R&D, Structural Transformation, and the Distribution of Income *

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Abstract

Placeholder

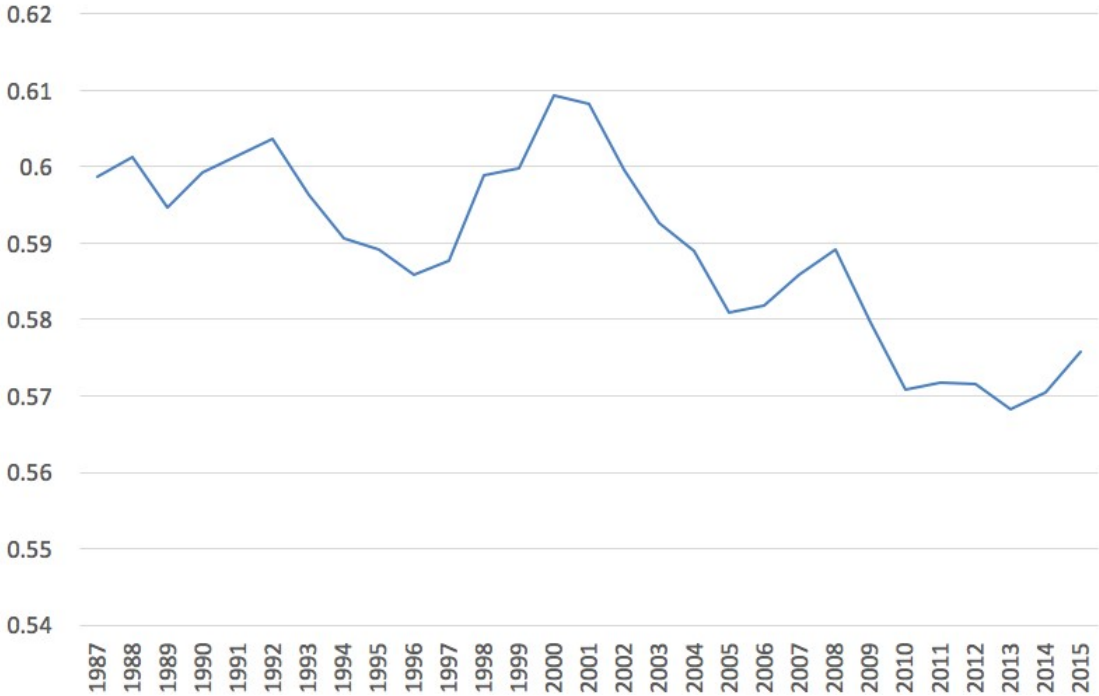**Keywords:** key1, key2, key3

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1 Introduction

Oh, for the days of balanced growth. In Solows growth model, labor-augmenting technical change at a constant rate produces long-term growth in output per capita and wages at the same constant rate. The returns to capital are stable, as are the factor shares of national income going to labor and capital. In the heyday of the Solow model, these were viewed by Kaldor (1957) and others as the stylized facts of long-term economic growth.

These stylized facts have visibly broken down since around 2000. There has been a striking disconnect between the continued growth of labor productivity (GDP per worker) and the stagnation of compensation per worker, resulting in a discernible decline in the labor share of income, as shown in Figure 1 for the non-farm business sector. The decline in labor share is widely, if not universally, attributed to automation - robots and other smart machines - displacing labor.

Figure 1: Labor Share of GDP at Factor Cost

Source: See Appendix

There are other possible culprits besides automation, including a conjectured rise in monopoly power, a fall in U.S. union coverage and power, and the effects of global trade on the distribution of income. Of course, several factors may be at play. My view is that automation - the replacement of human labor by machines and code - is likely to be the most important of the factors.

Indeed, my argument is that the decline in the labor share via automation has been occurring well before 2000 but that it has been obscured in the macroeconomic data by offsetting structural changes. Balanced growth, in short, was always a mirage. The difference now is that the imbalances are now showing more vividly, and are likely to intensify.

One reason that unbalanced growth was underemphasized before 2000 is that different sectors of the economy were affected by automation in different, and indeed offsetting, ways. It is useful, I believe, to disaggregate GDP into five major sectors:

- Goods-producing sectors: agriculture, mining, construction, and manufacturing;

- Basic business services: utilities, wholesale trade, retail trade, transport and warehousing;

- Personal services: arts, leisure, food, and accommodations, other personal;

- Professional services: information, finance, education, health, management, scientific and technical, other professional;

- Government services: federal, state, and local

These sectors are differentially susceptible to automation. Historically, there seem to have been two key dimensions to work tasks that determine their suitability for automation: degree of expertise required and repetitiveness/predictability of the task. Tasks requiring high expertise (for example, as measured by their educational requirements) and that have low predictability/repetitiveness in workflow have been less easily automated. Based on
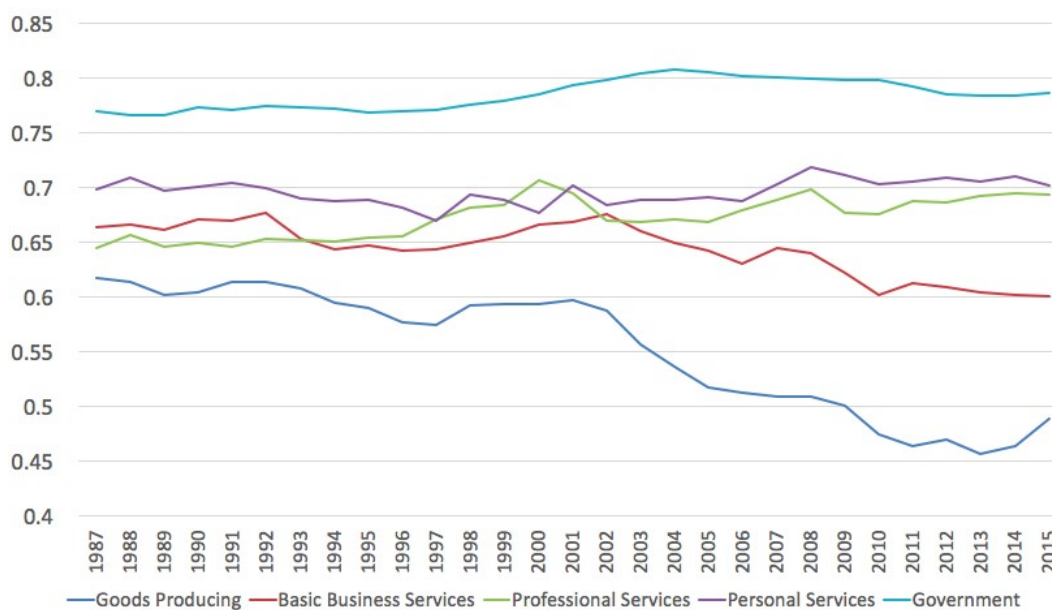
the occupational mix and production processes of the five sectors, we can place the sectors roughly as follows:

