/\rho}$$

where $\rho, \gamma < 1 \rightarrow$

$$i = \left[ (bn^mk')^{\gamma - \psi (\hat{k})^{\gamma}} \right]^{1/\gamma}$$

$$= \Phi (n^m, k', k, \bar{K}, K_0)$$

where $\bar{K} = n^m\bar{k}$ and $K_0 = n^m\kappa$
Utility function of agent born at date $t$ is given by

$$U = U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$$

Let $m$ and $F(k)$ be measure and skill distribution of present managers

$$2 - m = \int_{k_{\min}}^{k_{\max}} n(k) dF(k)$$

$$\int_{k_{\min}}^{k_{\max}} y(k) dF(k) + \int_0^{\bar{e}} e dG(e)$$

$$= \int_0^{\bar{e}} [c_t^o(e) + c_t^y(e)] dG(e) + \int_{k_{\min}}^{k_{\max}} i(k) dF(k)$$
Market Economy of No Financial Friction for Comparison

Managers offer two options to young workers

(i) Simple labor contract: pay \( w \) for one unit of labor

(ii) Career package: pay \( w^m \) and train to acquire skill \( k' \)

Assuming both being accepted, the participation constraint is

\[
w + \frac{w'}{R} = w^m + \frac{\pi(k', z')}{R}
\]

(\text{PC})

Each manager chooses \((n^n, n^m, k')\) to maximize the profit

\[
\pi(k, z) = Ak^\alpha(n^n + n^m)^{1-\alpha} - wn^n - w^mn^m - \Phi(n^m, k', k, K, K_0),
\]

subject to (PC) for a given \((w, R, \bar{k})\)
The first order conditions are

\[ MPL = (1 - \alpha)A \left( \frac{k}{n} \right)^\alpha = w \]

\[ = w^m + \frac{d\Phi}{dn^m_{MCn^m}} = w^m + \frac{i}{n^m}(1 + qs) \]

where

\[ q = \frac{\psi \left( \hat{k} \right)^\gamma}{i^\gamma} \quad \text{and} \quad s = \frac{\eta k^\rho}{\eta k^\rho + \eta \bar{K}^\rho + (1-\eta-\bar{\eta})K_0^\rho} \]

\[ \frac{\partial \Phi}{\partial k'_M C k'_V} = n^m \cdot \frac{1}{R} \cdot \frac{\partial \pi(k', z')}{\partial k'_M V k'_V} \]
The equilibrium profit is

\[ \pi(k, z) = y - wn - (w - w^m)n^m - i \]
\[ = \alpha y + iqs \]

The participation constraint \((PC)\) implies

\[ \frac{i}{n^m}(1 + qS) = \frac{1}{R} \left[ \alpha \frac{Y'}{m'} + i'q's' - (1 - \alpha)\frac{Y'}{2 - m'} \right] \]

**Claim:** Without financial friction, there is a competitive equilibrium in which

(a) All managers have the same skill within the generation

(b) the allocation is efficient if no externality, \( \bar{\eta} = \bar{s} = 0 \)
No Financing Constraint

A: simple worker’s income
B: manager’s income
C: consumption of both

\[ e + w^m \]
\[ w' \]
\[ c' \]
\[ \pi' \]
Market Economy with Financing Constraint as Main Case

Each future manager can borrow against only up to $\theta \in (0, 1)$ fraction of future profit.

When $\theta$ and $e$ are small, the borrowing constraint is binding and the discounted utility of future manager is

$$V(w^m, \pi'; e) = \ln \left( e + w^m + \frac{\theta}{R} \pi' \right) + \beta \ln \left[ (1 - \theta) \pi' \right]$$

The participation constraint becomes

$$V(w^m, \pi'; e) \geq V(w, w'; e)$$

$$= \ln \left( \frac{e + w + \frac{w'}{R}}{1 + \beta} \right) + \beta \ln \left( \frac{\beta R e + w + \frac{w'}{R}}{1 + \beta} \right)$$
With financing constraint, we get:

(a) All young workers with endowment \( e \geq e^* > 0 \) are trained. All young workers with \( e < e^* \) are not trained.

