Breakable commitments:
present-bias, client protection and bank ownership forms

Karna Basu & Jonathan Conning*

April 18, 2017

Abstract

When do financial intermediaries provide the commitment services to help present-biased consumers stick to long-term savings accumulation and/or debt management plans and when do they instead opportunistically pander to consumer biases. By how much might financial trade be reduced by consumer fears that the latter could happen? We study a consumer protection problem that survives even when consumers are sophisticated and fully informed. The consumer would like her future selves to be held to a balanced path of saving accumulation/debt repayment via a commitment contract, but her future selves can be tempted to raid savings or take on more debt. Investor-owned banks may find it costly to credibly commit to refrain from such opportunistic exploitation, leading to lost trade and lower ex-ante bank profits and/or consumer welfare. In such contexts strategic choices in bank ownership and governance forms may lower the cost of making credible (or renegotiation-proof) commitment contracts because explicit limitations on profit distribution and socially minded owners weaken the contract renegotiation incentives. This leads to a theory of commercial non-profits and endogenous client protection similar to Hansmann (1996) but on new behavioral microfoundations. The model helps understand the structure and evolution of ownership and contract forms in consumer banking, microfinance, and mortgage and payday lending, and frames debates over proposed consumer protection measures against excessive refinancing and 'overindebtedness' in these sectors. JEL Codes: O16, D03, D18

*Department of Economics, Hunter College & The Graduate Center, City University of New York. Email: kbasu@hunter.cuny.edu, jconning@hunter.cuny.edu. We are grateful to the Roosevelt House Public Policy Institute and a Small Business Administration research grant for support. For detailed comments and suggestions, we thank Abhijit Banerjee, Partha Deb, Alexander Karaivanov, and Eric Van Tassel; participants at the Chicago Booth conference on 'Economics of Social Sector Organizations', NBER Development Economics Summer Institute, CIDE-ThReD Conference on Development Theory, and NEUDC; and seminar participants at Michigan State University, IIES Stockholm, Delhi School of Economics, Queens College, Indian School of Business, Hunter College, and Florida Atlantic University. Kwabena Donkor provided excellent early research assistance. The most recent version of this paper can be found at: `https://goo.gl/2h4YUb`. Code for figures and simulations and supplemental materials at: `https://github.com/jhconning/renegotiation`.

Contents

1 Introduction

Hyperbolic discounters – consumers with present-biased preferences – are those who struggle to stick to their long-term asset accumulation or debt management plans and for this reason may benefit from the 'commitment services' or designed restrictions that a financial intermediary may build into multi-period financial contracts. Long-term consumption smoothing goals are more likely to be achieved where customers can enlist financial intermediaries to act as their partners in resisting later temptations to raid savings and/or increase debt in ways that could undermine responsible long-term plans. The properties of *costlessly*-enforced commitment banking contracts have been examined in several papers[1] and a number of randomized controlled trials and other empirical studies have found quite large impacts from introducing new commitment products.[2] This evidence suggests an unsatisfied real demand for commitment services in many places but also begs the question of why the market was not already providing them.

By costlessly-enforced commitment, we refer to contracts that will not be re-written even if all signatories wish to do so. This assumption is hard to sustain, especially with bilateral contracts. Commitments can be broken and the same person that at first demanded a commitment contract from a bank may later want to renegotiate its terms. A bank's ex-post profits can often be increased by pandering to such demands. For example, a present-biased consumer who takes out a loan that promises to balance repayments and expected

[1]See Laibson *et al.* (2003), Amador *et al.* (2006), and Basu (2014)

[2]See for example Ariely & Wertenbroch (2002), Thaler & Benartzi (2004), Ashraf *et al.* (2006), Bauer *et al.* (2012) and the studies mentioned in the survey by Bryan *et al.* (2010)

consumption across future periods may at a later period be unable to resist the bank's tempting offer of an expensive (but at this later date attractive) additional consumption loan that drives up immediate consumption but wrecks earlier laid consumption smoothing plans.

This type of problem raises obvious consumer protection concerns in the case of naive or partially naive present-biased consumers as they, by definition, do not understand how their own future preferences will evolve to make them vulnerable to such opportunistic exploitation by a bank. In the absence of consumer protection a bank might attempt to lure customers with attractive teaser contracts that it knows can be later renegotiated for additional gain.

Consumer protection issues also arise in the case of sophisticated present-biased consumers but take the less obvious and harder to measure form of reduced or lost financial trade. Sophisticated consumers anticipate the possibility of opportunistic contract renegotiation and therefore will be cautious to minimize their vulnerability by insisting on renegotiation-proof contracts that must be endogenously enforced. But imposing any such additional constraints can only reduce the feasible financial contract space, reducing gains from trade and therefore lowering bank profits and/or consumer welfare, and lost trade. The practical result in either case will be equilibria with less than optimal consumption smoothing due to saving 'too little' and/or borrowing 'too much' (in the estimation of their earlier period selves) in some periods compared to situations where full-commitment contracts could be costlessly offered.

Concerns about excessive refinancing and 'over-indebtedness' have been raised in the wake of recent financial crises in both the developed and the developing world. On the eve of the mortgage banking crisis in 2007, over 70 percent of new subprime mortgage loans were refinances of existing mortgages and approximately 84 percent of these were 'cash out' refinances (Demyanyk & Van Hemert, 2011). In the market for payday loans the concern of many economists and regulatory observers is often not so much that fees are high for one-time short-term loans (the typical cost is 15% of the amount borrowed on a 2 week loan) but rather that over 80% of payday loans are 'rolled over' or renewed rather than paid off, incurring new fees each time resulting in very high total loan costs and placing many people into very difficult debt management situations DeYoung (2015).