Sector	Typical Expertise /Education	Typical Workflow Predictability
Goods Producing	Low to Moderate	High
Basic Business Services	Moderate	Moderate to High
Personal Services	Low to Moderate	Low to Moderate
Professional Services	High	Low
Government	Moderate to High	Moderate to High

This suggests that the goods-producing sector has been easiest to automate and professional services the most difficult, with the other sectors somewhere in the middle, depending on the particular sub-sectors involved. As I describe later, Artificial Intelligence could change the character of automation in the future, leading to much more automation of high-skilled tasks.

These differences in susceptibility to automation show up in the sector trends in labor share of value added (measured at factor cost) since 1987, shown in Figure 2:

Figure 2: Labor Share by Sector

Source: See Appendix

We see a large drop in the labor share of value added in the good-producing sector, from 61.7 percent to 48.9 percent, consistent with the ease of automation in that sector, contrasted by an increase in the labor share of value added in professional services and government, consistent with the relative difficulty of automation in those two sectors. Basic business services also show a modest decline in the labor share, from 66.3 percent to 60.1 percent. The labor share of value added in personal services was unchanged, consistent with the relatively low workflow predictability of that sector, making it more difficult to automate.
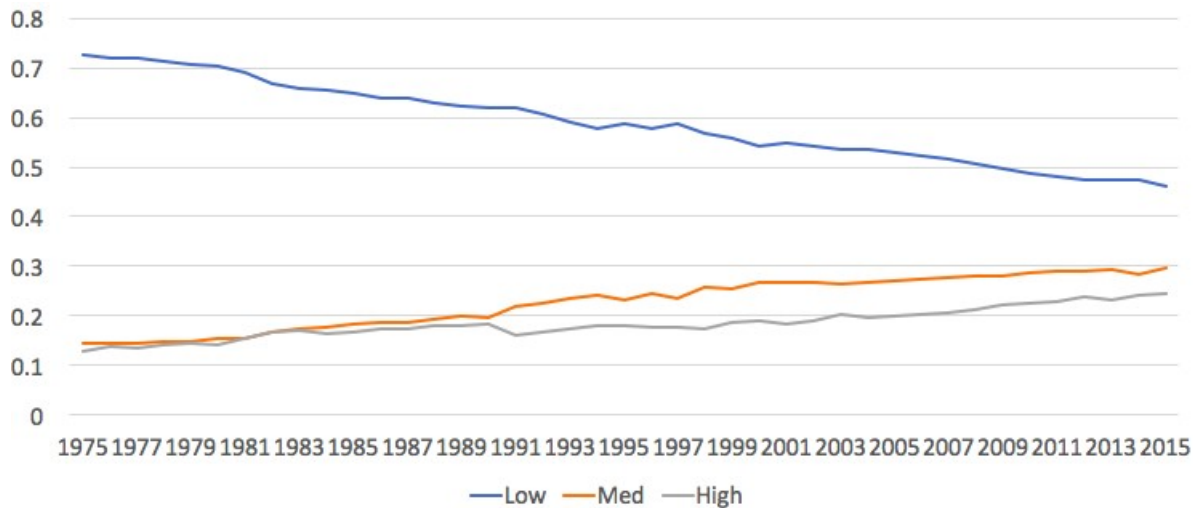
Figure 2 makes clear that in the goods-producing and basic-business-service sectors, automation has been taking place for decades, but the trends have been somewhat obscured by the relative lack of automation in the other sectors, and by the fact that both output and employment have been shifting from goods production to professional services, that is, from the broad sectors experiencing the most automation to the ones experiencing the least automation.

Even the significant observed decline in the labor share in the goods-producing sector understates the extent of structural change in that sector, since the composition of labor has also been shifting dramatically from production workers with relatively low levels of schooling to supervisory workers with higher levels of schooling. This too marks a rise in the share of capital income in value added, albeit the income earned by human capital rather than by business fixed capital.

Figure 3 offers a rough estimate of the overall share of labor income in the economy accounted for by different levels of educational attainment. For our purposes, I have grouped the educational attainment into three bins: low, compromising attainment up to some college including a two-year associate degree; medium, comprising a bachelors degree but no advanced degree; and high, comprising an advanced degree. Using census data on the mean income and number of workers at these levels of educational attainments, we can find the shares of labor income accruing to different categories, as shown in Figure 3.

Labor income accruing to workers with less than a bachelors degree plummeted from 72.7

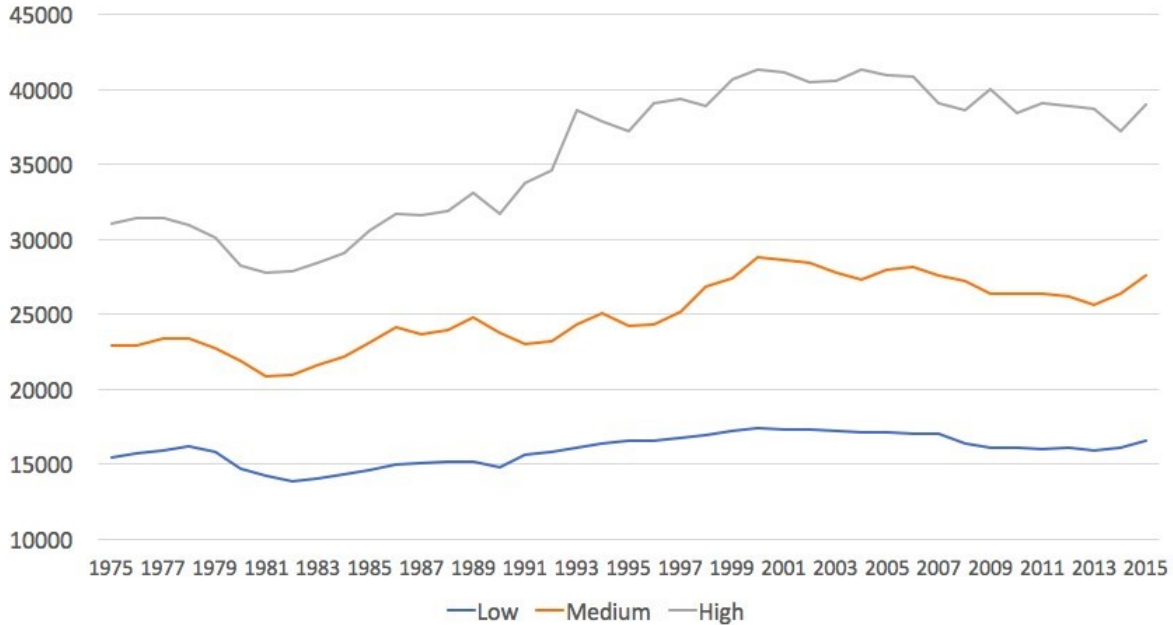Figure 3: Share of Earnings by Educational Attainment

