Intangibles, Inequality and Stagnation

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Questions

How does managerial skill (intangibles) accumulate across generations?

How does a small difference in initial endowment lead to a large inequality across people?

How does a negative shock to endowment generate a persistent stagnation and a rise in inequality?

Approach

Overlapping Generations Model with On-the-Job-Training + Financial Friction

Skilled managers can train young workers to become future managers

Training is costly : Investment in intangible capital

Young workers are heterogeneous in initial endowment

Managers offer two options

simple labor: pay wage for labor

career path: apprentice wage and training to be future manager

Without financing constraint, present value of income would be the same between two $\longrightarrow$ no inequality in permanent income

With financing constraint

intangible investment is lower for a given interest rate

rich young workers receive more training, while poor young workers receive no training - working as simple workers for life

skill level becomes diverse across managers

more skilled managers train richer young more intensively $\longrightarrow$ large inequality

temporary decrease in initial endowment $\longrightarrow$

intangible investment $\Downarrow$, aggregate production $\Downarrow$, inequality $\Uparrow$ persistently

Model

Overlapping generations: a unit measure of agents are born every period and lives for 2 periods

When young, each agent is endowed with goods e and initial skill κ

$$e \sim G(e) \text{ on } [0, \bar{e}]$$

$$G(e) = 1 - \omega + \omega \frac{e}{\bar{e}}$$

In Basic Model, κ is the same across young

No aggregate nor idiosyncratic uncertainty

Everyone is endowed with one unit of labor. Can work as a worker or a manager

When a manger of skill k hires n labor, output is

$$y = Ak^{\alpha}n^{1-\alpha}$$

Each manager can train n^m number of young workers to become future managers with skill k' by investing i goods

$$n^m k' = \frac{1}{b}\left[i^{\gamma} + \psi\left(\hat{k}\right)^{\gamma}\right]^{1/\gamma}$$

$$\hat{k} = \left[\eta k^{\rho} + \overline{\eta}(n^m\overline{k})^{\rho} + (1-\eta-\overline{\eta})\left(n^m\kappa\right)^{\rho}\right]^{\frac{1}{\rho}}$$

where $\rho, \gamma < 1 \rightarrow$

$$\begin{aligned} i &= \left[(bn^m k')^{\gamma} - \psi\left(\hat{k}\right)^{\gamma}\right]^{\frac{1}{\gamma}} \\ &= \mathbf{\Phi}\left(n^m, k', k, \overline{K}, K_0\right) \end{aligned}$$

where $\overline{K} = n^m\overline{k}$ and $K_0 = n^m\kappa$

Utility function of agent born at date t is given by

$$U = U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$$

Let m and $F(k)$ be measure and skill distribution of present managers

$$2 - m = \int_{k_{\min}}^{k_{\max}} n(k) dF(k)$$

$$\int_{k_{\min}}^{k_{\max}} y(k) dF(k) + \int_0^{\overline{e}} e dG(e)$$

$$= \int_0^{\overline{e}} [c_t^o(e) + c_t^y(e)] dG(e) + \int_{k_{\min}}^{k_{\max}} i(k) dF(k)$$

Market Economy of No Financial Friction for Comparison

Managers offer two options to young workers

(i) Simple labor contract: pay w for one unit of labor

(ii) Career package: pay w^m and train to acquire skill k'

Assuming both being accepted, the participation constraint is

$$w + \frac{w'}{R} = w^m + \frac{\pi(k', z')}{R} \qquad \text{(PC)}$$

Each manager chooses (n^n, n^m, k') to maximize the profit

$$\begin{aligned} \pi(k, z) \;=\; & Ak^{\alpha}(n^n + n^m)^{1-\alpha} - wn^n - w^m n^m \\ & -\Phi(n^m, k', k, \overline{K}, K_0), \end{aligned}$$

subject to (PC) for a given $(w, R, \overline{k})$

The first order conditions are

$$MPL = (1-\alpha)A\left(\frac{k}{n}\right)^{\alpha} = w$$

$$= w^m + \frac{d\Phi}{\underset{MCn^m}{dn^m}} = w^m + \frac{i}{n^m}(1+qs)$$

where

$$q = \frac{\psi\left(\hat{k}\right)^{\gamma}}{i^{\gamma}} \text{ and } s = \frac{\eta k^{\rho}}{\eta k^{\rho} + \overline{\eta}\overline{K}^{\rho} + (1\text{-}\eta\text{-}\overline{\eta})K_0^{\rho}}$$

$$\frac{\partial\Phi}{\underset{MCk'}{\partial k'}} = n^m \cdot \frac{1}{R} \cdot \frac{\partial\pi(k',z')}{\underset{MVk'}{\partial k'}}$$

The equilibrium profit is

$$\begin{aligned}\pi(k,z) &= y - wn - (w - w^m)n^m - i \\ &= \alpha y + iqs\end{aligned}$$

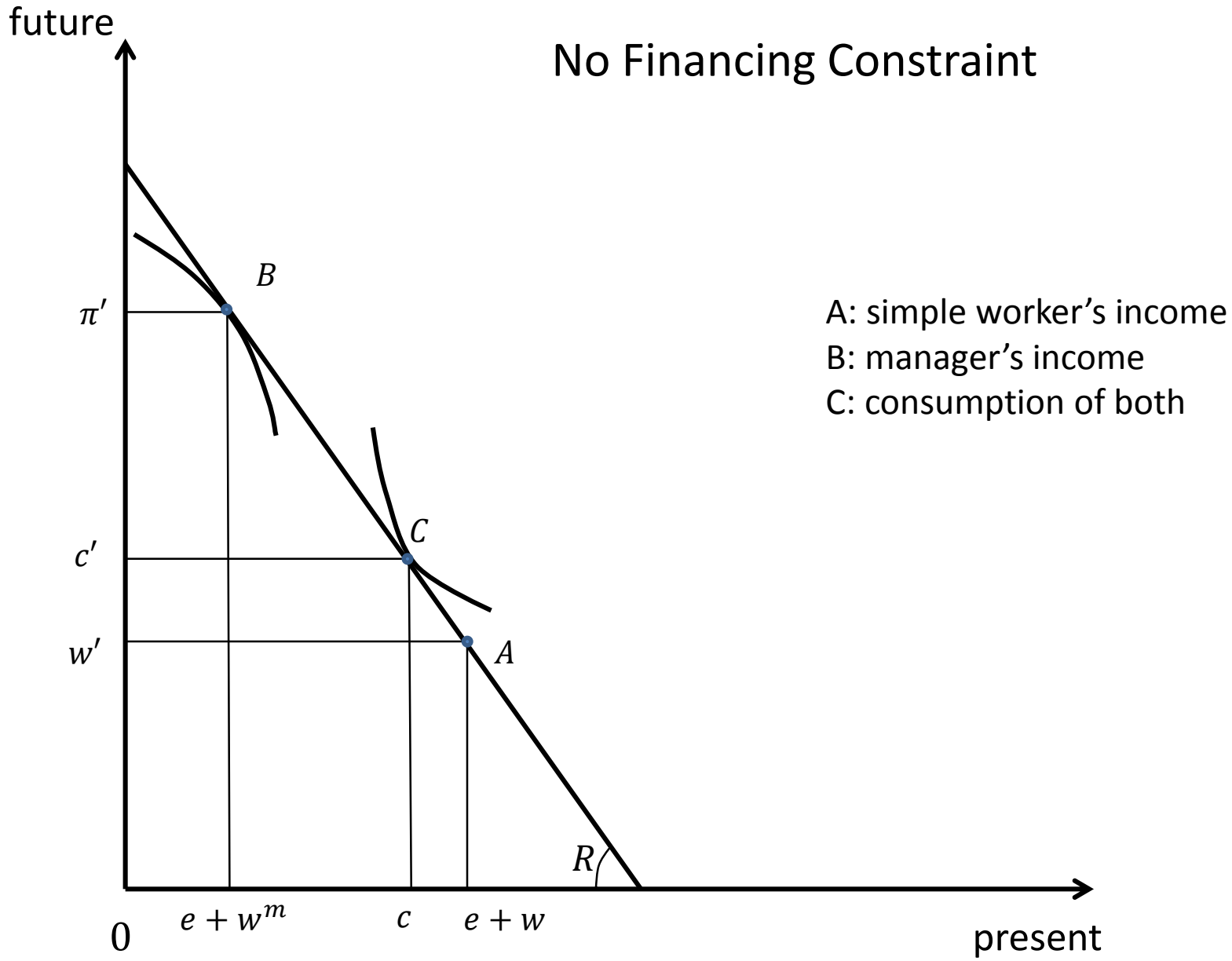
The participation constraint (PC) implies

