Empirical Analysis of Equilibrium in Over-the-counter (OTC) Trading Networks

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Overview

Market Structure: OTC Markets

- Frictions exist in markets for financial assets, Centralized or Decentralized (OTC)
- One important source of friction in OTC financial markets is imperfect competition
- Imperfect competition implies that agents are no longer price-takers, but their own actions do affect equilibrium prices, giving rise to strategic behavior in the markets
- Imperfect competition gives rise to price impact of agents' trading activity
- In OTC markets, trades occur in bilateral interactions among traders

OTC Markets cont'd

- Consequently, the imperative for strategic behavior is quite pronounced in OTC markets
- Various equilibrium models have been developed, including ones that reflect strategic behavior and imperfect competition driving trading frictions
- Empirical analyses of these models have only calibrated and/or only performed comparative statics

Research Objectives

- This study proposes direct econometric estimation of the parameters on an OTC trading network

- Estimates would enable characterization of interplay between trading network's properties, the attainment of equilibrium, and the strategic behavior of traders

Research Objectives

- Specifically, we would be able to gain the following insights:
  - does the transaction data reflect the theoretical equilibrium?
  - to what extent does private information drive observed transactions for the OTC network being studied?
  - does the theoretical model's representation of dealers' strategic profit motives reflect in the transaction data?

Literature Review

- Two main approaches exist in the literature for analyzing OTC markets
- There is the random search and match approach (Duffie, Garleanu, Pedersen (2005)), in which traders are randomly matched and trade occurs as a consequence of investors' divergent asset allocation needs
- This approach imparts dynamic behavior to the evolution of OTC markets, but ignores the reality that trading relationships form among traders, and these relationships are sticky
- The alternative is the network approach, which takes into account the reality that agents form long-lived trading relationships through which they trade
- This study focuses on the latter approach

Literature Review

- The network approach to OTC market microstructure analysis focuses more on the strategic nature of the bilateral interactions among traders in an OTC trading network

- Babus and Kondor (2018), henceforth BK, directly models bilateral strategic trading in networks where private information drives endogenous formation of information asymmetries and price impact

- Malamud and Rostek (2017); Giannikos (2012); Wang (1994) theoretically show implications for other plausible mechanisms also driving information asymmetries and price impact

- Kocagil and Shachmurove (1998); Bessembinder (2006); Hollifield et al (2017); Li and Schuroff (2019) have all performed empirical analyses that use trade quantities to proxy information content of price change

MODEL: Strategic Trading

- In the BK model, each dealer engaged in a bilateral OTC trade with another dealer also serves his own price-sensitive customer-base.

- Let s^i be trader i's private signal comprising the common valuation across dealers of the asset being traded, Θ, superimposed by an uncertainty process η^i and a signal observation error ϵ^i, so that, $s^i = \Theta + \eta^i + \epsilon^i = \theta^i + \epsilon^i$. Let ρ the square of the correlation of valuations across the various dealers within the trading network, σ^2 be a variance parameter, and ζ be a variance scaling of the measurement error, then $\Theta \sim N(0, \rho\sigma^2)$, $\eta^i \sim N(0, (1-\rho)\sigma^2)$, and $\epsilon^i \sim N(0, \zeta\sigma^2)$.

- Let (g, E) represent a graph with nodes g and edges E of the OTC interdealer trading network. Then any dealer i's trading network (g^i, E^i) is a subset of the entire network, so that $(g^i, E^i) \subseteq (g, E)$

Strategic Trading

- The demand function for trader i, trading with trader j on the bilateral trade link ij, is written as:

$$Q^i_{ij}(s^i; \mathbf{p}_{g^i}) = t^i_{ij}(y^i s^i + \sum_{k \in g^i} z^i_{ik} p_{ik} - p_{ij})$$

- $\mathbf{p}_{g^i}$ is a vector that contains prices on each bilateral trading link within trader i's set of network links, g^i
- t^i_{ij} is trader i's trading intensity on the link ij
- y^i is the weight trader i places on his own apriori signal in his (Bayesian) learning of the asset's value
- z^i_{ik} is the information asymmetry parameter, a weight trader i places on the signal he derives from the link ik within his trading network
- p_{ik} is the price on the bilateral link ik

Strategic Trading

- Market clearing on each bilateral trading link ij is as follows:

$$Q_{ij}^{i}(s^{i};\mathbf{p}_{g^{i}}) + Q_{ij}^{j}(s^{j};\mathbf{p}_{g^{j}}) + \beta_{ij}p_{ij} = 0$$

  Where $\beta_{ij} = \beta(1+\mu_{ij}) = \beta *$ (markups on ij), with β being slope of the demand curve for the non-dealer investors