Similar concerns have been raised in microfinance markets in developing countries but in such markets where consumer protection laws are weak to non-existent, the issues take on an interesting extra dimension. To state our hypothesis briefly and bluntly: microfinance markets around the world have been dominated by a mix of mission-driven and commercial non-profits and 'hybrid' firms – for example for-profit firms partly or entirely owned and controlled by not-for-profit foundations or governments (Conning & Morduch, 2011; Cull *et al.* , 2009). We argue that this is because, compared to investor-led for-profit firms, hybrid and non-profit firms can make more credible commitments to not engage in the type of opportunism described above, because their incentive to do so is muted by the fact that they cannot easily distribute any profits. This is a variation on Henry Hansmann's theory of commercial non-profits but set on new behavioral micro-foundations that we will model explicitly below. In the early stages of new microfinance markets one or more non-profit firms may dominate and successfully provide the commitment services in savings and credit contracts. However new entry and intensified market competition diminishes the capturable rents and hence the strategic advantage of non-profit status and this can lead to a 'commercialization' stage and the conversion of many non-profits to for-profit forms. This will be associated with a rise of of the type of consumer protection issues identified above and concomitant collapse of borrower and saver discipline.

In this paper we view the provision of commitment as an important element of consumer protection in banking. Our goal is to provide a simple dynamic framework for analyzing real-world settings where commitment is demanded but cannot be credibly provided at zero cost. We take seriously the idea that commitment contracts must satisfy a 'no-renegotiation' constraint to be credible. We work with a quite general three-period consumption smoothing model for a present-biased consumer with quasi-hyperbolic preferences that allows for saving (repayment) or borrowing (dissaving) in each period. In each contracting scenario the consumer's period Zero-self (henceforth 'Zero-self' or simply 'Zero') has a bias for present consumption but wants to smooth future consumption across periods one and two. She correctly anticipates that her later period-one 'One-self' will have a change of preferences that will lead her to want to 'raid savings' and/or take on new debt to drive up period one consumption at the expense of period two consumption, thereby undoing Zero's early intent to balance consumption across the two periods. In every case the equilibrium contract will be the subgame perfect Nash equilibrium of a game where Zero-self chooses a contract first anticipating One-self's reactions, possibly limited by the Bank's exogenously or endogenously enforced commitment to agree to not renegotiate with One-self.

We derive predictions about how contract terms are affected by a bank's costs of enforcing commitment, and find conditions under which a bank will voluntarily modify its governance structure as a means to making commitment more credible. We show how the results depend on consumer type (sophisticated or naive) and market structure (monopoly or competition). The model is indeed stylized and limited to one of many mechanisms, but we argue that it makes a number of compelling points relevant to ongoing policy debates and is able to explain some stylized facts while generating sometimes counterintuitive empirical predictions.

1.1 Outline of arguments

In Section 2 we describe a consumer who faces an income stream that, in the absence of a bank ('autarky'), can be rearranged to provide imperfect consumption smoothing at best. We describe banks that have access to funds at a competitive interest rate, and can offer the Zero-self consumer a 3-period contract. The extent to which contract terms can be enforced in future periods depends on some non-pecuniary renegotiation cost κ, which is borne by the bank and can be interpreted as a concern for the consumer's well-being or own reputation.

We then build a graphical framework for analyzing equilibrium contracts as the outcome of a Stackelberg-type game where Zero-self moves first while anticipating One-self's best response. As an example, we derive the equilibrium contract when the consumer faces competitive banks (so that surplus is returned to the consumer) and the banks have high renegotiation costs (so that contract terms are always respected). This yields the first-best contract from Zero-self's perspective–she is able to allocate consumption across periods 1 and 2 in accordance with her own preferences without conceding to One-self's present-bias. In other words, the contract achieves 'full-commitment'.

Section 3 formalizes the renegotiation problem. If $\kappa = 0$, neither a monopolist nor competitive banks can offer credible commitment contracts, as any contract terms can be costlessly renegotiated in period 1. Since both the One-self consumer and the bank could gain from renegotiation, any full-commitment contract must get modified in period 1. More generally, we derive a 'no-renegotiation' constraint that any credible contract must satisfy. We show that if renegotiation costs are below a cutoff level $\bar{\kappa}$, competitive full-commitment contracts can never be credible while monopoly full-commitment contracts are credible only for consumers whose autarky utility is low. This difference–the relatively greater feasibility of full-commitment under monopoly–is not due to a monopolist's superior ability to commit. Rather, it is because monopoly contracts offer less consumption than do competitive contracts. At lower levels of consumption, the potential gains from renegotiation too are lower, thus making commitment more feasible.

In Section 4, we derive contracts when the 'no-renegotiation' constraint binds. We first focus on sophisticated hyperbolic discounters. The Zero-self can enter into a multi-period contract that helps bind her One-self to contract terms only to the extent that the bank's commitment can be endogenously enforced (i.e. the bank's ex-post gain in profits from breaking their commitment must fall short of any direct renegotiation costs κ). The 'imperfect commitment' contract represents a compromise between Zero-self's and One-self's preferences–consumption allocations between periods 1 and 2 must be tilted sufficiently in favor of period 1 that any further renegotiation would be unprofitable. This reduces the potential gains to trade between consumers and banks, and consumers with sufficiently high autarky utility will avoid contracting with banks. For others, contracts will result in lower bank profits (monopoly) or lower consumer discounted utility (competition).

We also make predictions about the shapes of contracts.[3] Under monopoly, imperfect commitment contracts will offer larger loans (or reduced savings) than full-commitment contracts. Under competition, the comparison is ambiguous. We explain this contrast between monopoly and competition using the intuition of income and substitution effects (from the consumer's perspective, a weakening of commitment has only substitution effects under monopoly while it has both substitution and income effects under competition).

Section 4 finally turns to naive hyperbolic discounters who fail to anticipate the extent to which contracts may be renegotiated. Now, Zero-self is offered a contract in which consumption in periods 1 and 2 strongly tilted towards period 2. This maximizes the potential gains from renegotiation. Under monopoly, this is achieved through a small loan (or high savings) since the consumer believes her future to be better than it will turn out to be. So, the naive consumer is not targeted with large loans; instead, she is offered a small teaser loan that will subsequently be rolled over in a manner that resembles some aspects of payday lending. Under competition, again initial loan/savings sizes are ambiguous since anticipated gains from renegotiation must be distributed back to the consumer.

