Stable Matching with Incomplete Information

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Introduction

- The matching literature has been very successful:
  - Focus almost exclusively on complete information.
  - Incomplete Information is largely unexplored.
- We do not have a satisfactory solution concept to fill in the role of stability.
  - The literature on asymmetric information core (started by Wilson (1978)).
    - Issues of information aggregation.
    - A mix of non-cooperative and cooperative approaches.
- We propose a notion of stability for matching games with incomplete information.

Example

- We build on the standard matching model with transferrable utility (Shapley-Shubik (1971)).
- Worker types: $w \in \{1, 2, 3\}$.
- Firm types: $f \in \{2, 4, 5\}$.
- A match of a worker of type w to a firm of type f generates premuneration values (Mailath-Postlewaite-Samuelson 2012a,b).
  - worker (job satisfaction, value of human capital):

$$\nu_{w,f} = wf.$$

  - firm (output, reputation):

$$\phi_{w,f} = wf.$$

Complete-Information Stability: Efficiency

The following matching is not stable for any transfers:

worker payoffs, π_i^w:	π_a^w	π_b^w	π_c^w
worker types, w:	1	3	2
firm types, f:	2	4	5
firm payoffs, π_j^f:	π_a^f	π_b^f	π_c^f

- The total surplus for the red matches is

$$\pi_b^w + \pi_b^f + \pi_c^w + \pi_c^f = 2 \cdot 3 \cdot 4 + 2 \cdot 2 \cdot 5 = 44.$$

- This implies

$$\pi_b^w + \pi_c^f < 2 \cdot 3 \cdot 5 = 30 \quad \text{or} \quad \pi_c^w + \pi_b^f < 2 \cdot 2 \cdot 4 = 16.$$

Incomplete Information

- Firm types are commonly known.
- Worker types are private information, but a firm knows the type of its matched worker.
- A candidate matching outcome, with payments (wages) from firm to worker:

worker payoffs, π_i^w:	2	16	6
worker types, $\mathbf{w}$:	1	3	2
payment, $\mathbf{p}$:	0	4	-4
firm types, $\mathbf{f}$:	2	4	5
firm payoffs, π_j^f:	2	8	14

- Is this outcome “stable” under incomplete information?

Incomplete Information: Belief

Candidate matching:

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worker types, **w**:	1	3	2
payment, **p**:	0	4	-4
firm types, **f**:	2	4	5
firm payoffs, π_j^f:	2	8	14

- Under complete information: worker *c* (with type 2) and firm *b* (with type 4) form a blocking pair .

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- But firm does not know type of deviating worker.
- Worker c (type 2) prefers this deviation only if $p > -2$.

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- But firm does not know type of deviating worker.
- Worker *c* (type 2) prefers this deviation only if $p > -2$.
- But worker *c* (type 1) also prefers this deviation if $p > -2$.
- And firm *b* (type 4) is strictly worse off from matching with the type 1 worker at $p > -2$.

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- But worker *c* (type 1) also prefers this deviation if $p > -2$.
- And firm *b* (type 4) is strictly worse off from matching with the type 1 worker at $p > -2$.
  $\Longrightarrow$ Firm 2's beliefs matter.

Incomplete Information: Inference

An easy case, 1

- Candidate matching:

worker payoffs, π_i^w:	6	4	π_c^w
worker types, $\mathbf{w}$:	2	1	3
payment, $\mathbf{p}$:	2	0	$\mathbf{p}_{cc}$
firm types, $\mathbf{f}$:	2	4	5
firm payoffs, π_j^f:	2	4	π_c^f

- Firm b happy to match with any worker if $p < 0$.
- Worker a also prefers to match with firm b at any $p > -2$.

Incomplete Information: Inference

An easy case, 2

- Candidate matching:

worker payoffs, π_i^w:	8	0	π_c^w
worker types, $\mathbf{w}$:	2	1	3
payment, $\mathbf{p}$:	4	-4	$\mathbf{p}_{cc}$
firm types, $\mathbf{f}$:	2	4	5
firm payoffs, π_j^f:	0	8	π_c^f

- Worker *a* and firm *b* cannot "block".
- But, firm *a* happy to match with any worker if $p < 2$, and
- worker *b* also prefers to match with firm *a* at any $p > -2$.

Incomplete Information: Inference

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- Worker *a* and firm *b* cannot “block”.
- But, firm *a* happy to match with any worker if $p < 2$, and
- worker *b* also prefers to match with firm *a* at any $p > -2$.
- Applying one round of “rationality” implies that the lowest attribute worker is matched with the lowest attribute firm.