- The risk-neutral dealer i solves the following OTC game to produce a linear Nash Equilibrium in demand functions $(\mathbf{Q}^{i}(s^{i};\mathbf{p}_{g^{i}}))$, the Bayesian Nash Equilibrium (BNE):

$$\max_{(Q_{ij}^{i})_{j\in g^{i}}} E\left\{\left[\sum_{j\in g^{i}} Q_{ij}^{i}(s^{i};\mathbf{p}_{g^{i}})(\theta^{i}-p_{ij})\right]\middle| s^{i},\mathbf{p}_{g^{i}}\right\}$$

- Solving the maximization problem above, subject to market clearing for each link, the equilibrium price and demand quantities are:

$$p_{ij} = \frac{t_{ij}^{i}E(\theta^{i}|s^{i},\mathbf{p}_{g^{i}}) + t_{ij}^{j}E(\theta^{j}|s^{j},\mathbf{p}_{g^{j}})}{t_{ij}^{i}+t_{ij}^{j}-\beta_{ij}}$$

Equilibrium

- The equilibrium values, $E(\theta^i|s^i, \mathbf{p}_{g^i}) = e_i$, still remain to be resolved
- The conventional approach implies solving for a fixed-point in the space of NxN matrices for a network containing N dealers to obtain the OTC game's BNE
- But, BK introduces the *Conditional Guessing Game Equilibrium* (CGGE), a methodological innovation to supplant the conventional approach and solve for OTC game's BNE
- BK proves that the CGGE converges to the OTC game's BNE, and that the OTC game captures all features of the OTC price-discovery process but its potentially dynamic nature; i.e. bidding strategies being static and not depending on information from all previous bidding rounds but just the last

Equilibrium

- In the *Conditional Guessing Game* equilibrium (CGGE):

$$e_i = \bar{y}^i s^i + \bar{z}_{g^i} e_{g^i}$$

where

$$\bar{y}^i = \frac{y^i}{\left(1 - \sum_{k \in g^i} z^i_{ik} \frac{2 - z^k_{ki}}{4 - z^i_{ik} z^k_{ki}}\right)}$$

and

$$\bar{z}^i_{ij} = z^i_{ij} \frac{\frac{2 - z^i_{ij}}{4 - z^i_{ij} z^j_{ji}}}{\left(1 - \sum_{k \in g^i} z^i_{ik} \frac{2 - z^k_{ki}}{4 - z^i_{ik} z^k_{ki}}\right)}$$

Working Form of the Model

- Equilibrium (posterior) expectation of each dealer i's valuation, e_i, of the traded asset, conditioned on private and network information can be shown to have the form:

$$\vec{e} = \bar{y}\vec{s} + \bar{z}\vec{e}$$

OR,

$$\vec{e} = (I - \bar{z})^{-1}\bar{y}\vec{s} = U^{-1}\bar{y}\vec{s}$$

  - $\vec{e}$ is an N-column vector of e_i's, the equilibrium conditional expectation of each trader's valuation of the asset
  - $\bar{y}$ is a NxN diagonal matrix with $\bar{y}^i$'s on its diagonal
  - $\vec{s}$ is an N-column vector of s^i's, the traders' private signals
  - $\bar{z}$ is a NxN matrix with zeros on its diagonal and the information asymmetry parameters, $\bar{z}_{ij} = \bar{z}_{ij}{}^i = \bar{z}_{ji}{}^i$ and $\bar{z}_{ji} = \bar{z}_{ij}{}^j = \bar{z}_{ji}{}^j$, as its off-diagonal elements

Inverting the System Matrix

- The matrix U has all ones on its diagonal, and proving its invertibility is one major hurdle that must be cleared to establish an estimation methodology that has general applicability for this problem.
- A sketch of this proof proceeds by identifying that U has a special structure and exploits this structure for the proof, as follows:

$$U = (I + (-\bar{z}_l)) + (-\bar{z}_r)$$

$$U = \begin{pmatrix} 1 & & & \\ & \cdot & & \mathbf{0} \\ & & \cdot & \\ & [-\bar{z}_{ij}]_{\{i>j\}} & & \cdot \\ & & & & 1 \end{pmatrix} + \begin{pmatrix} 0 & & & \\ & \cdot & & [-\bar{z}_{ij}]_{\{i<j\}} \\ & & \cdot & \\ & \mathbf{0} & & \cdot \\ & & & & 0 \end{pmatrix}$$

Inverting the System Matrix

- A special case of U is when all traders in the network are connected to every other trader, and they each place the same amount of information asymmetry weights on all of their links (an undirected complete network with weighted or unweighted links). U becomes a symmetric matrix.