$$\frac{i}{n^m}(1+qs) = \frac{1}{R}\left[\alpha\frac{Y'}{m'} + i'q's' - (1-\alpha)\frac{Y'}{2-m'}\right]$$

Claim: Without financial friction, there is a competitive equilibrium in which

(a) All managers have the same skill within the generation

(b) the allocation is efficient if no externality, $\overline{\eta} = \overline{s} = 0$


future
No Financing Constraint
A: simple worker's income
B: manager's income
C: consumption of both
B
π'
C
c'
w'
A
R
0
$e + w^m$
c
$e + w$
present

Market Economy with Financing Constraint as Main Case

Each future manager can borrow against only up to $\theta \in (0,1)$ fraction of future profit

When θ and e are small, the borrowing constraint is binding and the discounted utility of future manager is

$$V(w^m, \pi'; e) = \ln\left(e + w^m + \frac{\theta}{R}\pi'\right) + \beta \ln\left[(1-\theta)\,\pi'\right]$$

The participation constraint becomes

$$V(w^m, \pi'; e) \geq V(w, w'; e)$$

$$= \ln\left(\frac{e + w + \frac{w'}{R}}{1+\beta}\right) + \beta \ln\left(\beta R \frac{e + w + \frac{w'}{R}}{1+\beta}\right)$$

With financing constraint, we get:

(a) All young workers with endowment $e \geq e^* > 0$ are trained. All young workers with $e < e^*$ are not trained

(b) Skill level is diverse across managers in equilibrium

$$k \sim F(k), \text{ on } [k_{\text{min}}, k_{\text{max}}]$$

(c) A more productive manager is matched with richer workers than less productive managers

Manager of skill k trains n^m number of workers with endowment $e(k)$ to acquire skill k' and hires n^n simple workers to maximize the profit

$$\begin{aligned}\pi(k,z) \;=\; & Ak^{\alpha} f(n^n + n^m) - wn^n - w^m n^m \\ & -\mathbf{\Phi}(n^m, k', k, \overline{K}, K_0)\end{aligned}$$

subject to the constraint of competitive package

$$V(w^m, \pi(k', z'); e(k)) \geq V^*(e(k))$$

$\longrightarrow$

$$\frac{\partial \mathbf{\Phi}(n^m, k', k, \overline{K}, K_0)}{\partial k'}$$

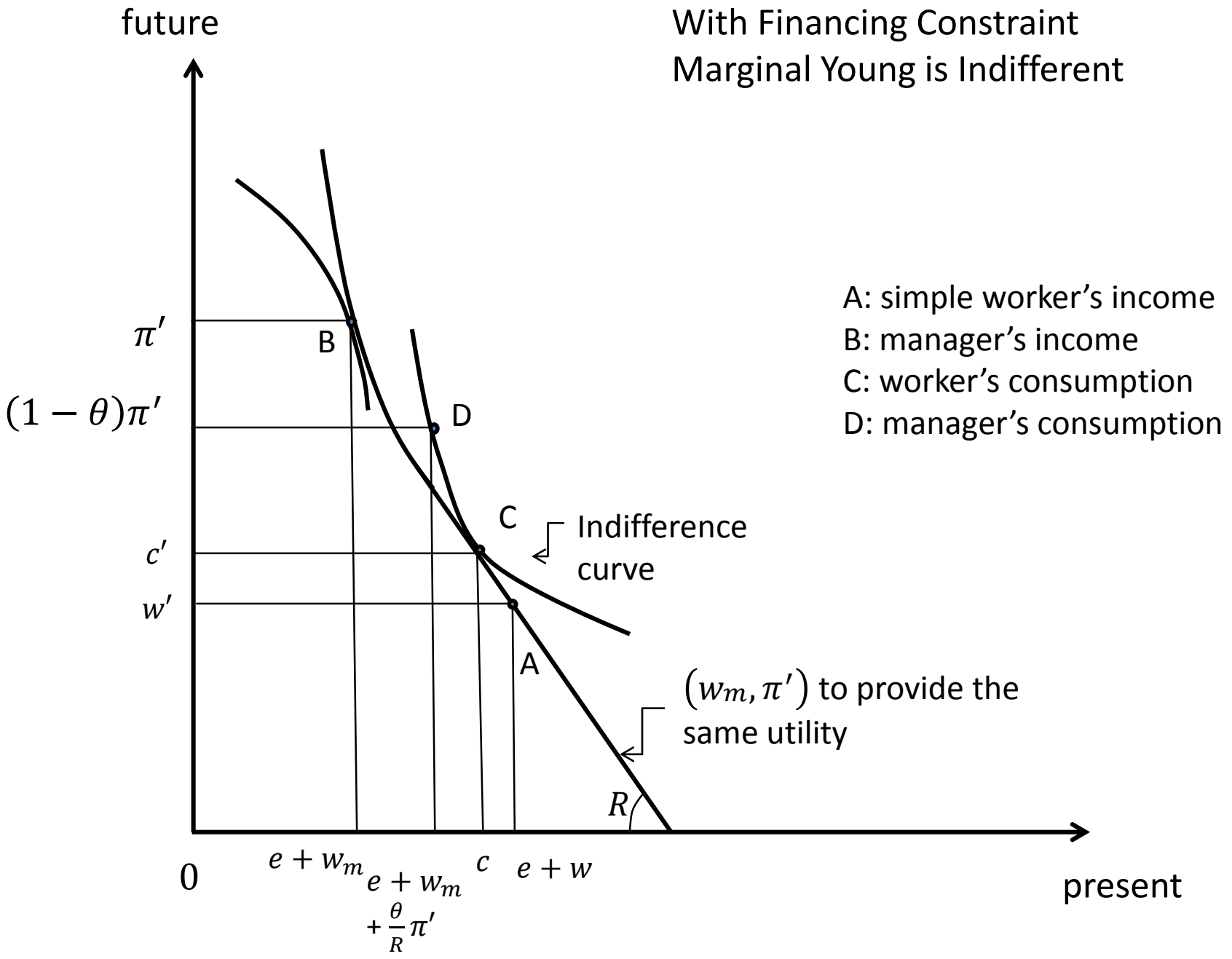
$$= n^m \cdot \frac{\frac{\partial V(w^m, \pi(k', z'); e(k))}{\partial \pi'}}{\frac{\partial V(w^m, \pi(k', z'); e(k))}{\partial w^m}} \cdot \frac{\partial \pi(k', z')}{\partial k'}$$