Another Example

Information Revealing Prices

- Candidate matching:

worker payoffs, π_i^w:	4	0	π_c^w
worker types, **w**:	2	1	3
payment, **p**:	0	-4	$\mathbf{p}_{cc}$
firm types, **f**:	2	4	5
firm payoffs, π_j^f:	4	8	π_c^f

- Consider a match between worker *a* (type 2) and firm *b* (type 4) at a $p = -3$:
  - firm *b* knows that worker *a* cannot have type 1 $(4 - 3 < 2)$,
  - and so match is acceptable.

Incomplete Information

- Applying one round of “rationality” implies that the lowest attribute worker is matched with the lowest attribute firm.
- If agents understand that all the other agents are rational, then agents should understand that in a “stable” matching, the lowest attribute worker is necessarily matched with the lowest attribute firm.
- When rationality is mutually known, can apply again.

Incomplete Information

- Applying one round of “rationality” implies that the lowest attribute worker is matched with the lowest attribute firm.
- If agents understand that all the other agents are rational, then agents should understand that in a “stable” matching, the lowest attribute worker is necessarily matched with the lowest attribute firm.
- When rationality is mutually known, can apply again.
- A modest exercise: exploring the implications of the common knowledge of
  1. rationality,
  2. firm type assignments (to firm names),
  3. payments,
  4. the match (by name), and
  5. lack of blocking.

General Formulation

- Finite set of workers, $I \ni i$, and firms, $J \ni j$.
- Finite set of worker types, $W \subset \mathbb{R}$, and firm types, $F \subset \mathbb{R}$.
- Worker type assignment $\mathbf{w} : I \to W$; drawn from a fixed dsn with support $\Omega \subset W^I$.
- Firm type assignment $\mathbf{f} : J \to F$.
- Worker premuneration values: $\nu_{wf} > 0$ for all $wf \in W \times F$.
- Firm premuneration values: $\phi_{wf} > 0$ for all $wf \in W \times F$.
- Transfer to worker i from firm j: $p \in \mathbb{R}$.
- Allocation $(\mu, \mathbf{p})$:
  - one-to-one matching function $\mu : I \to J$, and
  - associated payments $\mathbf{p} = (\mathbf{p}_{i,\mu(i)})$.
- Observables: i, j, $\mu(i)$, $p_{i,\mu(i)}$, $\mathbf{f}(j)$.
- Matching outcome: allocation $(\mu, \mathbf{p})$ plus realization of $\mathbf{w}$ ($\mathbf{f}$ is suppressed, since it is fixed and common knowledge).

Individual Rationality

Worker i:

$$\nu_{\mathbf{w}(i),\mathbf{f}(\mu(i))} + \mathbf{p}_{i,\mu(i)} \geq 0.$$

Firm j:

$$\phi_{\mathbf{w}(\mu^{-1}(j)),\mathbf{f}(j)} - \mathbf{p}_{\mu^{-1}(j),j} \geq 0.$$

Complete-Information Stability

Shapley and Shubik (1971).

Definition

A matching $(\mu, \mathbf{p}, \mathbf{w})$ is a complete-information stable matching outcome if it is individually rational, and there is no unmatched worker-firm combination (i, j) and a payment $p \in \mathbb{R}$ such that

1. $\nu_{\mathbf{w}(i),\mathbf{f}(j)} + p > \nu_{\mathbf{w}(i),\mathbf{f}(\mu(i))} + \mathbf{p}_{i,\mu(i)}$, and
2. $\phi_{\mathbf{w}(i),\mathbf{f}(j)} - p > \phi_{\mathbf{w}(\mu^{-1}(j)),\mathbf{f}(j)} - \mathbf{p}_{\mu^{-1}(j),j}$.

- Properties: existence, efficiency, equal treatment of equals.

Blocking

Definition

Fix a set of individually rational matching outcomes, Σ. A matching outcome $(\mu, \mathbf{p}, \mathbf{w}) \in \Sigma$ is Σ-blocked if there is an unmatched worker-firm pair (i, j) and payment $p \in \mathbb{R}$ satisfying

1. $\nu_{\mathbf{w}(i),\mathbf{f}(j)} + p > \nu_{\mathbf{w}(i),\mathbf{f}(\mu(i))} + \mathbf{p}_{i,\mu(i)}$, and
2. $\phi_{\mathbf{w}'(i),\mathbf{f}(j)} - p > \phi_{\mathbf{w}'(\mu^{-1}(j)),\mathbf{f}(j)} - \mathbf{p}_{\mu^{-1}(j),j}$ for all $\mathbf{w}' \in \Omega$ satisfying
    1. $(\mu, \mathbf{p}, \mathbf{w}') \in \Sigma$,
    2. $\mathbf{w}'(\mu^{-1}(j)) = \mathbf{w}(\mu^{-1}(j))$, and
    3. $\nu_{\mathbf{w}'(i),\mathbf{f}(j)} + p > \nu_{\mathbf{w}'(i),\mathbf{f}(\mu(i))} + \mathbf{p}_{i,\mu(i)}$.