- Considering the general case, if trader i is not in trader j's network, then $z_{ij}^{\{i,j\}} = 0$

- Applying the Sherman-Morrison-Woodbury formula, a proof is established for the invertibility of the system matrix

Inverting the System Matrix

- Let $U^{-1} = W$. With the existence of W established, we have:

$$\vec{e} = W\bar{y}\vec{s} = \begin{bmatrix} e_1 \\ e_2 \\ . \\ . \\ . \\ e_n \end{bmatrix}$$

- e_i then gives the CGGE valuation of trader i

- This resolves traders' (posterior) valuations of the traded asset to enable us calculate the theoretical equilibrium prices and quantities

Inverting the System Matrix

- The equation for equilibrium price becomes:
$$p_{ij} = \frac{(2-z^j_{ij})e_i+(2-z^i_{ij})e_j}{4-z^i_{ij}z^j_{ij}} = e_{ij}, \text{ say}$$

- And, the equilibrium traded quantity becomes:
$$q^i_{ij} = \frac{-\beta_{ij}(2-z^j_{ij})[(2+z^j_{ij}-z^i_{ij}z^j_{ij})e_i-(2-z^i_{ij})e_j]}{(4-z^i_{ij}z^j_{ij})(z^i_{ij}+z^j_{ij}-z^i_{ij}z^j_{ij})}$$

Exogenous Demand and Trading Behavior in OTC Networks

- The demand function of Giannikos (2012) compared to that of BK reveals parallels suggesting (CARA) risk-aversion (α) inversely relates to the trading intensity; i.e. $\alpha \propto \frac{1}{t^i_{ij}}$
- Malamud and Rostek (2017) suggests price impacts arising from distribution of risk-preferences and initial risk endowments in the market
- Wang (1994) suggests individual and aggregate risks reflect in prices and the joint distribution of prices and traded quantities reflect heterogeneity of agents in the market place

Exogenous Demand and Trading Behavior in OTC Networks

- Empirically:
  - Kocagil and Shachmurove (1998) study the traded price/quantities dynamic of price discovery in several Futures markets
  - In the bond markets, Bessembinder (2006), Hollifield et al (2017), Li and Schuroff (2019) all have used traded quantities to empirically proxy information asymmetry
- Motivated by these additional concepts, an empirical component is added to BK's equilibrium relations for price and quantities to obtain final estimation relationships, as follows:

$$p_{ij}^{(a)} = e_{ij}^{(a)} + \Delta p_{ij}^{(a)} + intp^{(a)}$$

$$q_{ij}^{(a)} = qe_{ij}^{(a)} + \beta_{ij}^{(a)} * \Delta p_{ij}^{(a)}$$

Exogenous Demand and Trading Behavior in OTC Networks

- $\Delta p_{ij}^{(a)}$ is a trade-specific effect on the price for security a traded on the trading link ij
- $intp^{(a)}$ is a security-specific effect on the price for security a
- $\beta_{ij}^{(a)}$ is a trade-specific effect on the exogenous demand for security a traded on the trading link ij.
- The definition of $qe_{ij}^{(a)}$ for the pure private-information equilibrium is now modified to reflect the notion that the relevant β in that previous equation is fixed for the given bilateral trading link. Hence,

$$qe_{ij}^{(a)} = \frac{-\bar{\beta}_{ij}(2 - z_{ij}^{j})[(2 + z_{ij}^{j} - z_{ij}^{i} z_{ij}^{j})e_i^{(a)} - (2 - z_{ij}^{i})e_j^{(a)}]}{(4 - z_{ij}^{i} z_{ij}^{j})(z_{ij}^{i} + z_{ij}^{j} - z_{ij}^{i} z_{ij}^{j})}$$

ESTIMATION METHODOLOGY: Repeated Trading in the OTC Network

- Estimation is based on the premise that the network parameters are persistent across different trades
- If network parameters were changing across time or trades, Bayesian regime switching could be employed (see Agbeyegbe and Goldman (2005)). However, the necessary repeated occurrences in the data would still be scarce to find in the transaction data
- Trade dates are appended to CUSIP IDs of securities to control for potentially changing market environment and create unique securities that feed the estimation