Source: See Appendix

percent to 46.1 percent. Workers with a bachelors degree saw their share of labor income doubling from 14.3 percent to 29.6 percent. And workers with an advanced degree also saw their share of labor income doubling from 12.9 percent to 23.4 percent.

Real mean earnings per worker among these three categories shows a similar trend, in Figure 4. Earnings of low-skilled workers (defined here as all the way up to some college or an associates degree) began to stagnate in the mid-1970s, and have not risen since then. Mean earnings for workers with a bachelors or advanced degree continued to rise to around 2000, and then have been stagnant or even falling in real terms in the case of those with advanced degrees.

The relative numbers of workers at each educational attainment has responded to the changing market incentives and to outlays for education by governments at all level. As we see in Figure 5, the proportion of all workers at less than a bachelors degree declined from 83.4 percent to 64.3 percent, while those with a BA rose from 10.0 to 22.6 percent, and those with an advanced degree from 6.6 to 13.2 percent, between 1975 and 2016.

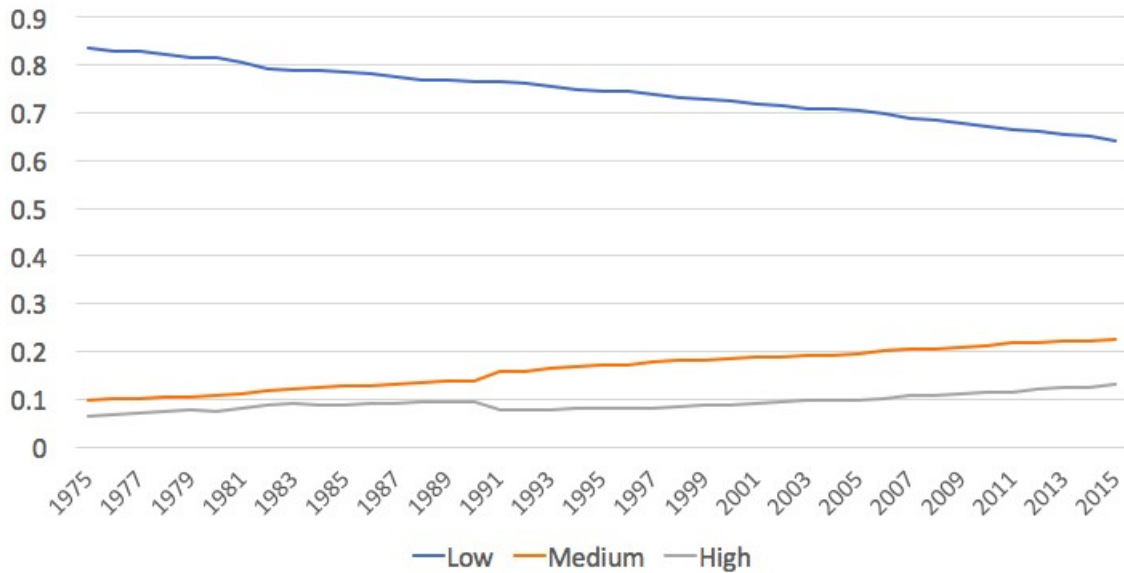Figure 4: Real Mean Earnings by Education in $1982-84

Source: See Appendix

What makes these trends especially important for us, I believe, is that the ability to automate tasks is likely to increase dramatically with the recent advances in big data, machine learning, and other forms of artificial intelligence. The trends to date - the falling share of labor income, rising share of earnings flowing to highly trained workers, and decline of real earnings of workers who are subject to automation - may soon be felt by a much wider swath of workers and sectors.

In a fundamental sense, we are witnessing the gradual unfolding of a fundamental General Purpose Technology, digital information, that is at least as fundamental as the steam engine and electrification. Digital information began to unfold with the theoretical breakthroughs of Alan Turing, John von Neumann, Claude Shannon and Norbert Weiner in the 1930s and 1940s, and then advanced dramatically with the first mainframe computers in the 1940s, the invention of the transistor in 1947, the invention of integrated circuits in the late 1950s, and the initiation of Moores Law at the end of the 1950s. Of course, the digital revolution now engages a vast range of science and technology, including solid-state physics, nanotechnology,

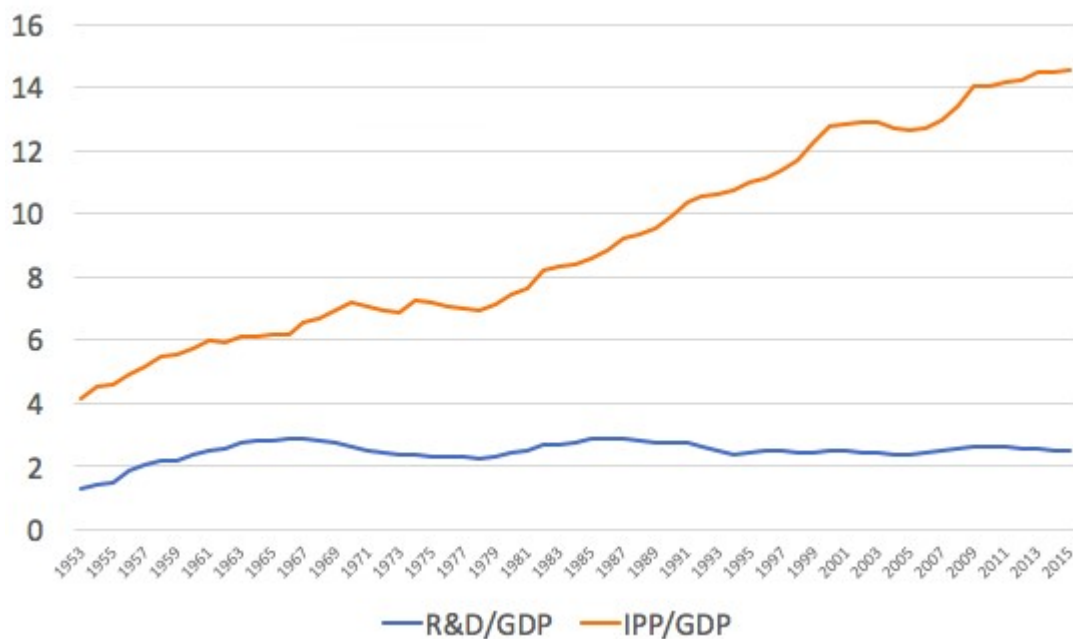Figure 5: Share of Employment by Education