In Section 5, banks may explore commercial nonprofit status as a mechanism to more credibly commit to not opportunistically exploiting the weaknesses of its sophisticated time-inconsistent clients. By operating as a nonprofit (or more broadly as a 'hybrid' bank), the bank agrees to face legal or governance restrictions on how any profits generated from any such opportunistic renegotiation can be distributed and enjoyed. The bank can now credibly convince the sophisticated consumer that it will be less likely to renegotiate the contract in the future. This allows the bank to offer the consumer an initial contract that maintains the restrictions on future consumption patterns that the consumer demands, raising the contracting surplus and therefore how much can be ultimately extracted by the bank's stakeholders.

A firm's decision about whether to adopt nonprofit status rests on a trade-off. As a non-profit, the firm has an opportunity to extract greater surplus from the consumer (by providing commitment), but now faces restrictions on the ability of managers and shareholders to enjoy this surplus. In the case of monopoly, the bank will adopt nonprofit status if the following is true: non-profit restrictions should be sufficiently severe that the bank is able to extract more surplus from the consumer, but should not be so severe that

[3]We are able to provide quite complete characterizations of optimal contracting scenarios under the assumption of monopoly or competition in the market for period-zero banking contracts with or without the assumption of enforceable exclusive contracts in later periods. We can provide exact closed-form solutions for contract terms for CRRA utility functions for most of these cases including renegotiation-proof contracts when $\kappa = 0$. For the $\kappa > 0$ cases where closed form solutions cannot be directly obtained we can nonetheless characterize some important contract properties and solve for contracts numerically.

it is unable to enjoy the surplus. We show how the details of the trade-off depend on the governance choices available to the bank and the consumer's reservation outcomes. That nonprofit firms may survive even in the absence of motivated agents or asymmetric information is, to the best of our knowledge, a novel result.

This trade-off is also sensitive to market structure. Under competition, a lender's ability to provide effective commitment through non-profit status depends on the exclusivity of contracts. When long-term contracts can be made exclusive, the tradeoff disappears and all active firms function as non-profits. This is because of the zero-profit condition–since firms do not make profits anyway, there is nothing to lose from switching to non-profit status. On the other hand, there are profits to be gained–if all other firms are for-profit, a firm could make positive profits by offering superior commitment as a non-profit (this is valuable even if its enjoyment of these profits is limited).

When contracts are not exclusive, commitment generated through non-profit status becomes impossible to achieve. Since non-profit firms would make zero profits anyway, each firm has an incentive to switch to for-profit status so it can take advantage of the opportunity to re-finance *other* banks' loans. As a result, for-profit firms must be active in equilibrium, and their presence will eliminate the possibility of non-profit commitment.

This can partly explain a key difference between traditional monopolistic non-profit microfinance, which is rigid, and say competitive commercial credit card lending which offers refinancing flexibility (credit card punishments gain salience because they are *less* strict, not more).

1.2 Context and Related Literature

1.2.1 Commitment as a form of Consumer Protection

Problems of consumer protection are typically analyzed through two channels: naive or uneducated consumers and their failure to correctly anticipate fees and punishments (see Gabaix & Laibson (2006), Armstrong & Vickers (2012), and Akerlof & Shiller (2015) for related arguments), and bank's moral hazard (see Dewatripont & Tirole (1999) and Oak & Swamy (2010)). We argue that, given the growing evidence of time-inconsistent preferences,[4] a bank's ability to provide credible commitment should also fall under this umbrella–sometimes consumers *want* punishments or fees to limit renegotiation.

In recent years, especially in light of crises in consumer credit markets, there has been renewed emphasis on consumer protection and better governance and regulation in banking.[5] One particular outcome of concern has been borrower over-indebtedness, an issue

[4]See, for example, Laibson *et al.* (2003), Ashraf *et al.* (2006), Gugerty (2007), and Tanaka *et al.* (2010).

[5]In the US, the Consumer Financial Protection Bureau was set up in 2011 under the Dodd–Frank Wall Street Reform and Consumer Protection Act. In India, the far-reaching Micro Finance Institutions Develop-

that has been at the center of recent microfinance repayment crises in places as far-flung as Morocco, Bosnia, Nicaragua and India, as well as the 2008 mortgage lending crisis in the United States. In each of these cases the issue of refinancing or the taking of loans from multiple lenders emerges.

Journalistic and scholarly analyses of such situations, including the recent mortgage crisis in the United States, have often framed the issues as problems of consumer protection, suggesting that many lenders designed products to purposefully take advantage of borrowers who have limited financial literacy skills and are naive about their self-control problems. Informed by such interpretations, new regulations introduced in the wake of these crises have swung toward restricting the terms of allowable contracts, for example by setting maximum interest rates and limiting the use of coercive loan recovery methods.

We place consumers' struggles with intertemporal self-control issues at the center of the analysis, but argue that borrowers may be more sophisticated in their understanding of their own time-inconsistency than is often assumed. From this perspective, 'predatory lending' is not primarily about tricking naive borrowers into paying more than they signed up for with hidden penalties or misleading interest rates quotes, but about offering excessive flexibility and refinancing of financial contracts in ways that limit or undermine the commitments to long term consumption and debt management paths that borrowers themselves may be attempting to put in place.[6]

A sophisticated hyperbolic discounter understands that her 'future selves' will attempt to take out new loans on top of old ones, or renegotiate the terms of existing loans to further defer debt repayment. If she is a saver, she will try to withdraw more rapidly in the future than she would have liked, or will deposit less than ideal. That such consumers should be and are willing to pay for commitment has been demonstrated in several theoretical and empirical papers. By entering into a contract that commits them to a specific time-path of repayments or deposits, the consumer attempts to ensure that future selves will not skew consumption patterns to privilege instant gratification in the following periods. Viewed this light, fees and punishments for failing to adhere to a schedule are not inherently undesirable to the consumer–for sophisticated hyperbolic discounters, threats of punishment can indeed serve as useful commitment devices.