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Blocking is “hard to do.” Difficult to form a successful block. Complete-info stable matching outcomes are not blocked.

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Definition

A matching outcome $(\mu, \mathbf{p}, \mathbf{w}) \in \Sigma$ is Σ-stable if it is not Σ-blocked.

Incomplete-Information Stability

$\Sigma^0 :=$ the set of individually rational matching outcomes.

$$\Sigma^k := \left\{(\mu, \mathbf{p}, \mathbf{w}) \in \Sigma^{k-1} : (\mu, \mathbf{p}, \mathbf{w}) \text{ is } \Sigma^{k-1}\text{-stable}\right\}.$$

Definition

The set of incomplete-information stable outcomes is given by

$$\Sigma^\infty := \bigcap_{k=1}^{\infty} \Sigma^k.$$

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Definition

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Theorem

For each $\mathbf{w}$ *there exists* $(\mu, \mathbf{p})$ *such that* $(\mu, \mathbf{p}, \mathbf{w}) \in \Sigma^\infty$.

A Variant of the Earlier Example

A candidate matching:

worker payoffs, π_i^w:	2	16	8
worker types, $\mathbf{w}$:	1	3	2
payment, $\mathbf{p}$:	0	4	-2
firm types, $\mathbf{f}$:	2	4	5
firm payoffs, π_j^f:	2	8	12

- Worker *c* and firm *b* not able to block.

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- Consider a match of worker *b* with firm *c* at $p = 1\frac{1}{2}$.

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- Worker *c* and firm *b* not able to block.
- Consider a match of worker *b* with firm *c* at $p = 1\frac{1}{2}$.
- Firm *c* knows that worker *b* is not of type 1 or 2 (since offer not profitable for such types), and firm *c* willing to join. $\Longrightarrow (\mu, \mathbf{p}, \mathbf{w})$ is not Σ^0-stable and so is not in Σ^1.

The Earlier Example

Candidate matching (note that only $\mathbf{p}_{cc}$ is different):

worker payoffs, π_i^w:	2	16	6	1
worker types, $\mathbf{w}$:	1	3	2	1
payment, $\mathbf{p}$:	0	4	-4	
firm types, $\mathbf{f}$:	2	4	5	
firm payoffs, π_j^f:	2	8	14	

- $(\mu, \mathbf{p}, \mathbf{w})$ with $\mathbf{w}(c) = 2$ is Σ^0-stable and so is in Σ^1.

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- $(\mu, \mathbf{p}, \mathbf{w})$ with $\mathbf{w}(c) = 2$ is Σ^0-stable and so is in Σ^1.
- But, $(\mu, \mathbf{p}, \mathbf{w}')$ with $\mathbf{w}'(c) = 1$ is not Σ^0-stable. (The match of worker c with firm a at $p = -\frac{1}{2}$ blocks the outcome.)

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- $(\mu, \mathbf{p}, \mathbf{w})$ with $\mathbf{w}(c) = 2$ is Σ^0-stable and so is in Σ^1.
- But, $(\mu, \mathbf{p}, \mathbf{w}')$ with $\mathbf{w}'(c) = 1$ is not Σ^0-stable. (The match of worker c with firm a at $p = -\frac{1}{2}$ blocks the outcome.)
- And so $(\mu, \mathbf{p}, \mathbf{w})$ with $\mathbf{w}(c) = 2$ is not Σ^1-stable.

Incomplete-Information Stability: Fixed Point Characterization

Definition

A set of individually rational matching outcomes E is self-stabilizing if it is E-stable.

Incomplete-Information Stability: Fixed Point Characterization

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Theorem

1. *A singleton set* $\{(\mu, \mathbf{p}, \mathbf{w})\}$ *is self-stabilizing if and only if* $(\mu, \mathbf{p}, \mathbf{w})$ *is complete-information stable.*
2. *If* E *is a self-stabilizing set, then* $E \subset \Sigma^\infty$.
3. Σ^∞ *is a self-stabilizing set, and hence is the largest self-stabilizing set.*

Premuneration Values

Assumption: Strict Supermodularity

For all $w > w'$ and $f > f'$,

$$\nu_{wf} + \nu_{w'f'} > \nu_{w'f} + \nu_{wf'},$$

and similarly for $\nu + \phi$.