Source: See Appendix

fiber optics, digital communication, and a startling range of applications across every domain of science and every sector of the economy.

The rising investments in R&D are therefore a fundamental part of the story and the fundamental driver of structural transformation. Figure 6 shows the national-accounts estimates of R&D annual outlays and the cumulative stock of intellectual property, both as a share of GDP. R&D as a share of GDP roughly doubles from the early 1950s to today, from around 1.3 percent to 2.6 percent. The stock of IP rises from around 4.5 percent to 14 percent of GDP. The point is that IP has risen far faster than GDP; the economy has become far more science-intensive.

Rather than the Solow-era stylized facts, I would therefore propose the following alternative stylized facts:

1. The share of national income accruing to capital rises over time in sectors experiencing automation, especially when capital is measured to include human capital;

2. The share of national income accruing to low-skilled labor drops while the share ac-

Figure 6: R&D and Intellectual Property (%GDP)

Source: See Appendix

cruing to high-skilled labor rises;

3. The dynamics across sectors vary according to the differential timing of automation, with automation spreading from low-skill and predictable tasks towards higher-skilled and less-predictable tasks;

4. Automation reflects the rising intensity of science and technology throughout the economy, in terms of R&D, IP, and scientific expertise in the labor force;

5. Future technological changes associated with artificial intelligence (e.g. machine learning) are likely to shift national income from medium-skilled and high-skilled workers toward owners of business capital (fixed capital and intellectual property products).

There are of course many unsolved problems of both theory and measurement, but I will now try to lay out some basic concepts in more formal terms.

2 A basic model

Consider the goods-producing sector of the economy (agriculture, mining, construction, and manufacturing), the first to automate. Let Q be output. Output is produced by capital and labor. I will distinguish two kinds of physical capital, buildings (B) and machines (M), and two kinds of non-physical capital, human capital and know-how embodied in machine technology.

Labor is organized into occupational tasks such as management, production, sales, and so forth. In general, these tasks require varying levels of expertise: unskilled (U), intermediate (I), and high (H), corresponding respectively to levels of education: less than a bachelors degree; a bachelors degree; and an advanced degree (masters, professional, or PhD).

To illustrate, suppose that there are just two tasks for labor: Production (P) and Non-production (N). The production task requires basic skills. The non-production task requires intermediate skills. High skills are needed for three purposes: R&D, professional services such as medicine, and university education. Tasks requiring basic skills can also be carried out by workers with intermediate or high skills, and tasks requiring intermediate skills can also be carried out by workers with high educational attainment.

Machines M can substitute for labor while buildings B are complementary to tasks. As a simple illustration, suppose that output Q is a Cobb-Douglas function of P, N, and B:

$$Q = P^a N^b B^{(1-a-b)} \tag{1}$$

Production P is produced either by labor L_P or machines M_P (such as assembly-line robots) assumed to be a perfect substitute, with t_P measuring the technological sophistication of the machines M_P.

$$P = L_P + t_P * M_P \tag{2}$$

Similarly, nonproduction tasks can be produced by labor L_N or machines M_N.

$$N = L_N + t_N * M_N \tag{3}$$

In the historical evolution of technology, it was easier to devise machines to carry out basic mechanical tasks (production) rather than intermediate tasks (nonproduction), so I start with the simplest assumption that $t_P > 0$ and $t_N = 0$. I note again, however, that as machines are getting "smarter," they are able to fulfill more nonproduction tasks.

Workers with basic education can work only in production, while workers with an intermediate education can work either in production or nonproduction tasks. Let L_U equal the number of workers with education U, and L_I the number of workers with educational attainment I. Then, with L_{ij} signifying the number of workers in task i (N,P) and skill j, full employment requires:

$$L_U = L_{PU} \tag{4}$$

$$L_I = L_{NI} + L_{PI}$$

The market equilibrium may involve a perfect sorting of tasks by skills (unskilled workers in production, intermediate skilled workers in nonproduction, with $L_{PI} = 0$), or may involve some intermediate-skilled workers employed in basic-skilled tasks, with $L_{PI} > 0$, a situation referred to as downskilling. In a dynamic context, the latter situation should be temporary, as workers will not generally invest in additional years of education for jobs that require a lower educational attainment.

In any period, the capital stock K is determined based on past savings, and is allocated between buildings and machines in production tasks:

$$K = B + M_P \tag{5}$$

Investors maximize their capital income by allocating K to equate the marginal products

of buildings and machines, or by setting $M_P = 0$ at a corner solution (when the marginal product of buildings is higher than that of machines for $B = K$ and $M_P = 0$).