Nevertheless, the fact that consumers value commitment does not automatically imply that firms will provide it in equilibrium. In fact, there remains an open question about

ment and Regulation Bill of 2012 was designed to increase government oversight of MFIs in response to the credit crisis in the state of Andhra Pradesh, and the perception that lax consumer protection and aggressive lending practices had led to rising over-indebtedness and stress.

[6]Bond *et al.* (2009) discuss evidence of predatory lending in the context of mortgages. In 2016 the Consumer Financial Protection Bureau put forth a proposal to protect consumers of payday loans with an emphasis on limiting the number and frequency of reborrowings, available at `http://files.consumerfinance.gov/f/documents/CFPB_Proposes_Rule_End_Payday_Debt_Traps.pdf`.

whether markets can be relied upon to supply commitment. The key consideration in this paper is the following: if a hyperbolic discounter is willing to pay to commit her future selves, her future selves are willing to pay to undo this commitment. Here, a bank that promises to be rigid and is then flexible could be seen as hurting, rather than helping, the consumer. We take seriously the bank's ex-post considerations and derive conditions under which it would renegotiate.

In this sense, our paper complements some others that demonstrate how commitment can be undone in related settings. Gottlieb (2008) shows how competition leads to inefficient outcomes in immediate rewards goods. Heidhues & Koszegi (2010) study the mistakes of partially naive borrowers in competitive credit markets. Mendez (2012) analyzes predatory lending with naive consumers. Our framework is encompassing, allowing us to study generic banking contracts with saving or borrowing: under both competition and monopoly (the latter being particularly relevant to informal banking in developing economies), for both sophisticated an naive consumers, and for multiple governance and ownership structures.

1.2.2 Commercial Non-profits in finance

The idea that firm ownership might be strategically chosen to solve or ameliorate 'contract failure' problems dates back at least to Arrow (1963) and is one that has been articulated most clearly in the work of Henry Hansmann (1996). Hansmann argued that in markets where the quality of a product or service might be difficult to verify, clients may rationally fear that investor-led firms will be tempted to opportunistically skimp on the quality of a promised product or service, or reveal a hidden fee, and this can greatly reduce or even eliminate contracting. In such circumstances becoming a 'commercial non-profit' may be a costly but necessary way to commit the firm to not act opportunistically, hence enabling trade.

Hansmann gives as a primary historical example the development of consumer saving, lending and insurance products in the United States and Europe. Life insurance in the United States for example has until quite recently always been dominated by mutuals. Rate payers could not trust investor-led firms to not act opportunistically by, for example, increasing premiums or by skimping or reneging on death benefit payouts. Mutuals on the other hand had little incentive to cheat clients to increase shareholder dividends as the clients themselves are the only shareholders. Mutuals therefore enjoyed a distinct competitive advantage until sufficient state regulatory capacity developed.

In the present analysis we begin by following Hansmann in defining nonprofits by the legal restrictions faced by them, setting aside other ways (such as motivation) in which they might be different from for-profit firms.[7] In this view "[a] nonprofit organization is, in

[7]Hence we abstract away from other considerations for nonprofits, as in Besley & Ghatak (2005), McIntosh

essence, an organization that is barred from distributing its net earnings, if any, to individuals who exercise control over it, such as members, officers, directors, or trustees."[8] Glaeser & Shleifer (2001) have formalized Hansmann's central argument to show that when a firm cannot commit to maintaining high quality, it might choose to operate as a commercial non-profit rather than as an investor-led for-profit in order to credibly signal that it has weaker incentives to cheat the consumer on aspects of unobserved product quality. As Hansmann describes it, firm ownership form adapts endogenously as a "crude form of consumer protection" in unregulated emerging markets where asymmetric information problems are rife. Bubb & Kaufman (2013) modify this model so that the non-contractible quality issue is on hidden penalties, which are incurred with certainty by some borrowers. All of these models are built rely on some form of asymmetric information or contract verification problem.

A contribution of our paper is to argue that a theory of ownership form can be built on behavioral micro-foundations even in environments with no asymmetric information and with sophisticated forward-looking agents. We believe this is an important element for understanding the development of consumer finance in developed countries historically as well as the current shape of microfinance today where non-profit and 'hybrid' forms still dominate the sector in most developing countries (Cull *et al.* , 2009; Conning & Morduch, 2011). Hybrid ownership forms include the many microfinance firms that, though technically incorporated as for-profit financial service providers, are in fact dominated by boards where, by design, social investors or client representatives exert substantial governance control. Hybrid forms such as these would appear to confer many of the benefits of non-profit status (specifically, credible commitment to consumer protection) with fewer of the costs (in particular, unlike a pure non-profit they can and do issue stock to outside investors although usually in a manner that does not lead to challenge control).

1.2.3 Market Structure and Governance Choice

Commenting upon a major microfinance crisis in the state of Andhra Pradesh in India, veteran microfinance investor and market analyst Elizabeth Rhyne (2011) describes the build up of "rising debt stress among possibly tens of thousands of clients, brought on by explosive growth of microfinance organizations . . ." fueled by the rapid inflow of directed private lending and new equity investors who, because they "paid dearly for shares in [newly privatized] MFIs . . . needed fast growth to make their investments pay off ."

She goes on to lay the blame on "poor governance frameworks" for behaviors that

& Wydick (2005), and Guha & Roy Chowdhury (2013). Nonetheless our modeling framework can be adapted to include these considerations and is the focus of related work.