Bayesian Estimation of the Network Parameters

- For a given trading network, $\bar{z}$ and $\bar{y}$ are considered fixed, owing to the various y^i's and $z_{ij}^{\{i,j\}}$'s being fixed.
- Using the closed-form relationships for p_{ij} and q_{ij}^i, a formulation for Bayesian estimation of these parameters is now pursued
- For a traded asset, A, recall: $s_r^{(A)} = \hat{\theta}^{(A)} + \eta_r + \epsilon_r$, where $\eta_r \sim N(0, \sigma_\eta^2)$, $\epsilon_r \sim N(0, \sigma_\epsilon^2)$, and $\hat{\theta}^{(A)} \sim N(\underline{\hat{\theta}}^{(A)}, \sigma_{\hat{\theta}}^2)$, $\sigma_\eta^2 = (1-\rho)\sigma^2$, $\sigma_{\hat{\theta}}^2 = \rho\sigma^2$, and $\sigma_\epsilon^2 = \zeta\sigma^2$
- η_r and ϵ_r are trader-specific uncertainty terms, whereas $(\hat{\theta}^{(A)} - \underline{\hat{\theta}}^{(A)})$ are security-specific uncertainty terms. The trader-specific uncertainty terms are assumed to be identical and independent across all traders within the network

Bayesian Estimation of the Network Parameters

- Instead of using a Gibbs sampler, which converges too slowly, especially in geometrically challenging parameter space manifolds, an Hamiltonian Monte Carlo (HMC) sampler implementing No-U-Turn-Sampling (NUTS) is used
  - see Gelman and Hoffman (2014), "The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo"
- A software implementation of HMC-NUTS is available via the Stan Probabilistic Programming Language `http://mc-stan.org`
- An interface to Stan is available through R, Python, and the Linux command line (cmdstan). The latter is used in this study

Bayesian Estimation of the Network Parameters

- Besides helping deal with challenging parameter space manifolds, the Bayesian approach also helped handle interval range constrains placed on the estimation parameters by the theory

- Important parameter constrains are: $z_{ij}^{\{i,j\}} \in (0,2)$, $y^i > 0$, $\rho \in (0,1)$, $\zeta > 0$, $-\beta_{ij} > 0$

- Whereas BK takes $\underline{\hat{\theta}}^{(A)}$ to be zero, I put a normal distribution on it, without loss of generality, so that $\underline{\hat{\theta}}^{(A)} \sim N(th_1, th_2)$, with th_1 and th_2 getting hyper-priors.

- The joint posterior distribution for the parameters, conditional on trade data, is derived as follows:

Bayesian Estimation of the Network Parameters

- Let a index securities, and i and j index links in the network as usual. Then the following distributions are posited for the various parameters and data:

$$p_{ij}^{(a)} \sim N\left(e_{ij}^{(a)} + \Delta p_{ij}^{(a)} + intp^{(a)}, \psi^{(a)}\right)$$

$$q_{ij}^{(a)} \sim N\left(qe_{ij}^{(a)} + \beta_{ij}^{(a)} * \Delta p_{ij}^{(a)}, \Gamma^{(a)}\right)$$

$$S^{(a)} = \begin{pmatrix} s_1^{(a)} \\ . \\ . \\ . \\ s_n^{(a)} \end{pmatrix} \sim N\left((1)_n \underline{\hat{\theta}}^{(a)}, diag(\sigma_{\hat{\theta}}^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\right)$$

Bayesian Estimation of the Network Parameters

$$y_i \sim Gamma(a^{(1)}, a^{(2)})$$

$$z_{ij}^{i} \sim TN_{(0,2)}(1, 1)$$

$$\beta_{ij} \sim Gamma(V, \gamma)$$

$$\sigma \sim Gamma(sd^{(1)}, sd^{(2)})$$

$$\zeta \sim Gamma(sth_1, sth_2)$$

$$\underline{\hat{\theta}}^{(a)} \sim N(th_1, th_2)$$

$$\rho \sim N(0.5, 1)$$

Bayesian Estimation of the Network Parameters

- $TN_{(a,b)}(m, n)$ is the Truncated Normal distribution with mean m and variance n, constrained to have non-zero density only in the interval (a, b).

- The parameters that follow are the (level 1) hyper-parameters in this hierarchical formulation (Gelman (2006)) of the estimation problem, together with their prescribed prior distributions:

$$\psi^{(a)} \sim Gamma(w_1, w_2)$$

$$\Gamma^{(a)} \sim Gamma(\tau_1, \tau_2)$$

$$V \sim Gamma(d_1, d_2)$$

$$\gamma \sim Gamma(e_1, e_2)$$

Bayesian Estimation of the Network Parameters

- The following are (level 2) hyper-parameters in this hierarchical model, and as I'm not interested in learning these parameters, I give them all the same $Gamma(1,1)$ non-informative prior distribution:
- $a^{(1)}, a^{(2)}, sd^{(1)}, sd^{(2)}, sth_1, sth_2, th_2, d_1, d_2, e_1, e_2, \tau_1, \tau_2$. With $th_1 \sim N(0,2)$

Bayesian Estimation of the Network Parameters

- Assuming $a = 1$ to A are unique CUSIP-time IDs that traded in the OTC network and (N_a, g_a) corresponds to the subnetwork in which a given a traded as described in section 3.2, then, applying Baye's rule, we have the following posterior joint distribution of the network parameters, conditional on the observed trade prices and quantities:

Hierarchical Probability Model

$$Pr\left(S^{(1:A)}, y_{(1:n)}, z^{(1:n)}_{(1:n)^2}, \bar{\beta}_{(1:n)^2}, \beta^{(1:A)}_{(1:n)^2}, \Delta p^{(1:A)}_{(1:n)^2}, intp^{(1:A)}, \{H\} \middle| p^{(1:A)}_{(1:n)^2}, q^{(1:A)}_{(1:n)^2} \right)$$

$$\propto$$

$$\left[\prod_{a=1}^{A} \left\{ N\left(S^{(a)} \middle| (1)_n \underline{\hat{\theta}}^{(a)}, diag(\sigma_A^2 + \sigma_\eta^2 + \sigma_\epsilon^2) \right) * N\left(\underline{\hat{\theta}}^{(a)} | th_1, th_2 \right) \right. \right.$$

$$\left[\prod_{i \in N_a} \prod_{j \in g^i} TN_{(0,\infty)} \left(p_{ij}^{(a)} \middle| e_{ij}^{(a)} + \Delta p_{ij}^{(a)} + intp^{(a)}, \psi^{(a)} \right) * \right.$$

$$\left. TN_{(0,\infty)} \left(q_{ij}^{(a)} \middle| \bar{q}e_{ij}^{(a)} + \beta_{ij}^{(a)} * \Delta p_{ij}^{(a)}, \Gamma^{(a)} \right) \right] *$$

Hierarchical Probability Model

$$\left[\prod_{ij\in g_a} G(\beta_{ij}^{(a)}|\alpha_1,\alpha_2) * N(\Delta p_{ij}^{(a)}|\mu,\lambda)\right] * G(\psi^{(a)}|w_1,w_2) * G(\Gamma^{(a)}|\tau_1,\tau_2)*$$

$$N(intp^{(a)}|intp_m, intp_s)\Big\}\Big]*$$

$$\left[\prod_{i\in\cup_{r=1}^{A}N_r} G(y^i|a^{(1)},a^{(2)})\right]*$$

$$\left[\prod_{\{ij\}\in\cup_{r=1}^{A}g_r} TN_{(0,2)}(z_{ij}^{i}|1,1) * TN_{(0,2)}(z_{ij}^{j}|1,1) * G(\bar{\beta}_{ij}|v,\gamma)\right]*$$

$$G(v|d_1,d_2) * G(\gamma|e_1,e_2) * G(\alpha_1|f_1,f_2) * G(\alpha_2|h_1,h_2) * G(\sigma|sd^{(1)},sd^{(2)})*$$
$$G(\zeta|sth_1,sth_2) * N(\mu|l_1,l_2) * G(\lambda|m_1,m_2) * N(intp_m|op_1,op_2)*$$
$$G(intp_s|om_1,om_2) * G(w_1|1,1) * G(w_2|1,1) * G(\tau_1|1,1) * G(\tau_2|1,1)*$$

Hierarchical Probability Model

$$G(d_1|1,1) * G(d_2|1,1) * G(e_1|1,1) * G(e_2|1,1) * N(th_1|0,2) * G(th_2|1,1) *$$
$$G(sd^{(1)}|1,1) * G(sd^{(2)}|1,1) * N(\rho|0.5,1) * G(sth_1|1,1) * G(sth_2|1,1) *$$
$$G(a^{(1)}|1,1) * G(a^{(2)}|1,1) * G(f_1|1,1) * G(f_2|1,1) * G(h_1|1,1) *$$
$$G(h_2|1,1) * N(l_1|0,2) * G(l_2|1,1) * G(m_1|1,1) * G(m_2|1,1) *$$
$$N(op_1|0,2) * G(op_2|1,1) * G(om_1|1,1) * G(om_2|1,1)$$

Data Description

- The Data for estimation is from the Municipal Securities (Munis) market, which researchers have documented to have a core-periphery trading network structure (Li and Schurhoff, 2019)

- The Municipal Securities Regulatory Board (MSRB), through its regulatory activities, requires Munis market participants to report their trade transactions. The resulting data-set of trade transactions is available through subscription

- The Academic Historical Transactions data-set (AHTD) is a version of the data subscription available for Academic research purposes only and includes data fields anonymized but unique for Dealers that in the sample period

Data Processing

- The data sample for this research spans the calendar period June 1, 2015 to May 31, 2016
- This data has been processed to filter out for use, only trades that occurred between Dealers, i.e. inter-dealer trades. Also, unique identifiers for Securities have been created by appending the calendar date of a trade to the Security's CUSIP identifier to create CUSIP-time identifiers
- As such, trades that occur on different calendar dates for the same CUSIP shall be considered as different Securities in the framework of this estimation