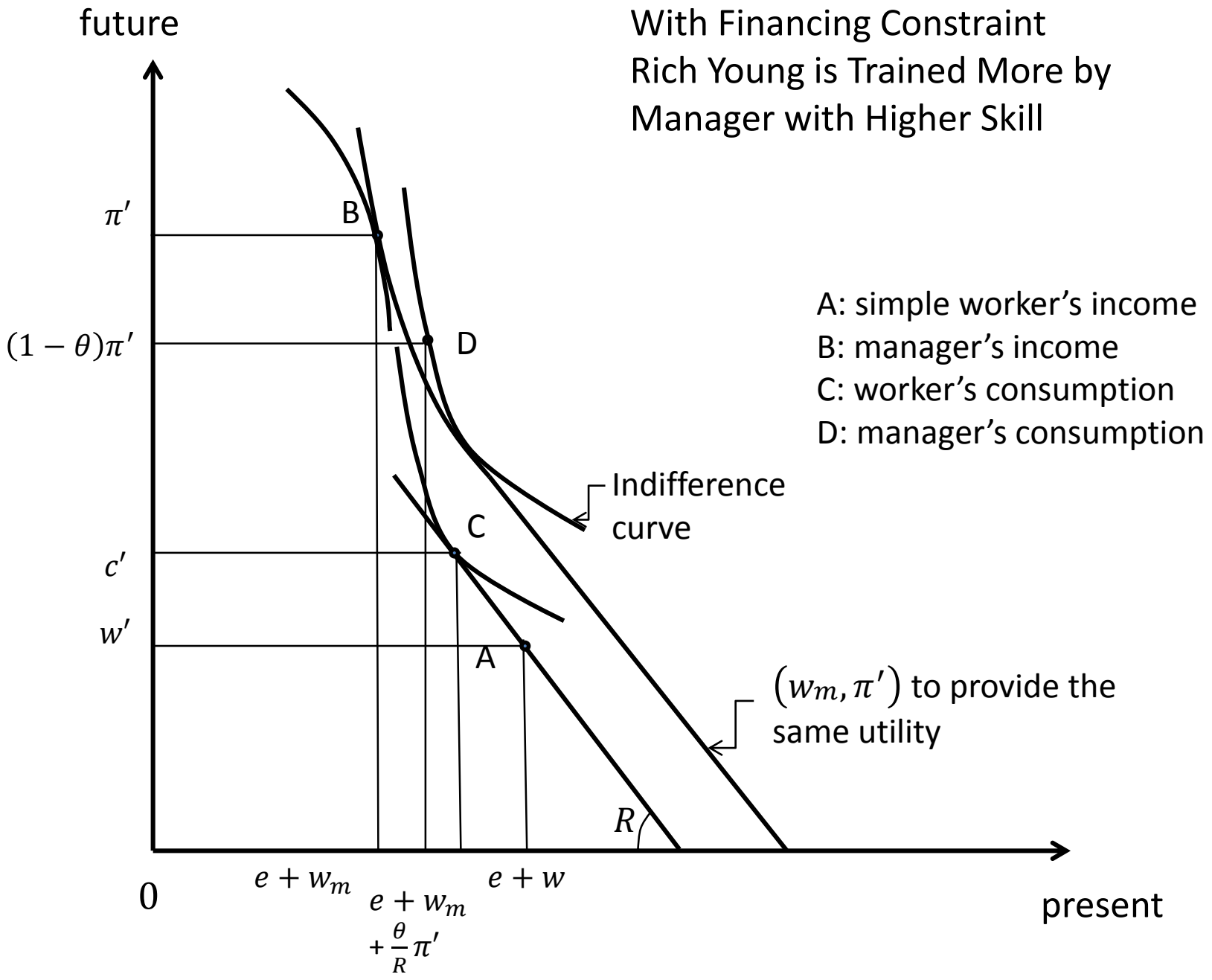
The indifference condition for marginal workers with endowment e^* is

$$V(w^m(k_{\text{min}}), \pi(k'(k_{\text{min}}), z'); e^*) = V(w, w'; e^*)$$

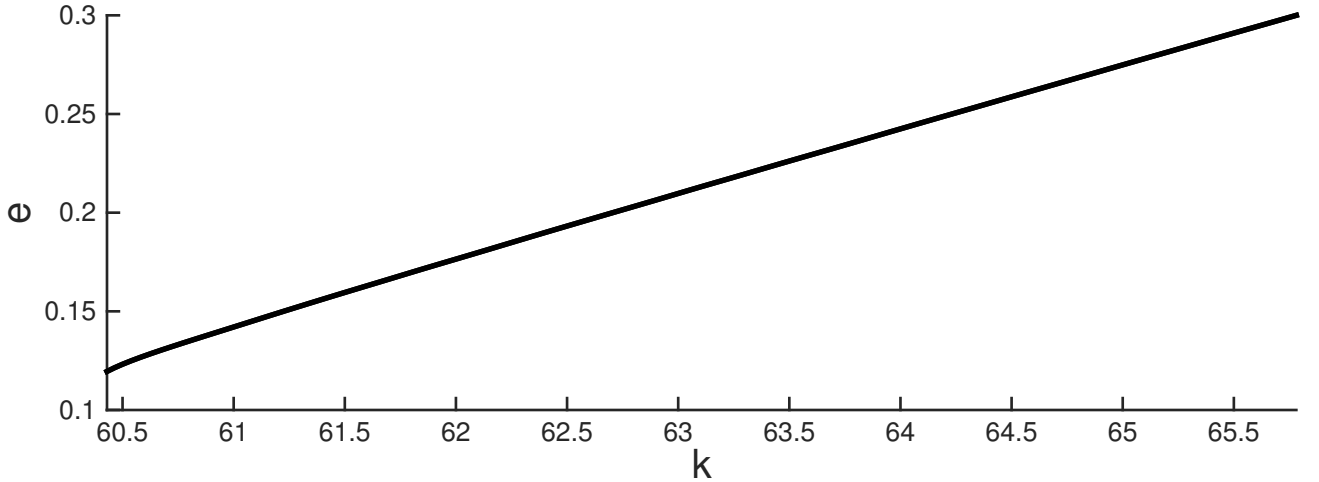
Saving of simple workers equals borrowing of future managers as