[8] In practice, nonprofit firms also enjoy certain benefits that are denied to for-profit firms (see, for example, Cohen, 2015). But for the purposes of Hansmann's (and our) argument, it is the *restrictions*, not benefits, that generate improved outcomes.

included “loan officers [that] often sell loans to clients already indebted to other organizations.” In her view, Indian MFIs might have avoided their problems and followed the model of leading microfinance organizations in other countries like Mibanco (Peru) and Bancosol (Bolivia) which “were commercialized with a mix of owners including the original non-governmental organization (NGO), international social investors (including development banks), and some local shareholders. The NGOs kept the focus on the mission, while the international social investors contributed a commercial orientation, also tempered by social mission.” These are the types of hybrid ownership forms, along with nonprofit firms, that we argue can provide surplus building consumer protection through a reduced incentive to renegotiate. Rhyne’s argument is that a number of Indian state regulations made it difficult for such hybrid ownership forms to rise organically in India. As our model makes clear, these governance choices are highly dependent on market structure, and nonprofits may survive better under monopoly than under competition.

2 The Model: Setup

There are three periods, $t \in \{0, 1, 2\}$. In any period t, the consumer’s instantaneous utility from consumption level c_t is given by a CRRA function defined over all nonegative consumption:[9]

$$u(c_t) = \begin{cases} \frac{c_t^{1-\rho}}{1-\rho} & \text{if } \rho > 0 \text{ and } \rho \neq 1 \\ ln(c_t) & \text{if } \rho = 1 \end{cases} \tag{1}$$

Given a consumption stream $C_t = (c_t, .., c_2)$, the period-t self’s discounted utility is:

$$U_t(C_t) \equiv u(c_t) + \beta \sum_{i=t+1}^{2} \delta^{i-t} u(c_i) \tag{2}$$

This describes quasi-hyperbolic preferences with a hyperbolic discount factor $\beta \in (0, 1]$. In any period t, the individual, or more accurately her period-t self, places greater relative weight on period-t consumption than her earlier selves would have done. The consumer could be sophisticated or naive about anticipating the time-inconsistency of her preferences (O’Donoghue & Rabin, 2001).[10]

Zero-self begins with an endowment of claims to an arbitrary positive income stream over the three periods, $Y_0 = (y_0, y_1, y_2)$, with a fixed present value $\sum_{t=0}^{2} \frac{y_t}{(1+r)^t} = y$. Her objective is to rearrange this into a preferable consumption stream $C_0 = (c_0, c_1, c_2)$ to maximize $U_0(C_0)$ in (2) using whatever financial contracting savings and/or borrowing

[9] When $\rho = 1$, the utility function is defined over all positive consumption (it is undefined at $c_t = 0$).

[10] A naive consumer believes her future selves to be exponential discounters with a discount factor of δ.

strategies that may be available.

In the absence of formal banking, she must rely on 'own-savings' or informal borrowing, if available. We call this her autarky utility, denoted $U_0^A(Y_0)$. Section 2.1.1 shows how her optimal autarky utility, U_0^F, can be achieved if she has access to borrowing and saving at competitive interest rates, plus commitment. In general, there are many reasons why optimal utility will not be achievable in autarky. If her income is back-heavy, borrowing constraints could limit her ability to smooth consumption, and in the absence of credit she must consume her income as it arrives. If her income is front-heavy, with available own-savings strategies the consumer may be able to construct a somewhat better smoothed autarky stream. However there may be technological restrictions to saving as well, with insecurity of storing cash at home being one obvious explanation. More relevant to our analysis, though, is the fact that even with access to perfectly secure savings, a consumer with time-inconsistent preferences cannot trust her later selves to follow her optimal consumption path Basu (see 2014, and Section 3 below). While remaining deliberately agnostic about autarky technologies, the rest of the paper focuses on the more reasonable and interesting case where $U_0^A \leq U_0^F$ and there are therefore potential gains to financial contracting with a new intermediary.

The consumer has the option of contracting with one or many risk-neutral banks, depending on whether the market structure is monopolized or competitive. Each bank can access funds that can be withdrawn from other investments at interest rate opportunity cost r, which for most of the analysis we normalize to $r = 0$. A bank will participate and offer a contract (a consumption path C_0 in exchange for which the bank retains the income stream Y_0) if and only if the profits it can earn, $\Pi_0(C_0; Y_0)$, are expected to be non-negative. Profits are defined as:

$$\Pi_t(C_t; Y_t) \equiv \sum_{i=t}^{2} \frac{(y_i - c_i)}{(1+r)^{i-t}} \tag{3}$$

Whether the contract is interpreted as loan or savings depends on the relative sizes of c_0 and y_0.

To simplify the analysis, we initially assume that contracts can only be initiated in period 0.[11] However, an existing contract may be renegotiated by the consumer and the bank in period 1. If this happens, we assume the bank would incur a non-monetary cost, $\kappa \geq 0$, which could be interpreted to include a concern for reputation or some other impact on the social preferences of its owners.[12]

Because the remainder of the analysis will be focused on contracts that attempt to

[11] The assumption is lifted in Section 6.

[12] The bank could incur additional monetary costs as well. However, we assume these to be 0 as they can be netted out and do not affect the analysis in any important way.

establish self-control, or that limit the consumer's later period selves' ability to recontract with the same or new financial intermediary, it will be important to distinguish legal or institutional environments that allow banks to establish exclusive contracts with customers and those that do not. We will analyze these cases in turn.

2.1 Full-commitment contracts

We first establish the benchmark 'full-commitment' contracts under perfect competition and monopoly, respectively. To characterize these familiar commitment contracts, we assume a bank can *costlessly* offer Zero-self a multi-period contract that binds the consumer's latter self(ves) to not renegotiate its terms with the same bank or other banks. Although we are not making the exact mechanisms explicit yet the bank's ability to credibly commit to not renegotiate such a contract must rest on the assumption that the bank has credibly bonded itself to paying a renegotiation penalty (κ) in the event of renegotiation and that this penalty is sufficiently high to deter the bank. In later sections of the paper we will examine what happens when κ is small. Then exogenously sustained commitment contracts can no longer be sustained and must be replaced by second-best self-enforcing renegotiation-proof contracts. We can then delve deeper into what in practice determines κ and how it might be endogenously determined via choices of bank ownership and governance forms, and shaped by the nature of the market structure.