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Incomplete-Information Stability: Efficiency I

- Efficiency $\equiv$ Maximizing total surplus.
- Efficiency requires that the “right” types match.

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- Efficiency $\equiv$ Maximizing total surplus.
- Efficiency requires that the "right" types match.
- When values are supermodular, this is implied by positive assortative matching and the appropriate cutoff for consummated matches (in that matching).

Theorem

If premuneration values are strictly monotonic and strictly supermodular, every incomplete-information stable outcome is ex post efficient (positively assortative).

Incomplete-Information Stability: Efficiency II

- When values are submodular, both incomplete-information stability and efficiency require negative assortative matching.
- But the interplay between incomplete-information stability and efficiency is more subtle.

	μ				μ'		
w$'$:	1	3	4	$\varnothing$	1	3	4
f:	$\varnothing$	3	2	1	3	2	1
$\nu_{wf} + \phi_{wf}$:	0	15	16	0	8	12	10

Incomplete-Information Stability: Efficiency III

Theorem

If premuneration values are strictly monotonic and strictly submodular, every incomplete-information stable outcome is negative assortative.
There are incomplete information stable outcomes that are not efficient.
If every match yields a nonnegative surplus, then every incomplete-information stable outcome is efficient.

How Small is the Incomplete-Information Stable Set?

Equal treatment can fail:

- Worker types i.i.d. $\{1, 2\}$, so that $\Omega = \{1, 2\} \times \{1, 2\}$.
- $\nu_{w,f} = \phi_{w,f} = w \cdot f$.
- An incomplete-information stable matching outcome:

π_i^w:	6	8
w:	2	2
p:	2	4
f:	2	2
π_j^f:	2	0

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- $\nu_{w,f} = \phi_{w,f} = w \cdot f$.
- $\{(\mu, \mathbf{p}, \mathbf{w}), (\mu, \mathbf{p}, \mathbf{w}')\}$ is self-stabilizing:

π_i^w:	6	8
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p:	2	4
f:	2	2
π_j^f:	2	0

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- $\nu_{w,f} = \phi_{w,f} = w \cdot f$.
- $\{(\mu, \mathbf{p}, \mathbf{w}), (\mu, \mathbf{p}, \mathbf{w}')\}$ is self-stabilizing:

π_i^w:	4	8
$\mathbf{w}'$:	1	2
$\mathbf{p}$:	2	4
$\mathbf{f}$:	2	2
π_j^f:	0	0

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- $\nu_{w,f} = \phi_{w,f} = w \cdot f$.
- $\{(\mu, \mathbf{p}, \mathbf{w}), (\mu, \mathbf{p}, \mathbf{w}')\}$ is self-stabilizing:

π_i^w:	4	8
$\mathbf{w}'$:	1	2
$\mathbf{p}$:	2	4
$\mathbf{f}$:	2	2
π_j^f:	0	0

In fact, $(\mu, \mathbf{p}, \mathbf{w}')$ is complete-information stable.
The argument also works if $\Omega = \{1,2\} \times \{2\}$.

Incomplete- vs. Complete-Information Stability

- Perturb the complete-information types.
- Other restrictions on Ω.

Incomplete- vs. Complete-Information Stability

- Suppose premuneration values are strictly monotonic and strictly supermodular.
- Suppose for any $\mathbf{w}, \mathbf{w}' \in \Omega$, there exists a one-to-one mapping $\iota : I \to I$ such that $\mathbf{w}(i) = \mathbf{w}'(\iota(i))$. That is, Ω is a set of permutations.
- Under permutations, worker type assignment is almost common knowledge,

Incomplete- vs. Complete-Information Stability

Permutations

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- Under permutations, worker type assignment is almost common knowledge,
- but almost is not good enough for all incomplete information stable outcomes to be complete information stable!

Incomplete- vs. Complete-Information Stability

Permutations

- Suppose $\mathbf{w} = (2, 2, 2, 4)$, and Ω consists of permutations.

π_i^w:	0	0	0	6
w:	2	2	2	4
p:	-4	-6	-6	-6
f:	2	3	3	3
π_j^f:	8	12	12	18

- Outcome is not complete information stable .
- Outcome is incomplete information stable: Rotating the high type worker through workers *b*, *c*, and *d* yields a self-stabilizing set.
- Note that this set does not contain a complete information matching outcome!