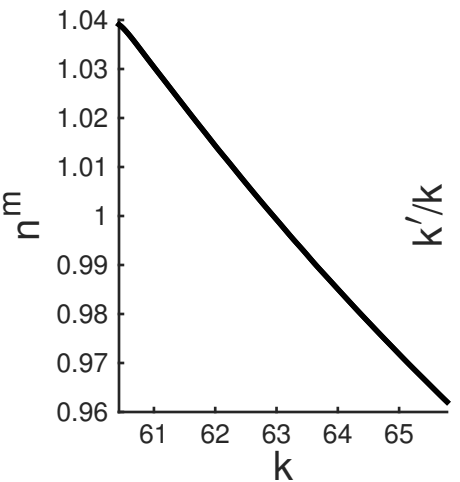
$$\begin{aligned} & \int_0^{e^*} \left(e + w - \frac{e + w + \frac{w'}{R}}{1 + \beta} \right) dG(e) \\ = \; & \frac{\theta}{R} \int_{k_{\text{min}}}^{k_{\text{max}}} n^m(k) \pi(k'(k), z') dF(k) \end{aligned}$$

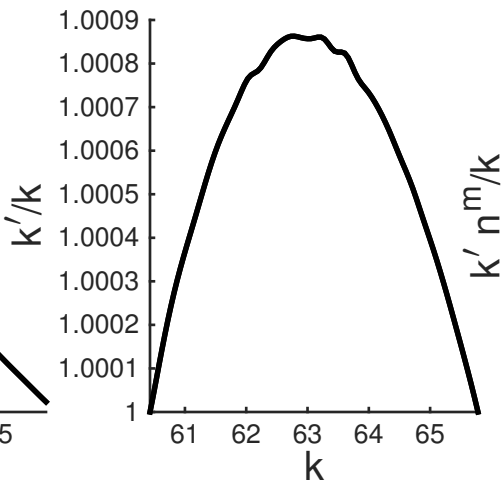

future
With Financing Constraint
Marginal Young is Indifferent
A: simple worker's income
B: manager's income
C: worker's consumption
D: manager's consumption
π'
$(1-\theta)\pi'$
c'
w'
B
D
C
Indifference curve
A
(w_m, π') to provide the same utility
R
0
$e + w_m$
$e + w_m + \frac{\theta}{R}\pi'$
c
$e + w$
present

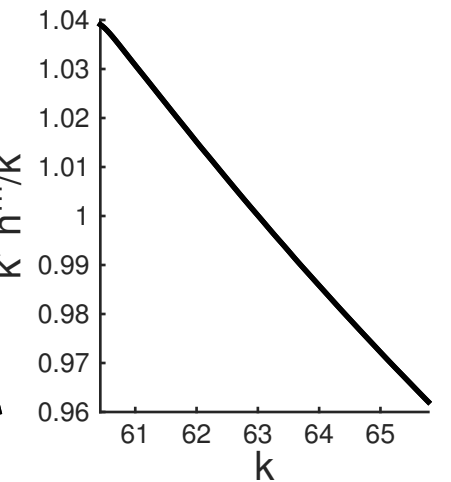

future
With Financing Constraint
Rich Young is Trained More by
Manager with Higher Skill
π'
B
$(1-\theta)\pi'$
D
A: simple worker's income
B: manager's income
C: worker's consumption
D: manager's consumption
Indifference
curve
C
c'
w'
A
(w_m, π') to provide the
same utility
R
0
$e + w_m$
$e + w_m$
$+\frac{\theta}{R}\pi'$
$e + w$
present

fraction of positive endowment ω	0.7
upper bound of endowment $\overline{e}$	0.3
initial skill κ	1
share of intangibles α	0.3
elasticity parameter in skill production γ	0.1
elasticity parameter in skill composite ρ	0.2
share parameter of skill composite ψ	2
share parameter of manager's skill η	0.4
share parameter of society's skill $\overline{\eta}$	0.2
utility discount β	0.75
borrowing constraint θ	0.2
initial skill of talented κ_h	1.1
initial skill of less talented κ_l	1
fraction of talented ε	0.07


0.3
0.25
0.2
0.15
0.1
e
60.5
61
61.5
62
62.5
63
63.5
64
64.5
65
65.5
k


1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
n^m
61
62
63
64
65
k


1.0009
1.0008
1.0007
1.0006
1.0005
1.0004
1.0003
1.0002
1.0001
1
k'/k
61
62
63
64
65
k