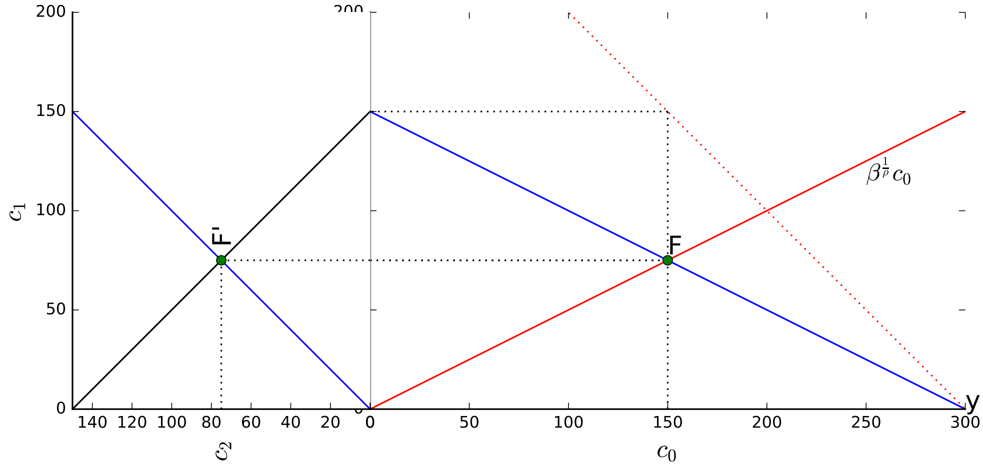
2.1.1 Full-commitment contracts under Competition

A consumer with time-inconsistent preferences cannot trust her later selves to stick to her preferred consumption plans. In this simple three-period setting Zero's concern is that her later One-self will try to divert resources earmarked for period 2 consumption to boost period 1 consumption instead. Like a Stackelberg-leader in a Cournot game, Zero's strategic saving/borrowing choices are affected by her anticipation of One's best response. A bank may be able to act as a strategic partner to Zero by offering contracts with commitment services to help restrict or otherwise control the consumer's later self(ves)'s best responses.

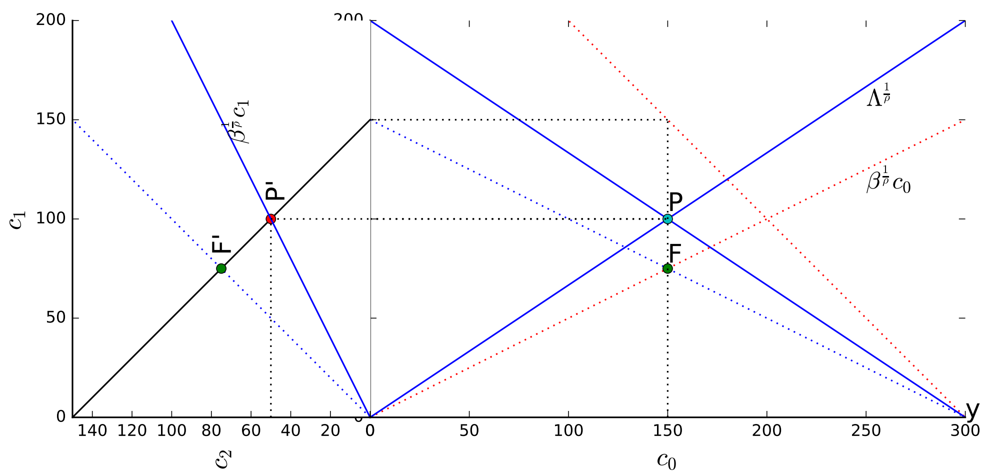
With an exclusive full-commitment contract the consumer faces no self-control problem. Zero chooses a contract that commits her One- and Two- selves to follow the chosen consumption plan. This contract design problem is solved as a standard utility maximization problem subject to an inter-temporal budget constraint (or subject to a financial intermediary's zero-profit condition). Zero-self chooses contract C_0 to solve:

$$\max_{C_0} U_0(C_0) \tag{4}$$

$$s.t. \quad \Pi_0(C_0; Y_0) \geq 0 \tag{5}$$



(a) Full-commitment



(b) Renegotiation-proof with $\kappa = 0$

Figure 1: Full-commitment and renegotiation-proof contracts under competition

The familiar first-order necessary conditions are:

$$u'(c_0) = \beta\delta(1+r)u'(c_1) = \beta\delta^2(1+r)^2u'(c_2) \tag{6}$$

An increase or decrease to the term $\delta(1+r)$, which enters each expression above, essentially 'tilts' consumption to be more generally rising or falling over time as $\delta \gtreqless 1/(1+r)$. As this across-the-board level of tilt will not alter key tradeoffs of interest (unlike the degree of present-bias β parameter which does) we shall impose the assumption that $\delta = \frac{1}{1+r}$ for the remainder of the analysis. This is without loss of generality but will greatly unclutter the math. The simplified first-order conditions are:

$$u'(c_0) = \beta u'(c_1) = \beta u'(c_2) \tag{7}$$

Along with a binding budget constraint the first-order conditions allow us to solve for the optimal competitive 'full-commitment' contract C_0^F, so called because latter period selves are committed to not change it. Conceptually the equilibrium contract will be found at the tangency between the highest iso-utility surface just touching the budget hyper-plane. With CRRA utility, a closed form solution for C_0 is easily found which we label the competitive full commitment contract C_0^F.[13] This contract has the property:

$$c_1^F = c_2^F = \beta^{\frac{1}{\rho}} c_0^F$$

Zero-self wants to indulge her present bias (by tilting consumption toward herself) and then allocate remaining resources evenly across the remaining two periods.

This solution can be seen graphically in Figure 1a drawn for a consumer with $\beta = 0.5$, $\rho = 1$ and an intertemporal budget constraint from endowment income with a present value of income $y = \sum y_t = 300$. The figure consists of two panels. The left panel depicts the (c_1, c_2) contract choice, conditional on $c_0 = c_0^F$. The allocation across periods 1 and 2 is determined by a budget constraint (the line through QF', satisfying $c_1 + c_2 = y - c_0$) and an optimal 'division rule' which in this case is $c_1 = c_2$ (the line through OF'). This division rule is attainable because full-commitment, by definition, allows Zero-self to force One-self to respect the first-order conditions of the maximization problem (4).