In the pure sorting equilibrium, the wages for LU and LI are given as follows:

$$W_U = a * (L_U + t_P * M_P)^{(a-1)} L_I^b S^{(1-a-b)} \tag{6}$$

$$W_I = b * (L_U + t_P * M_P)^a L_I^{b-1} S^{(1-a-b)}$$

K receives the rate of return r:

$$r = (1 - a - b) * L_U^a L_I^b M_P^{-(a+b)} \tag{7}$$

If t_P is below a threshold value t_P^T, then the entire capital stock K is allocated to buildings, so that $B = K$ and $M = 0$. In that case, there is no automation. If t_p is above t_P^T, then some capital is allocated to machines, with the added equilibrium condition:

$$r = t_P * W_U \tag{8}$$

The threshold t_P^T can be found by equating $t_P^T*W_U$ with the marginal product of structures when $B = K$, specifically: $t_P^T * L_U^{(a-1)} L_I^b * K^{(1-a-b)} = (1 - a - b) * L_U^a L_I^b K^{-(a+b)}$. With a little algebra, we find that:

$$t_P^T = (1 - a - b) * \left(\frac{L_U}{K} \right) \tag{9}$$

The capital share of income KS is given simply as:

$$KS = \frac{(r * K)}{Q} \tag{10}$$

Suppose now that the economy is operating in the range of automation, with $t_P > t_P^T$ and $M > 0$. The comparative static effects of a further rise in t_P are as follows:

$$\frac{\partial r}{\partial t_P} > 0 \tag{11}$$

$$\frac{\partial W_B}{\partial t_P} < 0$$

$$\frac{\partial W_I}{\partial t_P} > 0$$

$$\frac{\partial M_P}{\partial t_P} > 0$$

$$\frac{\partial KS}{\partial t_P} > 0$$

The incremental improvement in machine technology (automation) leads to a rise in the return to capital (a), a fall in the wage of basic labor (b), a rise in the wage of intermediate labor (c), a rise in automation (d), and a rise in the share of capital income (e). This is simply a case of skill-biased technical change, in the form of technological change that induces the substitution of less-educated workers by machines in the goods-producing sector.

3   Investing in Education

So far, we have taken the supplies of L_U and L_I as given, a reasonable assumption at a given moment of time but not in a dynamic context. The rise in the labor-market returns to schooling, $\frac{\partial (W_I - W_U)}{\partial t_P} > 0$, will lead to a rise in investment in schooling, either by household outlays or public outlays.

Remaining in a quasi-static context, suppose we start with initial levels of K, L_B, and L_I denoted by K(0), $L_B(0)$, and $L_I(0)$ and assume a given flow of saving (SV) that may be allocated to fixed business investment F or education E_I for upgrading basic skills to intermediate skills:

$$SV = F + E_I \tag{12}$$

$$K = K(0) * (1 - d) + F$$

$$L_I = L_I(0) + \frac{E_I}{c_I}$$

$$L_U = L_U(0) - \frac{E_I}{c_I}$$

The parameter c_I is the unit cost of producing one intermediate-skilled worker from one low-skilled worker, taken here to be fixed. This cost includes both the direct education outlays (such as tuition) as well as the opportunity costs, notably the reduction of a students labor-market participation and earnings during the years of study.

Once again, the marginal returns to alternative investments should be set equal, so that the marginal product of fixed capital, equal to r, should be set equal to the returns to education, measured as $W_I - W_U$. In equilibrium,

$$r * c_I = W_I - W_U \tag{13}$$

How, then, does a rise in t_P affect the investment in education? There are two effects. By raising the returns to fixed investment, r, the investment allocation can be shifted away from human capital to business fixed capital. On the other hand, by raising the wage of intermediate-skilled workers relative to basic-skilled workers, the net return to schooling is raised. In practice, the second effect is likely to dominate, especially if we also recognize that the rise in the return to capital will also likely increase the overall rate of saving SV.

If the education incentive effect indeed dominates, then the technological improvement increases the flow of students into higher education, thereby reducing the supply of basic-skilled workers and raising the supply of intermediate-skilled workers. The boost in the supply of skilled labor moderates the increase in wage inequality following the rise in t_P. In the extreme case that r remains constant, the wage differential would also remain unchanged by an offsetting increase in the skilled workforce sufficient to drive the wage differential back to the original level r*c.

4 Endogenous Growth

The model is greatly enriched by allowing the rate of technological advance to depend on the investments in R&D carried out by highly skilled scientists and engineers. Let us therefore now introduce a cadre of high-skilled professional workers in the number L_H. We will suppose that these workers are generally holders of advanced degrees in STEM fields.

The highly skilled workers L_H are employed in four major activities: (1) research and development, $L_{R\&D}$; (2) higher education, L_{ED}; (3) healthcare L_{HL} (medical doctors, medical equipment engineers, statisticians, etc.); and (4) professional consultancy services L_C. Other than health professionals and academic researchers, most workers with advanced degrees are employed in professional firms (engineering, consultancy, architecture, legal, etc.) that sell their research and consulting services to companies in other sectors, such as manufacturing.

$$L_H = L_{R\&D} + L_{ED} + L_{HL} + L_C \tag{14}$$

High-skilled professionals require an advanced degree, and therefore education at the post-bachelors level, denoted E_H. Thus, we revise the equations in (11) as follows:

$$SV = F + E_I + E_H \tag{15}$$

$$K = K(0) * (1 - d) + F$$

$$L_H = L_H(0) + \frac{E_H}{c_H}$$

$$L_I = L_I(0) + \frac{E_I}{c_I} - \frac{E_H}{c_H}$$

$$L_U = L_U(0) - \frac{E_I}{c_I}$$

The benefit of investing in advanced training depends, of course, on the productivity of high-skilled workers in their four respective activities: R&D, education, healthcare, and consultancy. We need, therefore, to specify production functions for these four activities.

One of the main fruits of R&D will be to improve automation, meaning a rise in t_P. A plausible relationship might be something like:

$$t_P(t+1) = t_P(t) * (1 - dep_{t_P}) + R\&D(t) \tag{16}$$

R&D(t) in turn would be produced with some combination of skilled labor, smart machines, and buildings in the R&D sector, such as:

$$R\&D(t) = (\Theta_{R\&D} * L_{R\&D})^g * B_{R\&D}^{(1-g)} \tag{17}$$

The parameter $\Theta_{R\&D}$ signifies the efficiency of research by high-skilled workers. A high value of $\Theta_{R\&D}$ would signify a fruitful period for scientific research, for example, due to a significant breakthrough in scientific knowledge. The inventions of the transistor and integrated circuit in the 1940s and 1950s, and the design of modern computers around the same time, meant that the productivity of applied physicists and engineers rose markedly after World War II, ushering in the information revolution and a golden age for R&D that lasts till today, and that is indeed accelerating.

The parameter $t_{R\&D}$ signifies the possibility of artificial intelligence substituting for researchers in new R&D. This is of course already happening in areas such as drug discovery, where machine learning can scan through vast libraries of drug candidates for potential research targets. To date, advanced machines have mostly complemented rather than substituted for high-skilled researchers, yet it is not hard to envision the day soon when smart machines excel at research in biochemistry, genomics, code writing, and machine design. The inventors of ultra-smart machines will eventually put themselves out of business, or at least drastically lower their own wages as $t_{R\&D}$ rises significantly.