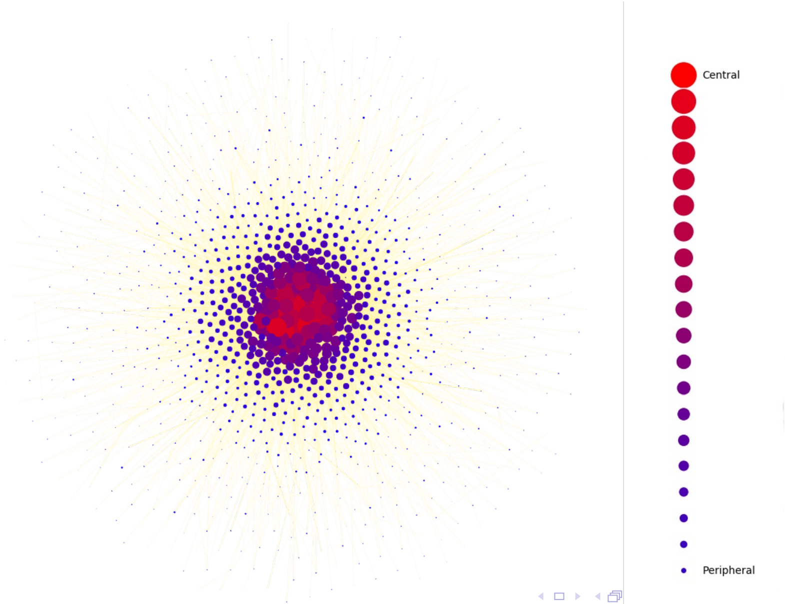
Data Processing

- A sub-sample of the data with only trades of \$1MM par-amount or larger was used to generate the main results. This reflects Hollifield et al (2017)'s choice of cumulative trade amounts that connote the existence of trading relationships

- This sub-sample contained:
  - Total trade count of 221,153
  - Unique dealer count in the network totaled 1,398
  - Unique number of bilateral trading interactions totaled 12,307
  - Unique number of CUSIP-time IDs totaled 15,5051
- For each trade (CUSIP-time) analyzed in the sample, a subnetwork containing only traders that traded that CUSIP-time was created

- The method for finding the CGGE posterior valuation signal, $e_i = E(\theta^i | s^i, \mathbf{p}_{g^i})$, of each trader i is then applied to this subnetwork

Network Visualization



Distribution of Connectedness

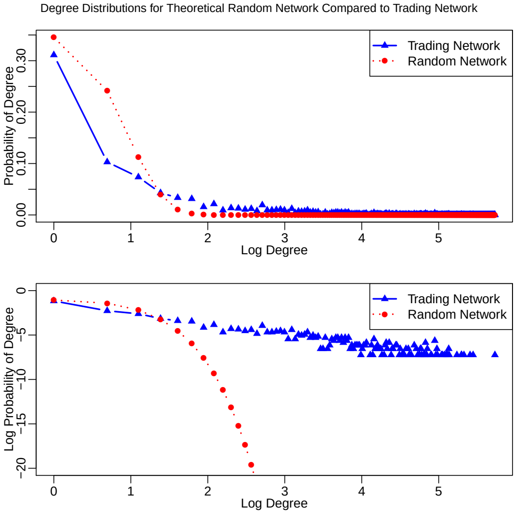


Figure: Degree Distributions

Trading Network's Degree Distribution

- The notion that most real-life networks are scale-free networks (Jackson (2008)) is checked in the preceding plots
- Scale-free networks have degree distributions that follow a power-law
- Log-log plots of power-law distributions must be linear, which is what we see in the plot above
- Hence, we can conclude that the trading network is a scale-free network

Kolmogorov-Smirnov Test Statistic

- Test result: D $=$ 0.47139, p-value $<$ 2.2e-16
- $\alpha = 0.005$ implies c(α) $=$ 1.731. For n $=$ m $=$ 1398, we get a D_{min} $=$ 0.0655
- Hence, the null hypothesis can be rejected at a significance level of 0.005. The theoretical degree distribution is different from the trading network's

General Network-level Results

Table: Select Estimated Parameters

Parameter	Mean	StdDev	5^{th} %	50^{th} %	95^{th} %
ρ	0.55	0.20	0.26	0.62	0.80
σ	3.40	2.00	0.79	3.30	6.40
ζ	1.20	0.98	0.18	1.30	2.70
$intp_m$	-1.00	0.39	-1.40	-1.00	-0.37
$intp_s$	1.60	2.20	0.24	0.36	5.40
v	3.90	2.40	0.46	4.10	7.10
γ	1.50	1.80	0.24	0.63	4.70
μ	0.92	0.75	-0.061	1.40	1.90
λ	1.80	1.30	0.18	2.80	3.20
α_1	2.30	1.20	0.99	2.10	4.30
α_2	2.00	1.00	0.42	2.80	3.00