(b) Skill level is diverse across managers in equilibrium

\[ k \sim F(k), \text{ on } [k_{\text{min}}, k_{\text{max}}] \]

(c) A more productive manager is matched with richer workers than less productive managers.
Manager of skill $k$ trains $n^m$ number of workers with endowment $e(k)$ to acquire skill $k'$ and hires $n^n$ simple workers to maximize the profit

$$\pi(k, z) = Ak^\alpha f(n^n + n^m) - wn^n - w^m n^m$$

$$-\Phi(n^m, k', k, K, K_0)$$

subject to the constraint of competitive package

$$V(w^m, \pi(k', z'); e(k)) \geq V^*(e(k))$$

$$\rightarrow$$

$$\frac{\partial \Phi(n^m, k', k, K, K_0)}{\partial k'}$$

$$= n^m \cdot \frac{\partial V(w^m, \pi(k', z'); e(k))}{\partial \pi'} \cdot \frac{\partial \pi'(k', z')}{\partial w^m} \cdot \frac{\partial \pi(k', z')}{\partial k'}$$
The indifference condition for marginal workers with endowment $e^*$ is

$$V(w^m(k_{min}), \pi(k'(k_{min}), z'); e^*) = V(w, w'; e^*)$$

Saving of simple workers equals borrowing of future managers as

$$\int_0^{e^*} \left( e + w - \frac{e + w + \frac{w'}{R}}{1 + \beta} \right) dG(e)$$

$$= \frac{\theta}{R} \int_{k_{min}}^{k_{max}} n^m(k) \pi(k'(k), z') dF(k)$$
With Financing Constraint
Marginal Young is Indifferent

A: simple worker’s income
B: manager’s income
C: worker’s consumption
D: manager’s consumption

\[(1 - \theta)\pi'\]

\[(w_m, \pi')\] to provide the same utility
With Financing Constraint
Rich Young is Trained More by Manager with Higher Skill

A: simple worker’s income
B: manager’s income
C: worker’s consumption
D: manager’s consumption

\( (w_m, \pi') \) to provide the same utility

\[
\pi' = e + \frac{\theta}{R} \pi' + w_m
\]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>fraction of positive endowment $\omega$</td>
<td>0.7</td>
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<tr>
<td>upper bound of endowment $\bar{e}$</td>
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<td>initial skill $\kappa$</td>
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<td>share of intangibles $\alpha$</td>
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<td>elasticity parameter in skill production $\gamma$</td>
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<td>elasticity parameter in skill composite $\rho$</td>
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<td>share parameter of skill composite $\psi$</td>
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<td>share parameter of manager’s skill $\eta$</td>
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<td>share parameter of society’s skill $\overline{\eta}$</td>
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<td>utility discount $\beta$</td>
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<td>borrowing constraint $\theta$</td>
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<td>initial skill of talented $\kappa_h$</td>
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<tr>
<td>initial skill of less talented $\kappa_l$</td>
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</tr>
<tr>
<td>fraction of talented $\varepsilon$</td>
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</table>
Present value of income and consumption when young

Income and consumption of the young

Income and consumption of the old

percentile in terms of present value income when young
Dynamic Response to Negative Shock to Endowment

When the fraction of young workers with positive endowment decreases unexpectedly and recovers gradually

(a) Managers with high skill train a smaller number of rich trainees more intensively

(b) Managers with low skill train a larger number of trainees less intensively

(c) Number of managers and aggregate intangibles decrease persistently

(d) Wage rate decreases and real interest rate increases persistently
Suppose the initial skill (talent) of workers are heterogeneous, and that
\[ \kappa = \kappa_h, \text{ for a fraction } \varepsilon \text{ of workers,} \]
\[ \kappa = \kappa_l < \kappa_h, \text{ for a fraction } 1 - \varepsilon \text{ of workers.} \]

We assume \( \varepsilon \) is small.

The initial endowment is independent of initial skill and \( e \sim G(e) \).
With financing constraint

(a) Young talented workers are trained iff $e \geq e_h^* \geq 0$

(b) Young less talented workers are trained iff $e \geq e_l^* > e_h^*$

(c) The skill of managers is diverse in equilibrium

(d) A more productive manager is matched with richer workers than less productive managers, controlling the worker talent

(e) A more productive manager is more likely to train more talented workers

(f) Poor talented workers are more likely to receive training but receive less training than less talented rich workers
Density of firms training skilled/unskilled workers

- **Unskilled Workers**: Blue line
- **Skilled Workers**: Red line
Conclusion

Training (and higher education) are accumulation of intangible capital across generations

With financing constraint, richer young people have an advantage in receiving better training

→ Small difference in endowment leads to a large inequality: Propagation across people

Temporary decrease in endowment causes a persistent decrease in intangible investment and rise in inequality: Propagation across time
Policy Implications

Government can provide loan for workers to receive training

But, government must be better than private lenders in enforcing debt repayment

Government can provide subsidy for training poor young

But, efficient allocation needs rationing of the training

→ People start investing earlier to acquire better initial skill

Government can improve basic education to improve the initial skill: Equal opportunity instead of equal outcome