Incomplete- vs. Complete-Information Stability

Theorem

Suppose premuneration values are strictly monotonic and strictly supermodular. Suppose for any $\mathbf{w}, \mathbf{w}' \in \Omega$*, there exists a 1:1 mapping* $\iota : I \to I$ *such that* $\mathbf{w}(i) = \mathbf{w}'(\iota(i))$*.*
Incomplete-information stability coincides with complete-information if

1. *different workers have different types* $(i \neq i' \Rightarrow \mathbf{w}(i) \neq \mathbf{w}(i'))$*, or*
2. *different firms have different types* $(j \neq j' \Rightarrow \mathbf{f}(j) \neq \mathbf{f}(j'))$*.*

Incomplete- vs. Complete-Information Stability

Permutations

Theorem

Suppose premuneration values are strictly monotonic and strictly supermodular. Suppose for any $\mathbf{w}, \mathbf{w}' \in \Omega$, there exists a 1:1 mapping $\iota : I \rightarrow I$ such that $\mathbf{w}(i) = \mathbf{w}'(\iota(i))$. Incomplete-information stability coincides with complete-information if

1. *different workers have different types ($i \neq i' \Rightarrow \mathbf{w}(i) \neq \mathbf{w}(i')$), or*
2. *different firms have different types ($j \neq j' \Rightarrow \mathbf{f}(j) \neq \mathbf{f}(j')$).*

Idea of proof

1. workers can separate themselves.
2. payments are fully revealing.

Stability and Pricing

- Have seen that inefficient outcomes (under some assumptions) cannot persist as incomplete information stable outcomes.
- Can one rely on a price system to ensure that inefficient outcomes will similarly not persist.

Price Sustainable Outcomes

- A commodity is a match between worker i and firm j, so since all goods "should" be priced.
- We have now price matrix $\mathbf{P} : I \times J \to \mathbb{R}$.

Price Sustainable Outcomes

Incomplete Information

Definition

An individually rational price-taking matching outcome $(\mu, \mathbf{P}, \mathbf{w}) \in \Psi$ is Ψ-price sustainable if there is no pair $i \in I$ and $j \in J$ such that

1. $\nu_{\mathbf{w}(i),\mathbf{f}(j)} + \mathbf{P}_{i,j} > \nu_{\mathbf{w}(i),\mathbf{f}(\mu(i))} + \mathbf{P}_{i,\mu(i)}$, or
2. $\phi_{\mathbf{w}'(i),\mathbf{f}(j)} - \mathbf{P}'_{i,j} > \phi_{\mathbf{w}'(\mu^{-1}(j)),\mathbf{f}(j)} - \mathbf{P}'_{\mu^{-1}(j),j}$ for all $\mathbf{w}' \in \Omega$ and $\mathbf{P}'$ satisfying
   1. $(\mu, \mathbf{P}', \mathbf{w}') \in \Psi$,
   2. $\mathbf{w}'(\mu^{-1}(j)) = \mathbf{w}(\mu^{-1}(j))$,
   3. $\mathbf{P}'_{i',\mu(i')} = \mathbf{P}_{i',\mu(i')}$ and $\mathbf{P}'_{i',j} = \mathbf{P}_{i',j}$ for any $i' \in I$.

Price Sustainable Outcomes

Incomplete Information

$\Psi^0 :=$ individually rational price taking outcomes.

$$\Psi^k := \left\{ (\mu, \mathbf{P}, \mathbf{w}) \in \Psi^{k-1} : (\mu, \mathbf{P}, \mathbf{w}) \text{ is } \Psi^{k-1}\text{-price sustainable} \right\}.$$

Definition

The set of incomplete-information price sustainable outcomes is given by

$$\Psi^\infty = \bigcap_{k=1}^{\infty} \Psi^k.$$

Price Sustainable Outcomes vs. Stability

Theorem

Every incomplete information stable outcome is price sustainable, but not conversely.

Example

$\Omega = \{\mathbf{w}, \mathbf{w}'\}$, $\nu_{w,f} = wf$, $\phi_{wf} = 2 + wf$.

π_i^{wd}:	6	6
π_i^{w}:	6	6
$\mathbf{w}$:	3	2
$\mathbf{f}$:	1	2
π_j^{f}:	2	4
π_j^{fd}:	0	(4)

π_i^{wd}:	2	6
π_i^{w}:	4	6
$\mathbf{w}'$:	1	2
$\mathbf{f}$:	1	2
π_j^{f}:	0	4
π_j^{fd}:	0	4

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 4 & 2 \end{pmatrix}$$