General Network-level Results

- The table above shows that the posterior distribution has a fairly low variance. Each network-level parameter reported has its standard deviation lower in magnitude than its mean, except for $intp_s$ and γ. Also, the 5^{th} to 95^{th} percentile ranges of each parameter is also narrow, with most of the parameters having their mean and median almost aligned

- The first parameter, ρ, which is the square of correlation has a value of 0.55 and thus indicates a correlation of about 0.742 among dealers' asset valuations across the network, indicating just over 20% private information in the network. $\rho = 1$ puts us in a common values model

- This supports the Li and Schurhoff (2019) observation that trading in Municipal bonds is driven more by liquidity needs and less by private information.

General Network-level Results

- σ of 3.40 implies a variance of 11.56, reflects high dispersion of valuations across the unique traded assets (prices ranged from \$1 to \$189 per \$100 par amount traded)
- ζ of 1.20 represents a slightly larger dealer-level valuation signal noise relative to the dispersion of valuations across securities
- The mean of the distribution for security fixed-effect, $intp_m$, is -1.00 indicating a downward bias, with a standard deviation of 0.39 (variance of 0.152) indicating fairly low dispersion across securities
- The trade-specific price effects, Δp, have a mean (μ) of 0.92 and standard deviation (λ) of 1.80 which represents a fairly low variance of 3.24

Trades, the CGGE, and Valuation Signals

Table: CGGE and Valuation Signal Correlations With Trades

Measurement	Value
Cor(ActualPrice, Seller CGGE Signal)	-0.0969
Cor(ActualPrice, Buyer CGGE Signal)	0.0254
Cor(ActualPrice, MeanCommonVal (Θ_m))	0.2933
Cor(ActualPrice, SecurityCommonVal (Θ))	0.0645
Cor(ActualQuantity, Seller CGGE Signal)	0.1280
Cor(ActualQuantity, Buyer CGGE Signal)	0.4157
Cor(ActualQuantity, MeanCommonVal (Θ_m))	-0.1072
Cor(ActualQuantity, SecurityCommonVal (Θ))	-0.1177

- CGGE (BNE), a fundamental premise is assessed in the table above. Correlation measurements between CGGE quantities and other network quantities are assessed against intuition

Trades, the CGGE, and Valuation Signals

- Trade prices correlate weakly positively with Buyer CGGE signals and weakly negatively with Seller CGGE signals (respectively 0.0254 and -0.0969)

- Traded quantities correlate positively with both Buyer and Seller CGGE signals (respectively 0.4157 and 0.1280)

- This observation aligns with the intuition that both buying and selling dealers want to maximize profit, so buyers buy more aggressively (in par amount; likewise, sellers sell less aggressively) when the CGGE signal is stronger.

- More aggressive buying induces the weakly positive correlation of the Buyer CGGE signal with price; the less aggressive selling induces the weakly negative correlation between prices and the Seller CGGE signal.

Trades, the CGGE, and Valuation Signals

- Dealers' common valuations of assets traded within the network, as well as the mean of this its distribution both correlate positively with actual prices (respectively, 0.0645 and 0.2933)
- The same quantities both correlate negatively with actual traded quantities (respectively, -0.1177 and -0.1072)
- This is a manifestation of the residual demand curve being negatively sloping.

Trades, the CGGE, and Valuation Signals

Table: Common Valuations and CGGE Signals

Measurement	Value
Cor(SecurityCommonVal (Θ), Seller CGGE Signal)	-0.1775
Cor(SecurityCommonVal (Θ), Buyer CGGE Signal)	0.0485

- Dealers' common valuations of assets traded within the network correlate positively with Buyer CGGE signals and negatively with Seller CGGE signals (respectively 0.0485 and -0.1775)