The right panel depicts the (c_0, c_1)choice conditional on the division rule being followed in period 1. The budget constraint (40) is the line going through yF and the first-order condition (38) is the line going through $0F$.

The optimal contract C_0^F lies at the intersections F and F'. To illustrate, at these parameters, Zero's preferred contract is $C_0^F = (150, 75, 75)$. Whether the consumer borrows

[13] All CRRA derivations and closed-form solutions are in the appendix.

or saves (or pays down debts) in any given period depends on how this consumption stream matches her autarky consumption stream. If, for example, the total income of 300 arrives evenly across periods as $Y_0 = (100, 100, 100)$ then this consumption plan would imply borrowing $c_0 - y_0 = 50$ in period 0 to be repaid as installments of 25 in each of periods 1 and 2. If the stream had instead been $Y_0 = (200, 50, 50)$ the consumer would be seen as saving 50 in period 0 to raise consumption by 25 in each of periods 1 and 2.[14]

This approach will be useful for further analyzing situations where Zero-self loses control over the division rule (i.e. loses commitment).

2.1.2 Full-commitment contracts under Monopoly

If there is monopoly instead of competition for banking services in period 0, the analysis is similar except that now we solve for the optimum full-commitment contract by maximizing bank profits subject to a consumer participation constraint. The bank solves:

$$\max_{C_0} \Pi_0 \left(C_0; Y_0\right) \tag{8}$$

$$s.t.\ U_0 \left(C_0\right) \geq U_0^A(Y_0) \tag{9}$$

The first-order tangency conditions are the same as competitive case given by expressions 7, which for CRRA utility again implies $c_1 = c_2 = \beta^{\frac{1}{\rho}} c_0$. Along with the fact that Zero's participation constraint must bind at a monopoly optimum these equations allow us to solve for the contract C_0^{mF} and corresponding bank profits $\Pi_0 \left(C_0^{mF}; Y_0\right)$. Closed form solutions for the CRRA utility case appear as appendix equations 44 and 45, respectively. Conceptually, the optimum contract will be at the tangency point where the highest iso-profit plane still touches the iso-utility surface associated with Zero's reservation utility.

The terms of the optimal monopoly contract, and hence also the level of bank profits achieved will be dependent on the consumer's autarky utility $U_0^A(Y_0)$. The present value of C_0^{mF} rises and profits fall with U_0^A. Since the monopolist retains the gains from trade and consumer iso-utility (or indifference) surfaces do not cross, consumption in each period under monopoly will be strictly lower than under competition at all Y_0 except in the extreme case where autarky utility is already optimal ($U_0^A = U_0^F$), in which case the monopolist will trivially offer the utility maximizing contract to the consumer and will make zero profits.

[14]These parameter values are chosen for expositional purposes. In particular $\rho = 1$ implies that period zero consumption will be the same with or without self-control but the analysis can be easily adapted to other cases.

3 The Renegotiation Problem

We now get to the central questions of the paper: when is commitment credible, how is it sustained, and at what cost? At issue is the fact that One-self always prefers higher period 1 consumption than Zero wants to build into a contract, so in any period 1 subgame between One-self and the (same or a different) bank there will be tempting potential gains to trade from breaking earlier contract commitments to Zero. The credibility of the full-commitment contract must rest rest on the threat of a sufficiently costly punishment deterring the bank from engaging in such renegotiation. In this section we determine the minimum size of the punishment required to deter the bank from breaking the full-commitment contract.

The fraught nature of this potential renegotiation problem is depicted in Figure 2, for a case where renegotiation costs are set to $\kappa = 0$. In other words, One-self and a bank are free to costlessly rewrite the terms of any contract. Assume – just for the sake of argument now – that the consumer had (naively as it will turn out) accepted the a full commitment contract C_0^F or C_0^{mF} in period zero, indicated as point F in the figure. This contract satisfies Zero-self's optimality condition $u'(c_1) = u'(c_2)$ as seen by the fact that Zero-self's indifference curve is tangent to the bank's iso-profit line. For One-self's preferences however this consumption bundle involves too little period 1 consumption since $u'(c_1) > \beta u'(c_2)$ and as can be seen from the fact that at F One-self's indifference curve is steeper than the iso-profit line. With zero renegotiation costs there are gains-to-trade that can be shared from recontracting from F to any new tangency point (along the $c_2 = \beta^{\frac{1}{\rho}} c_1$ ray where One-self's first-order conditions are met) between point R which is the contract least favorable to One (chosen if the bank could act as monopolist in period 1) and point P which is the contract most favorable to One (chosen with competitive refinancing). Zero is clearly worse off with any such refinancing away from its preferred optimal contract F.

3.1 Renegotiated contracts

Consider the subgame entered in period 1, with $C_1^0 = (c_1^0, c_2^0)$ denoting the terms of any continuation contract inherited from period 0. We now formally derive conditions under which the contract will survive the threat of renegotiation in period 1. Below, we characterize the 'best response' contract chosen in the subgame of period 1, conditional on the contract being renegotiated. Note that the terms of the renegotiated contract and therefore also the division of the gains to trade will depend on the period 1 market for refinances is competitive or monopolized (or something in between). The contract will be renegotiated if the gains to trade are sufficient to cover the bank's renegotiation costs κ.