1.04
1.03
1.02
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0.99
0.98
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0.96
$k' n^m/k$
61
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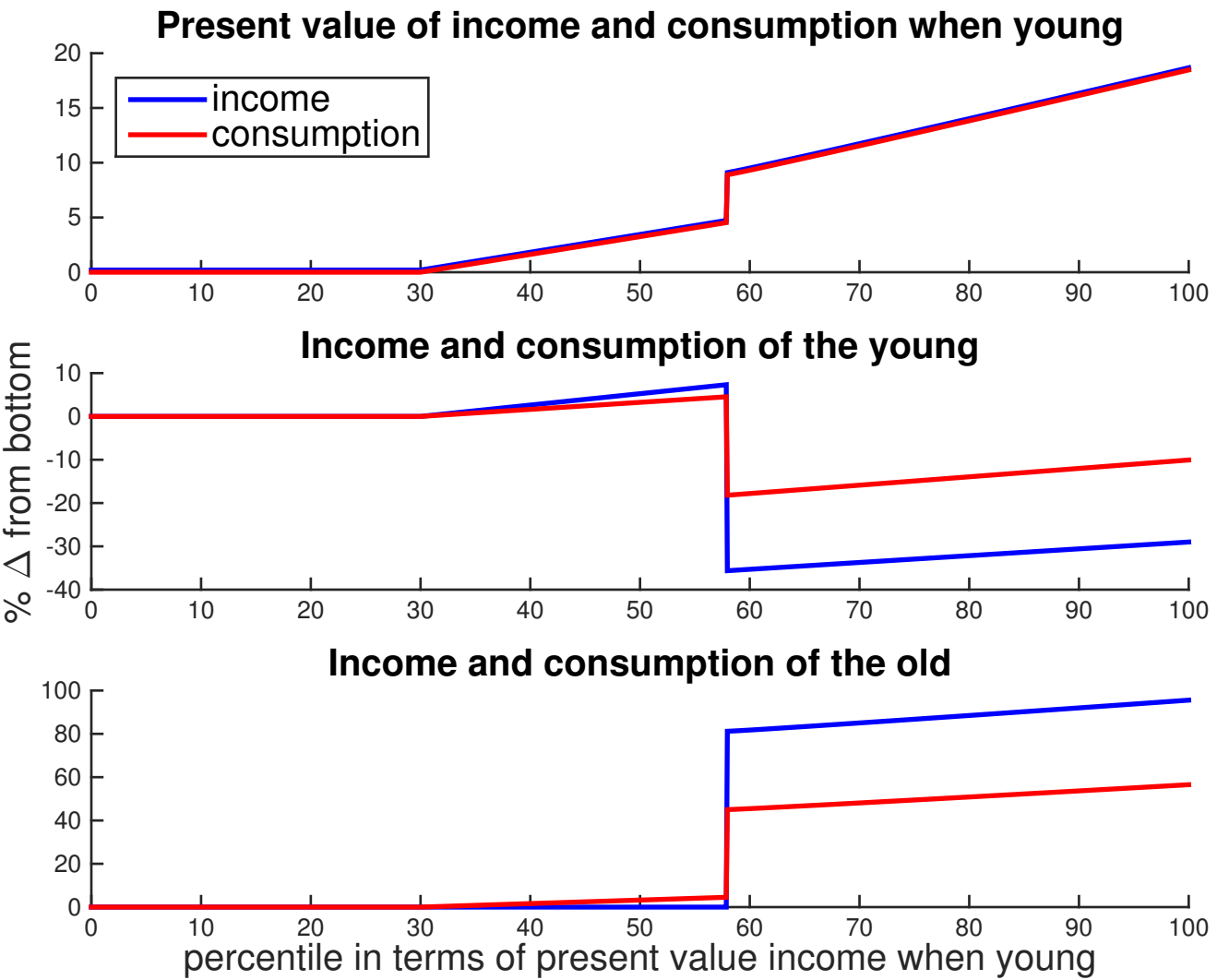
65
k


Present value of income and consumption when young
income
consumption
20
15
10
5
0
0
10
20
30
40
50
60
70
80
90
100
Income and consumption of the young
% Δ from bottom
10
0
-10
-20
-30
-40
Income and consumption of the old
100
80
60
40
20
0
percentile in terms of present value income when young

Dynamic Response to Negative Shock to Endowment

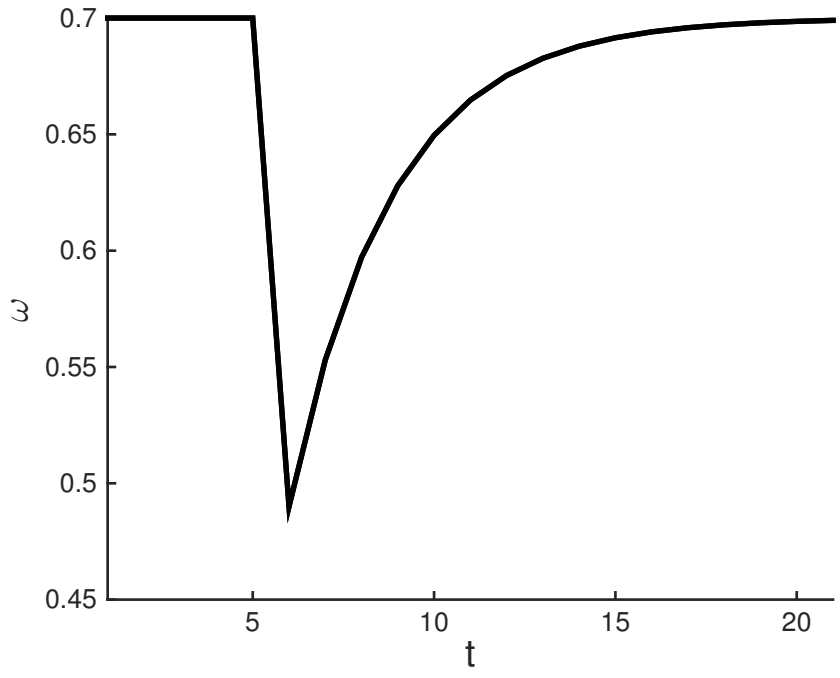
When the fraction of young workers with positive endowment decreases unexpectedly and recovers gradually

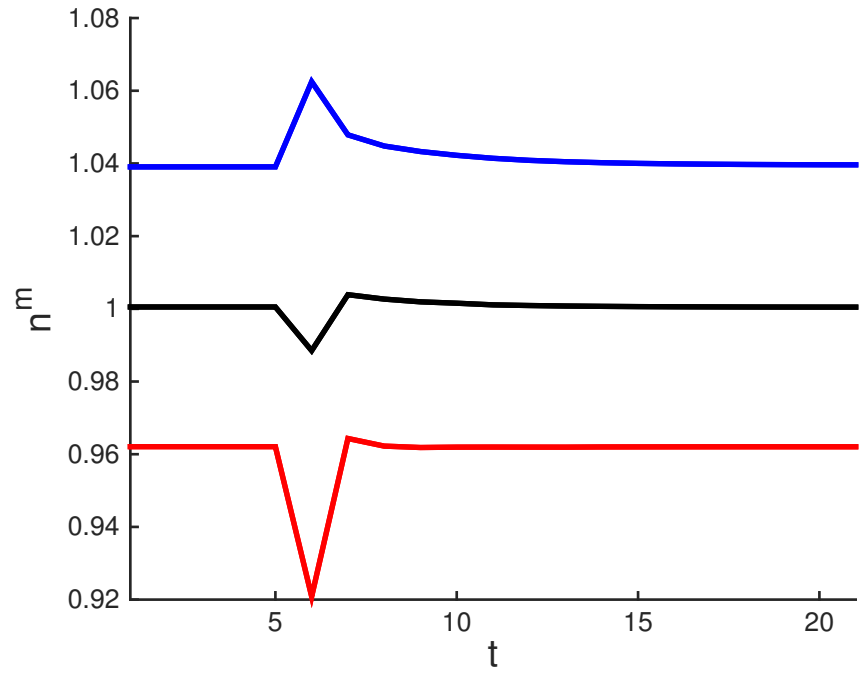
(a) Managers with high skill train a smaller number of rich trainees more intensively