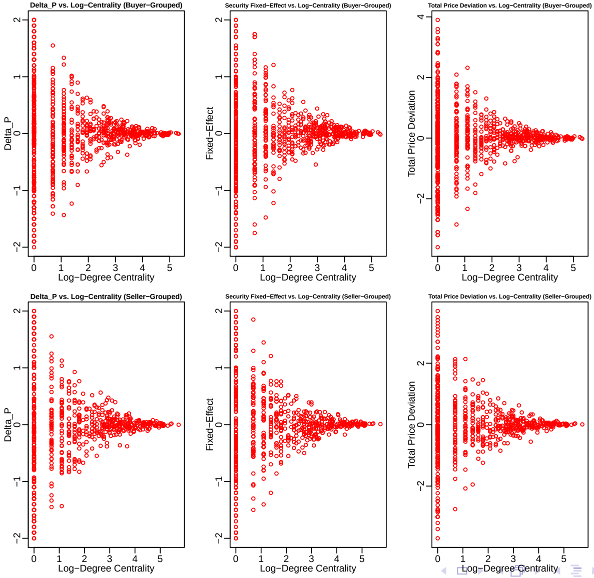
- Again this aligns with the intuition that agents want to maximize profits. Hence, they would sell when their valuations of the asset is unfavorable and buy when their valuations of the asset is favorable.

Centrality and Equilibrium

- Define centrality as the number of other traders that a particular trader has traded with in the sample
- The charts in Figure 2 show that the more central a dealer is in the network the closer, on average, that dealer's trade prices are to the OTC Bayesian Nash Equilibrium (BNE) price obtained by BK
- Recall $p_{ij}^{(a)} = intp^{(a)} + \Delta p_{ij}^{(a)} + \bar{p}_{ij}^{(a)}$ where $\bar{p}_{ij}^{(a)}$ represents the BNE price, $intp^{(a)}$ represents a CUSIP-time ID specific constant, and $\Delta p_{ij}^{(a)}$ represents a trade specific constant.
- Each chart in Figure 2 has log(centrality) on its horizontal axis, and respectively on its vertical axis $\Delta p_{ij}^{(a)}$, $intp^{(a)}$, and the total deviation from the equilibrium trade price given by $intp^{(a)} + \Delta p_{ij}^{(a)}$, respectively grouped by Buyers and Sellers.

Centrality and Equilibrium: Convergence of Prices

Figure 2: Plots of Equilibrium Behavior vs. Centrality



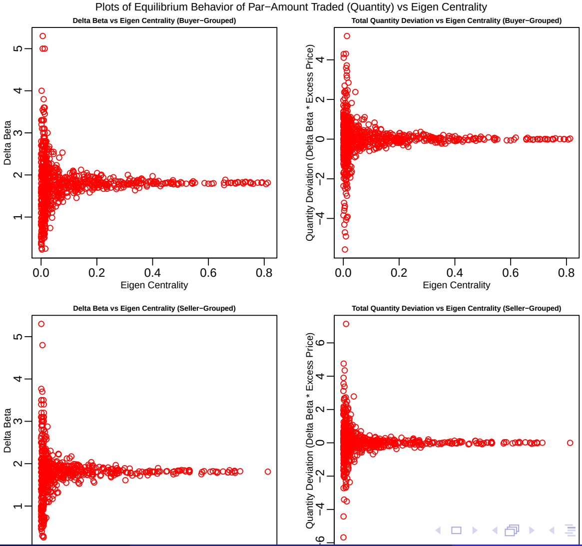
Centrality and Equilibrium

- The charts in the next figure show that the more central a dealer is in the network the closer, on average, that dealer's traded quantities are to the OTC Bayesian Nash Equilibrium (BNE) quantities obtained by BK

- Recall $q_{ij}^{(a)} = \Delta p_{ij}^{(a)} * \beta_{ij}^{(a)} + qe_{ij}^{(a)}$ where $qe_{ij}^{(a)}$ represents the BNE quantity, and $\beta_{ij}^{(a)}$ represents a trade specific "excess" exogenous demand factor.

- Each chart in Figure 2 has eigen-centrality on its horizontal axis, and respectively on its vertical axis $\beta_{ij}^{(a)}$ and the total deviation from the equilibrium traded-quantity given by $\Delta p_{ij}^{(a)} * \beta_{ij}^{(a)}$, respectively grouped by Buyers and Sellers.

Centrality and Equilibrium: Convergence of Trade Quantities



Centrality and Equilibrium

- The charts in the preceding figures show a consistent pattern of values on the vertical axis going towards zero as centrality increases
- This picture is an indication that deviations from the BNE price or quantities disappear as centrality increases
- This observation suggests that more central dealers are exposed less to information asymmetries when they trade, if the latter is characterized as deviation from the BNE (efficiency benchmark).
- That again aligns with the finding by Li and Schurhoff (2019), which also states that central dealers are exposed less to information asymmetries in the Munis market.

Conclusion

- In the foregone, it has been shown that the equilibrium parameters of an OTC trading network can be estimated, using a Bayesian inference techniques and applying the Stan software's implementation of HMC-NUTS for the computations.
- It is quite evident that the ability to estimate such endogenous parameters of the OTC trading network provides more insightful observations than would otherwise be possible.

Thank you!