The health sector output HL would have a similar production function, such as:

$$HL(t) = (\Theta_{HL} * L_{HL} + t_{HL} * M_{HL})^g * S_{HL}^{(1-g)} \tag{18}$$

A rise in Θ_{HL} would increase the supply of health services and the demand for health workers. But what of the demand for health services? We might suppose that the demand would also increase with Θ_{HL}. As health technology breakthroughs are made, these tend to become part of a minimum basic package of health services guaranteed by law and backed by public outlays. Thus, the public outlays on health services would tend to rise with Θ_{HL}.

5 Parameterizing the Model for the US Economy

The practical longer-term goal of this model will be to create a computable general-equilibrium (CGE) model of the US economy that can analyze the past and future effects of technological change, especially artificial intelligence and robotics, on the distribution of incomes, wealth, jobs, and sectors. A primary purpose will be to analyze the likely progress of AI in substituting for many occupations that currently have high educational requirements, such as in healthcare (remote patient monitoring, advanced imaging, machine-led diagnostics), education (online education, expert systems for teacher training and pedagogy), and various areas of research and development. This is a work in progress.

At this stage, it will have to suffice to present some early simulation results of an illustrative model not yet parameterized for US conditions. I will present two such simulations, to examine: (1) a rise in the productivity of R&D; and (2) a rise in automation for middle-skilled tasks (jobs currently requiring BA-level workers).
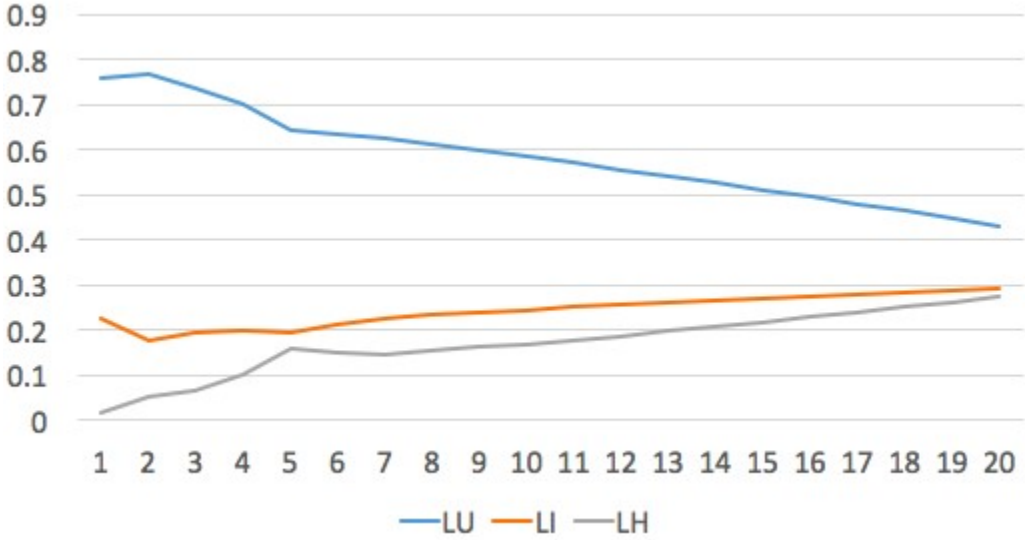
6 Rise in R&D Productivity

What happens to the structure of an economy when the returns to R&D rise because of a new general purpose technology such transistors and computers in the 1950s or machine learning and artificial intelligence in the 2020s. The experiment, to be precise, is a permanent, one-time step increase in $\Theta_{R\&D}$, the productivity of high-skilled R&D workers. In this first variant, I assume that only low-skilled workers face the competition from automation. In

a sense, this illustration tracks the experience of the 1950s 2010s, when the breakthroughs of the digital revolution enabled the automation of low-skilled tasks. The full model and specific parameters are available in the supplementary materials. For the purposes here, I emphasize the qualitative results.

In the numerical illustration, the rise in $\Theta_{R\&D}$ occurs in period 5 yet is anticipated from period 1. Even before the rise in R&D takes hold, workers begin to raise their educational attainment in anticipation of the widening gap between low-skilled and higher-skilled wages. After the rise in $\Theta_{R\&D}$ the shift in educational attainment is even stronger. The end result is a sharp decline in the proportion of low-skilled workers and a commensurate rise in middle-skilled and high-skilled workers, as shown in Figure 7, which qualitatively tracks the same empirical pattern we saw for the US economy in Figure 5.

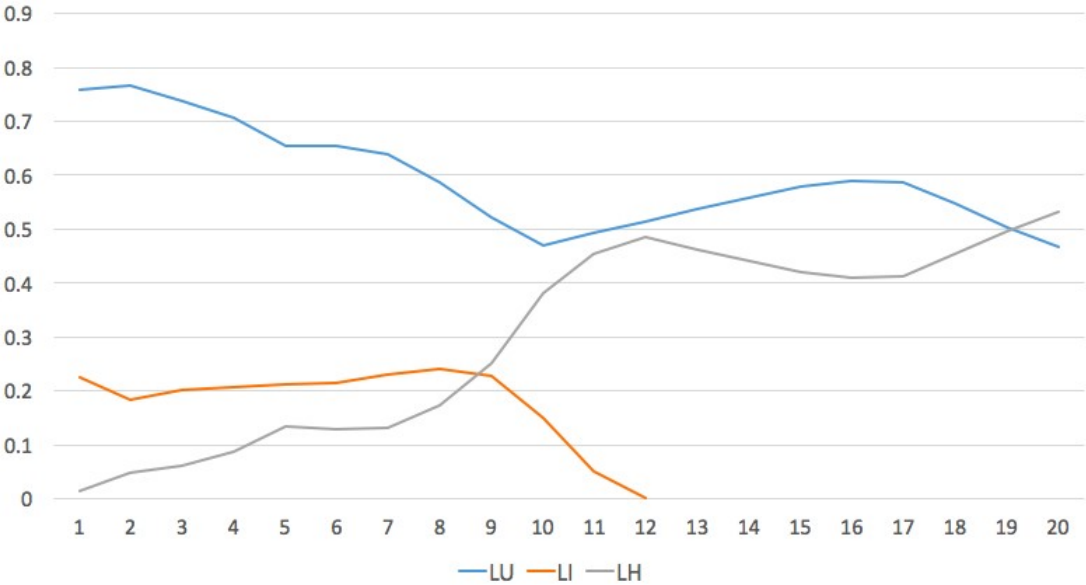Figure 7: Labor by Educational Attainment

Source: See Appendix

Automation initially gives rise to a fall in wages for unskilled workers, and a rise in wages for the intermediate and high-skilled sectors. The wage gap between high-skilled and low-skilled workers therefore opens, but then leads to the shift in educational attainment in Figure 7, thereby tending to restore the pre-shock relative wages across skill levels.