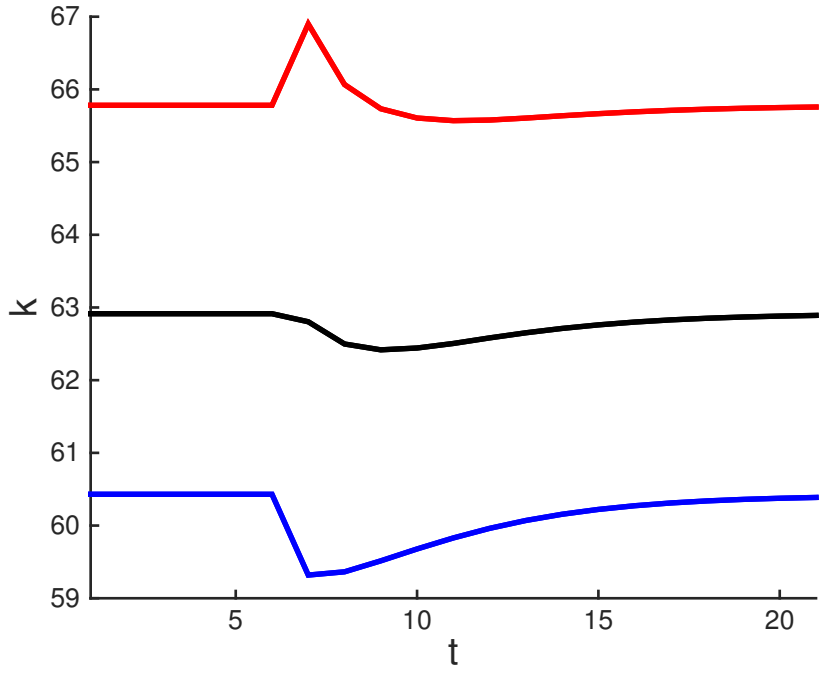
(b) Managers with low skill train a larger number of trainees less intensively

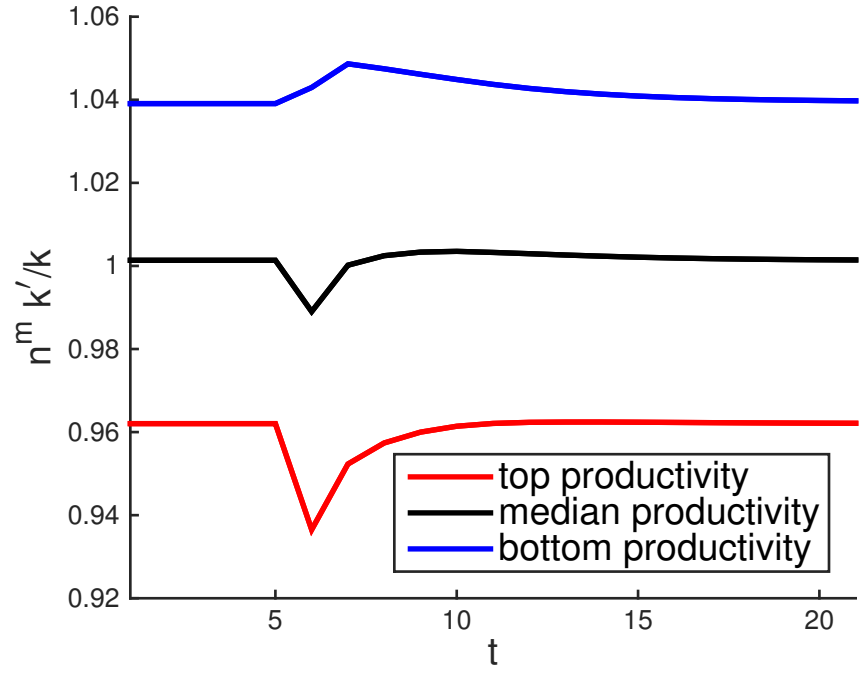
(c) Number of managers and aggregate intangibles decrease persistently

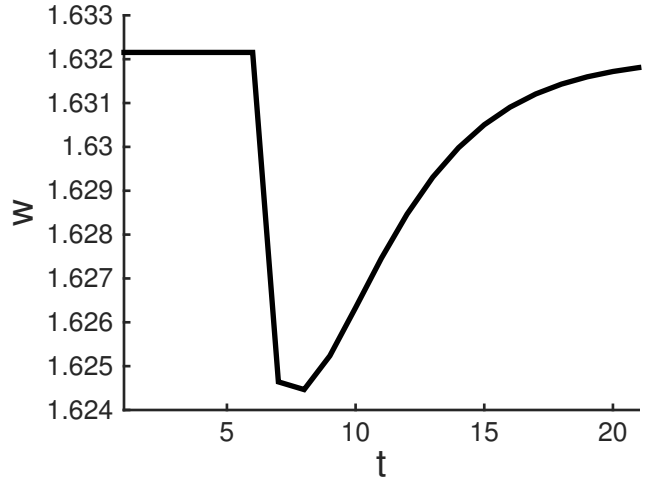
(d) Wage rate decreases and real interest rate increases persistently

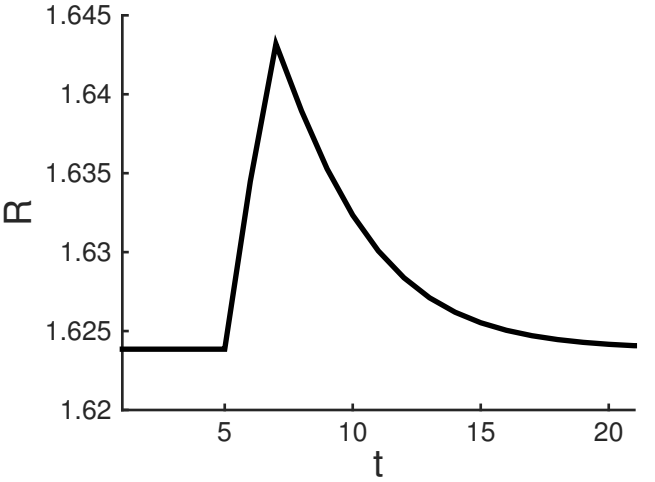

0.7
0.65
0.6
0.55
0.5
0.45
ω
5
10
15
20
t


1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
n^m
5
10
15
20
t


67
66
65
64
63
62
61
60
59
k
5
10
15
20
t


1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
n^m k′/k
5
10
15
20
t
top productivity
median productivity
bottom productivity


w
t
1.633
1.632
1.631
1.63
1.629
1.628
1.627
1.626
1.625
1.624
5
10
15
20


R
t
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1.64
1.635
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1.625
1.62
5
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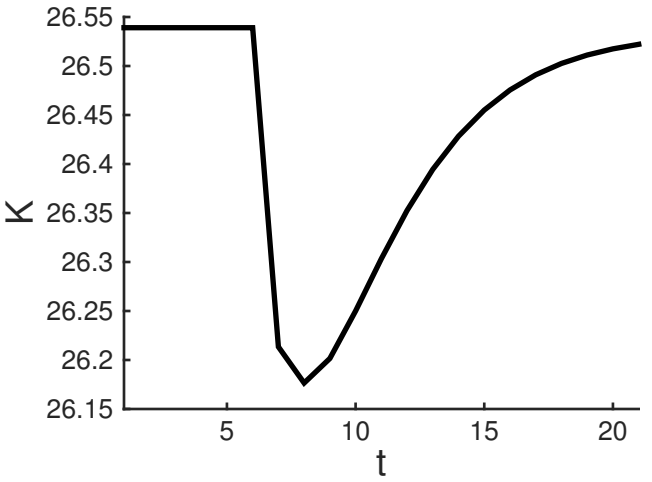
15
20


K
t
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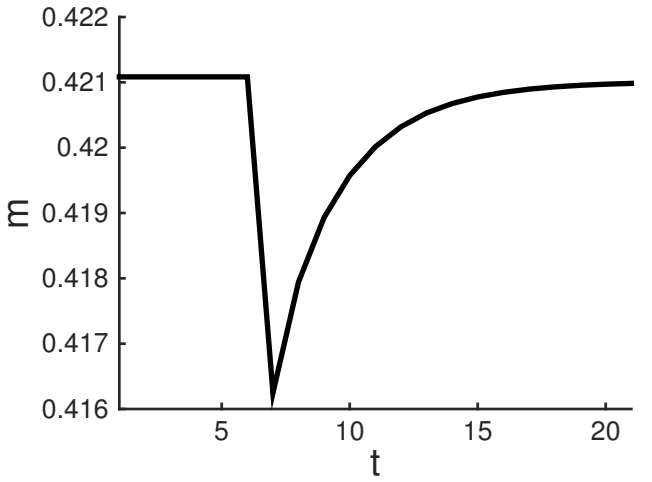
20


m
t
0.422
0.421
0.42
0.419
0.418
0.417
0.416
5
10
15
20

Full Model: Heterogeneous Initial Skill and Endowment

Suppose the initial skill (talent) of workers are heterogeneous, and that

$$\begin{aligned} \kappa &= \kappa_h, \text{ for a fraction } \varepsilon \text{ of workers,} \\ \kappa &= \kappa_l < \kappa_h, \text{ for a fraction } 1-\varepsilon \text{ of workers.} \end{aligned}$$

We assume ε is small

The initial endowment is independent of initial skill and $e \sim G(e)$

With financing constraint

(a) Young talented workers are trained iff $e \geq e_h^* \geq 0$

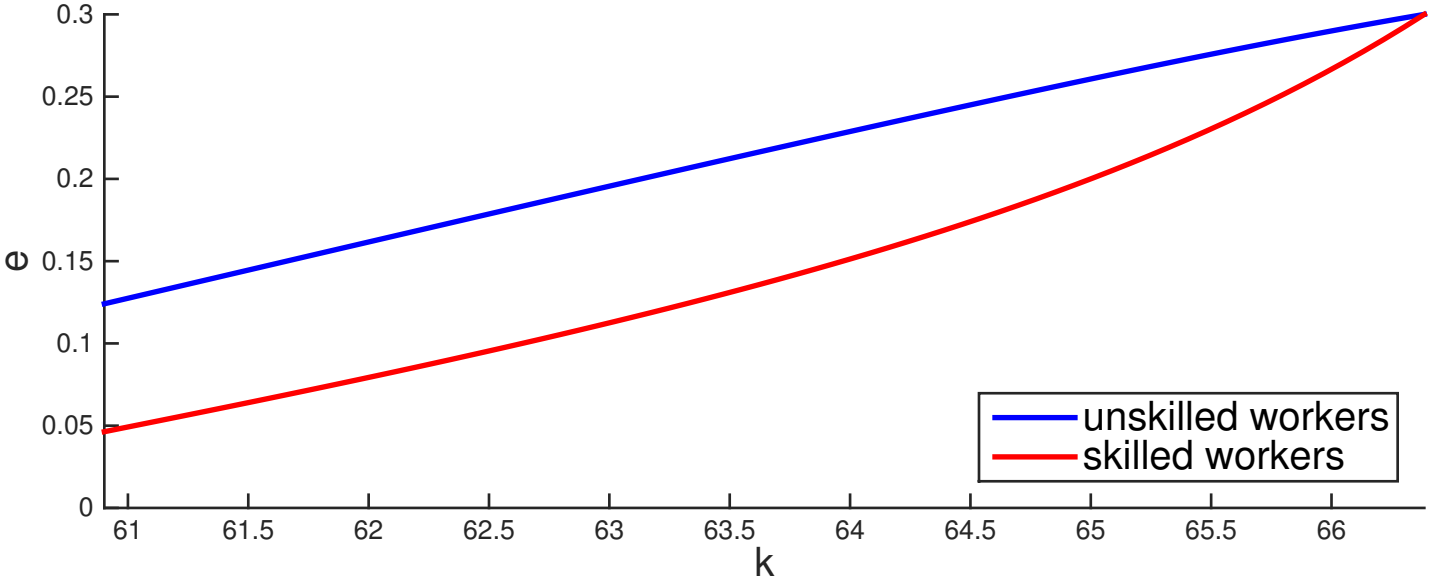
(b) Young less talented workers are trained iff $e \geq e_l^* > e_h^*$