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In the second simulation, the rise in $\Theta_{R\&D}$ for low-skilled tasks (again starting in period 5) is now accompanied by a similar rise in R&D productivity for automation in intermediate-skilled tasks (starting in period 10). Thus, automation replaces both low-skilled and intermediate-skilled workers. The result, of course, is to give an added boost to the attainment of advanced degrees, so that both L_U and L_I decline, while L_H rises. The pattern is shown in Figure 8, which may usefully be compared with Figure 7.

Figure 8: Labor by Educational Attainment: Automation for Low-Skill and Intermediate-Skill Tasks
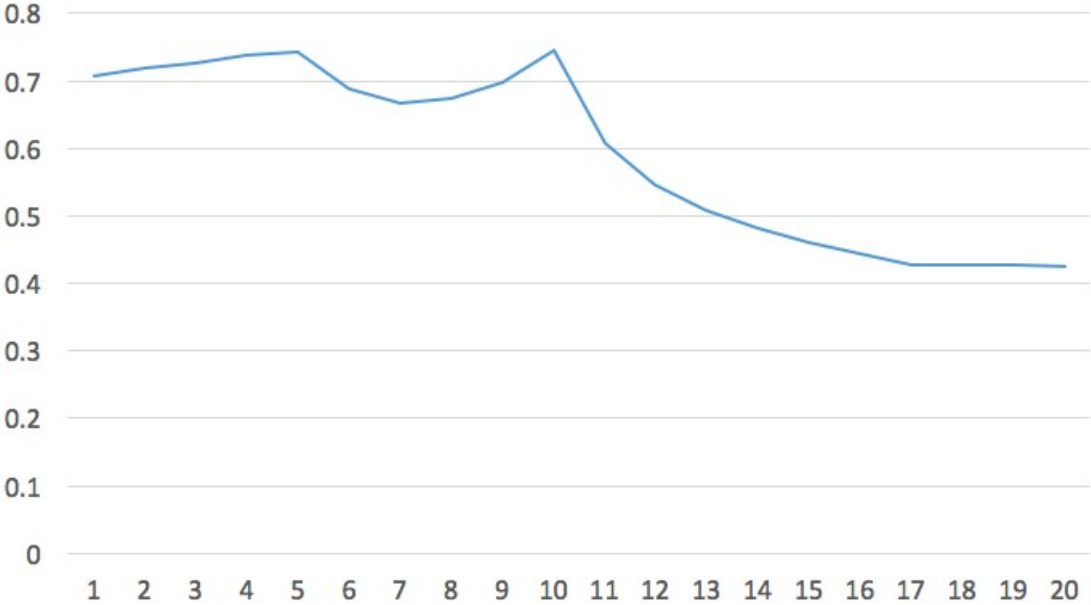

Source: See Appendix

In the case of automation of both unskilled and intermediate-skilled tasks, the main result is that market forces induce those receiving a bachelors degree to continue on to an advanced degree. The labor market ends up with just two kinds of labor, unskilled and highly skilled, with intermediate-skilled workers disappearing from the scene. Note that the model so far assumes that all workers are equally endowed with the skills needed for all levels of education; there is no "scarcity" value of STEM skills, for example, that would limit the supply of high-skilled workers. In a more realistic model, we would grapple with the obvious fact that not all students have the aptitude for an advanced degree for high-skilled work. Instead of the

wage differentials being offset by highly elastic shifts in educational attainment, a premium on higher education would be sustained in the long run as a kind of natural rent on high educational aptitude.

In both scenarios, the labor share of GDP declines markedly, as jobs are lost to automation. Figure 9 shows the time path of the labor share of GDP in the second scenario, in which automation for low-skilled workers takes off after period 5, and for intermediate-skilled workers after period 10. The labor share of income begins to dip around period 5 but then soars again around period 10 as the wages of skilled workers increases. Over time, as workers raise their educational attainment, wages decline and the overall labor share of income falls sharply under the pressures of automation.

Figure 9: Labor Share of GDP

Source: See Appendix

7   Next steps

The conclusions so far of the simulations are wholly qualitative. The next steps in modeling will be to parameterize the model according to the main structural features of the US economy. Of course, there are many difficult modeling and conceptual choices ahead, both in validating a parametrized model according to recent history and using the model to project the implications of future technological changes. Some of the difficulties are the following:

1. Modeling the automation process with empirical detail, for example by identifying the classes of machines that are complementarity to versus substitutional with various skills and occupations;

2. Estimating the returns to automation-inducing R&D, and the implications for the earnings of advanced technical workers;

3. Characterizing the supply and demand for higher education as a function of wage differential, borrowing costs, and educational aptitudes;

4. Characterizing the relative roles of private and public financing in determining the investments in R&D and in education;

5. Creating realistic scenarios for the future evolution of smart machines and their interaction with occupations at various skill levels;

6. Modeling the inter-generational dynamics of automation, as in Sachs and Kotlikoff (2012), and Benzell, Kotlikoff, LaGarda, and Sachs (2015);

7. Accounting for monopoly rents on patents and other changes in market structure associated with smart machines and artificial intelligence;

8. Accounting for the income distributional consequences of big data and network externalities, e.g. for giants such as Google and Amazon;

9. Accounting for the distributional implication of dematerialized production (e-commerce, e-books, e-payments) and the sharing economy (e.g. vehicles on demand);

10. Modeling the changes in past and future labor force participation and leisure time as the result of smart machines, artificial intelligence, and automation.