(c) The skill of managers is diverse in equilibrium

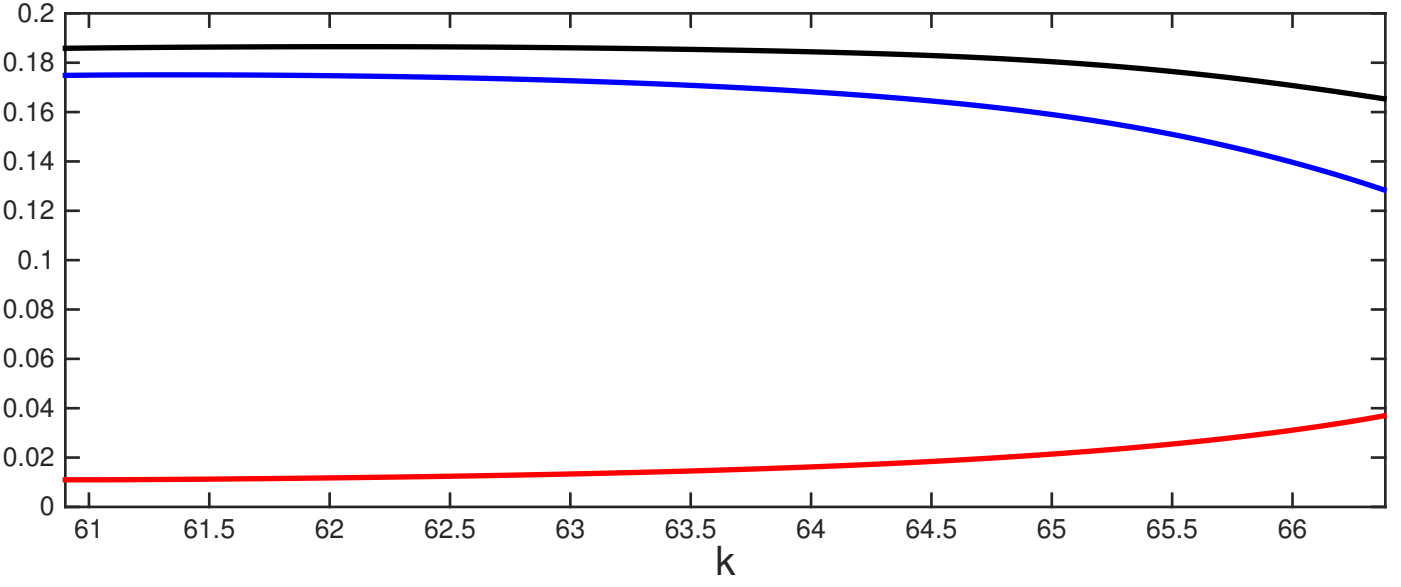
(d) A more productive manager is matched with richer workers than less productive managers, controlling the worker talent

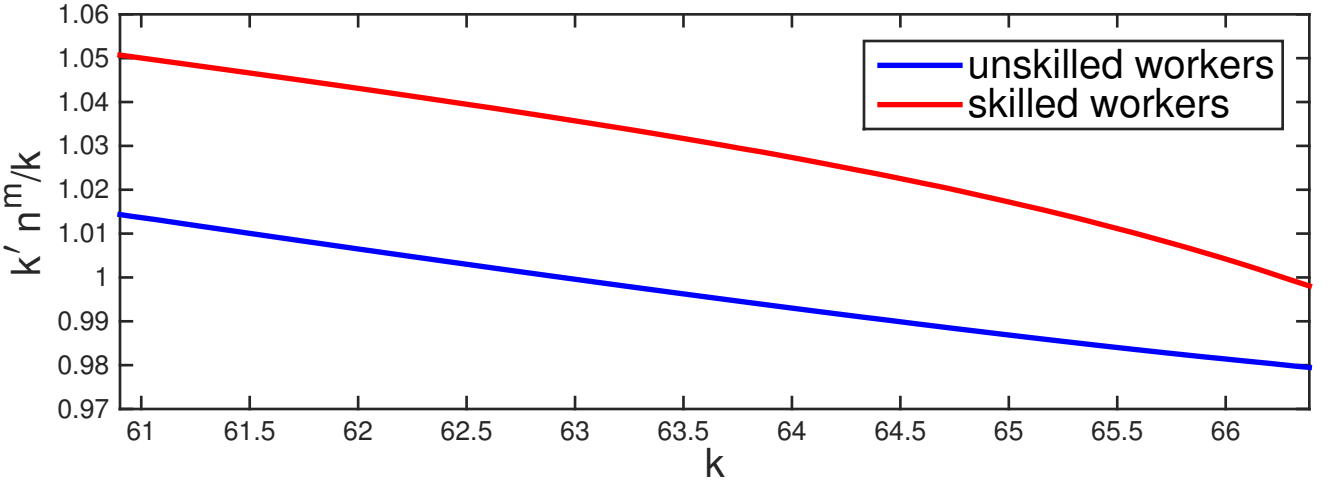
(e) A more productive manager is more likely to train more talented workers

(f) Poor talented workers are more likely to receive training but receive less training than less talented rich workers



Density of firms training skilled/unskilled workers




unskilled workers
skilled workers
k′ n^m/k
k
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1.05
1.04
1.03
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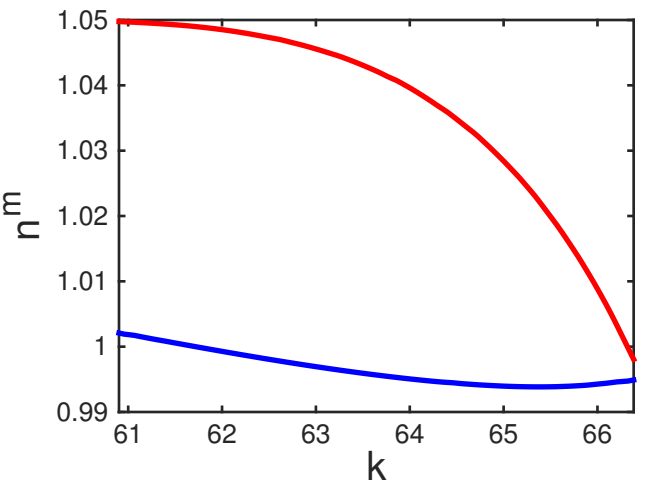
n^m
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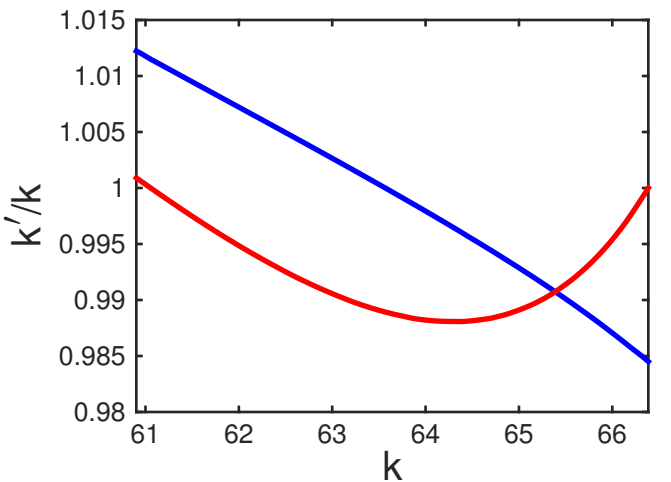
k′/k
k
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
61
62
63
64
65
66

Conclusion

Training (and higher education) are accumulation of intangible capital across generations

With financing constraint, richer young people have an advantage in receiving better training

$\longrightarrow$ Small difference in endowment leads to a large inequality: Propagation across people

Temporary decrease in endowment causes a persistent decrease in intangible investment and rise in inequality: Propagation across time

Policy Implications

Government can provide loan for workers to receive training

But, government must be better than private lenders in enforcing debt repayment

Government can provide subsidy for training poor young

But, efficient allocation needs rationing of the training

$\longrightarrow$ People start investing earlier to acquire better initial skill

Government can improve basic education to improve the initial skill: Equal opportunity instead of equal outcome