Appendix B. GAMS Equations

Kf(tf).. K(tf) =e= K0;
Hf(tf).. H(tf) =e= H0;
Uf(tf).. U(tf) =e= U0;
Sf(tf).. S(tf) =e= S0;
IPPAf(tf).. IPPA(tf) =e= IPPA0;
IPPAIf(tf).. IPPAI(tf) =e= IPPAI0;
Output(t).. Q(t) =e= TA(t)**Alpha*M(t)**(1-Alpha);
BAprod(t).. BA(t) =e= MBA(t)**.2*SBA(t)**.2*HBA(t)**.6;
PROFprod(t).. PROF(t) =e= MPROF(t)**.2*ProdPROF(t)*HPROF(t)**.8;
*PROFprod(t).. PROF(t) =e= ProdPROF(t)*HProf(t);
Health(t).. HL(t) =e= MHL(t)**.2*LUHL(t)**.1*SHL(t)**.2*HHL(t)**.5;
*HealthD(t).. HL(t) =e= HLmin*IPP(t)**.2;
HealthD(t).. HL(t) =e= .01;
Capital(t).. K(t) =e= M(t) + MBA(t) + MPROF(t) + MHL(t) + RA(t) + RAI(t);
Task(t).. TA(t) =e= (LU(t)+A(t))**Beta*(LS(t)+AI(t))**(1-Beta);
Robot(t).. A(t) =e= ThetaA(t)*HA(t)**Gamma*IPPA(t)**Delta*RA(t)**(1-Gamma-Delta);
ArtInt(t).. AI(t) =e= ThetaAI(t)*HAI(t)**Gamma*IPPAI(t)**Delta*RAI(t)**(1-Gamma-Delta);
RDA(t+1).. IPPA(t+1) =e= IPPA(t)*(1-depRD) + PRODRDA(t)*HRD(t);
RDAI(t+1).. IPPAI(t+1) =e= IPPAI(t)*(1-depRD) + PRODRDAI(t)*HRD(t);
HighS(t).. H(t) =e= HAI(t) + HA(t) + HRD(t) + HBA(t) + HPROF(t) + HHL(t);
KNext(t+1).. K(t+1) =e= K(t)*(1-dep) + FINV(t);
Saving(t).. C(t) =e= Q(t) - FINV(t) ;
UNext(t+1).. U(t+1) =e= U(t)*(1-n) - BA(t) + n*(U(t)+S(t)+H(t)) ;
SNext(t+1).. S(t+1) =e= S(t)*(1-n) + BA(t) - PROF(t);
HNext(t+1).. H(t+1) =e= H(t)*(1-n) + PROF(t);
LaborU(t).. U(t) =e= LU(t) + BA(t) + LUHL(t);
LaborS(t).. S(t) =e= LS(t) + 0.2*PROF(t) + SBA(t) + SHL(t);
Utils(t).. Ut(t) =e= log(C(t));
KLast(tl).. KL(tl) =e= K(tl)*(1-dep)+ FINV(tl);
CLast(tl).. CL(tl) =e= Q(tl) - dep*KL(tl);
Utility.. Util =e= sum(t,disc(t)*Ut(t)) + sum(tl,disc(tl)*log(CL(tl))/Discrate);

* Output

Parameter WageU(t), WageS(t), WageH(t), Rrate(t), IPPArate(t), IPPAIrate(t), Lshare(t), Kshare(t),
HAshare(t), RArate(t), Income(t), Lshare(t), LUshare(t), LSshare(t), LHshare(t);
Parameter Kshare(t), IPshare(t), LULF(t), LSLF(t), LHLF(t), LF(t);

WageU(t) = Alpha*Q.L(t)/TA.L(t) * Beta * TA.L(t)/(LU.L(t) + A.L(t));
WageS(t) = Alpha*Q.L(t)/TA.L(t) * (1-Beta) * TA.L(t)/(LS.L(t) + AI.L(t));
Rrate(t) = (1-Alpha)*Q.L(t)/M.L(t) ;
WageH(t) = ThetaA(t)*Gamma*(A.L(t)/HA.L(t))*WageU(t);
HAshare(t) = HA.L(t)/H.L(t);
RArate(t) = (1-Gamma-Delta)*(A.L(t)/RA.L(t))*WageU(t);
IPPArate(t) = Gamma*(A.L(t)/IPPA.L(t))*WageU(t);

IPPAIrate(t) = Gamma*(AI.L(t)/IPPAI.L(t))*WageS(t);
Income(t) = WageU(t)*LU.L(t) + WageS(t)*LS.L(t) + WageH(t)*H.L(t) + Rrate(t)*K.L(t) + IPPArate(t)*IPPA.L(t) + IPPAIrate(t)*IPPAI.L(t);
Lshare(t) = (WageU(t)*LU.L(t)+WageS(t)*S.l(t)+WageH(t)*H.L(t))/Income(t);
LUshare(t) = WageU(t)*LU.L(t)/Income(t);
LSshare(t) = WageS(t)*LS.L(t)/Income(t);
LHshare(t) = WageH(t)*H.L(t)/Income(t);
Kshare(t) = Rrate(t)*K.L(t)/Income(t);
IPshare(t) = (IPPArate(t)*IPPA.L(t)+IPPAIrate(t)*IPPAI.L(t))/Income(t);
LF(t) = LU.L(t) + LS.L(t) + H.L(t);
LULF(t) = LU.L(t)/LF(t);
LSLF(t) = LS.L(t)/LF(t);
LHLF(t) = H.L(t)/LF(t);

Appendix C. Parameter Values

Parameters Gamma, Alpha, Beta, Delta, Disc(t), dep, depRD, HLmin, Discrate;

Gamma = .5;
Alpha = .7;
Beta = .7;
Gamma = .3;
Delta = .3;
Discrate = .06;
Disc(t) = (1/(1+Discrate))**(ord(t)-1);
dep = 0.05;
depRD = .05;
HLmin = .1;

Parameters ThetaA, ThetaAI, tfpRA(t), tfpRAI(t);

ThetaA(t) = 1;
ThetaAI(t) = 1;
*tfpRA(t) = .01;
*tfpRA(t)$(ord(t) ge 10) = 1;
*tfpRAI(t) = .01;
*tfpRAI(t)$(ord(t) ge 15) = 1;
tfpRA(t) = 1;
tfpRAI(t) = 1;

Appendix D. Initial Values

Parameter K0, U0, S0, H0, ProdRDA(t), ProdRDAI(t), ProdPROF(t), IPPA0, IPPAI0, n, Start(t);

K0 = 21.9;
U0 = 7.3;
S0 = 2.25;
H0 = 0.15;
ProdRDA(t) = .01;
ProdRDAI(t) = .01;
*ProdRDAI(t)$(ord(t) ge 10) = 1;
ProdPROF(t) = 2;
IPPA0 = 0.001;
IPPAI0 = 0.001;
n = 0.05;