Consider first the situation where the market for refinanced contracts is competitive (i.e. the period 0 bank cannot enforce an exclusive contract). For the present analysis it

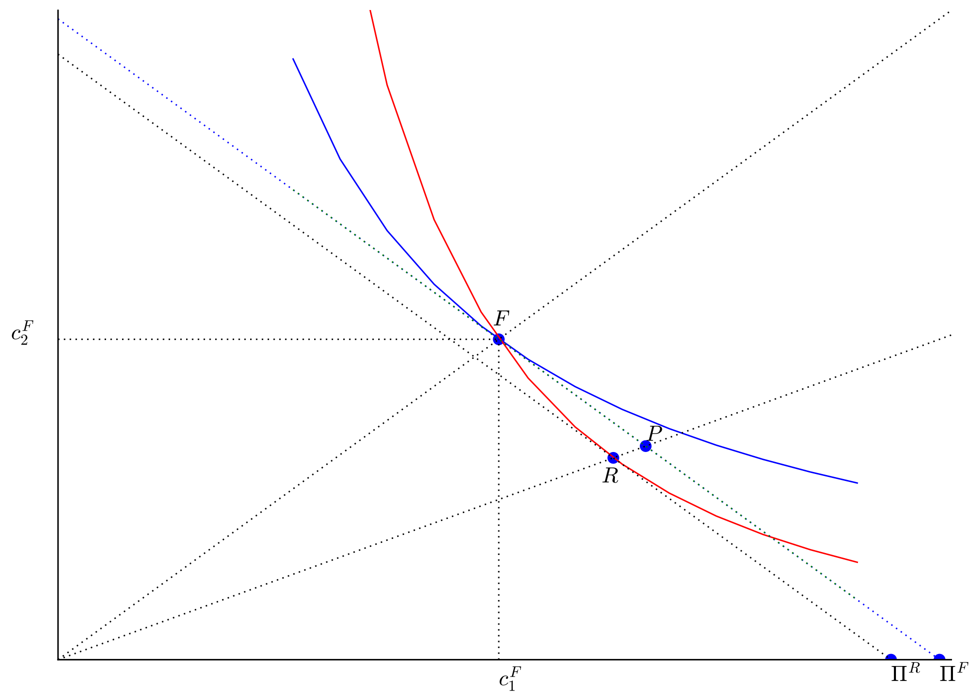


Figure 2: Full-commitment and renegotiation possibilities in period 1 (drawn for $c_0 = c_0^F, \beta = \frac{1}{2}, \rho = 1, \kappa = 0$)

turns out that it does not matter whether there was competition or monopoly in period 0. Competition in period 1 will lead banks to offer to renegotiate the existing continuation contract C_1^0 for a new contract $C_1^1(C_1^0)$, allowing the (new or existing) bank to just break even after covering any renegotiation costs:

$$C_1^1(C_1^0) = \arg\max_{C_1} \; U_1(C_1)$$

$$\Pi_1(C_1; C_1^0) \geq \kappa \tag{10}$$

As long as U_1 is well behaved this can be solved for an interior $C_1^1(C_1^0)$ using the first-order condition $u'(c_1^1) = \beta u'(c_2^1)$ and the binding condition 10. As drawn in figure ?? for the CRRA case (46), the contract C_1^0 at **P** would be renegotiated to point $R(\kappa)$ where the ray $c_2^1 = \beta^{\frac{1}{\rho}} c_1^1$ (along which the FOA are met) intersects the period 1 zero profit line 10. One-self makes a utility gain. Note that had $\kappa = 0$ the contract would have been renegotiated to $R(0)$.

If the bank instead finds itself in a monopoly position in period 1 then it will renegotiate existing contract C_1^0 to a new contract $C_1^{m1}(C_1^0)$ to increase profits by offering the smallest possible enticement for One-self to accept the renegotiated contract.

$$C_1^{1m}(C_1^0) = \arg\max_{C} \; \Pi_1(C_1; C_1^0)$$

$$U(C_1) \geq U(C_1^0) \tag{11}$$

Solving for an interior solution using first-order conditions and the binding condition **??**, the CRRA case yields a closed form solution (47). In figure ?? contract C_1^0 at **P** would be renegotiated to point R^m where the ray $c_2^1 = \beta^{\frac{1}{\rho}} c_1^1$ intersects One-self's participation constraint 10. As drawn the bank gains.

3.2 The 'no-renegotiation' condition

Under what conditions would the original contract *not* be renegotiated in period 1? Assuming a tie-breaking rule in favor of Zero-self's preferences, the no-renegotiation condition under period 1 competition is:

$$U_1\left(C_1^0\right) \geq U_1\left(C_1^1(C_1^0)\right) \tag{12}$$

And the no-renegotiation condition under period 1 monopoly is:

$$\Pi_1(C_1^1\left(C_1^0\right); C_1^0) \leq \kappa \tag{13}$$

Notice that these two conditions are in fact identical: a period 0 contract is credible if and only if there is no way for a bank to offer One-self a new contract that simultaneously (a) leaves One-self with at least as much discounted utility as in the original contract, and (b) generates additional profits of at least κ to the bank. In short the contract is renegotiation-proof as long as renegotiation costs are large enough to make sure no Pareto-gain is possible between One-self and the bank. This results in a single no-renegotiation condition that can be applied to any contract C_1^0 and renegotiation cost κ (a closed-form solution representation is given in Equation 49).

This condition can be illustrated using Figure 2. Let F denote *any* inherited contract in period 1. The horizontal distance between the isoprofit lines through P and R denotes the maximum possible profit gains a bank could earn through renegotiation (equal to the cost savings from renegotiating the contract $c_1^0 + c_2^0 - c_1^1 - c_2^1$. So, if this distance is less than or equal to κ, the consumer and the bank will not find it worthwhile to renegotiate.

3.3 At what cost are full-commitment contracts kept credible?

Now, we turn to the survival of full-commitment contracts in particular. What minimum renegotiation cost is sufficient to deter the renegotiation of full-commitment contracts? This is easily derived by setting $c_1^0 = c_1^F = c_2^0$ in the no-renegotiation condition (49). A competitive full-commitment contract will survive if and only if

$$\kappa \geq \bar{\kappa} \equiv c_1^F \cdot \Upsilon \tag{14}$$

and a monopolistic full-commitment contract will survive if and only if:

$$\kappa \geq \bar{\kappa}^m \equiv c_1^{mF} \cdot \Upsilon \tag{15}$$

where

$$\Upsilon = \left(2 - \left(\frac{1+\beta}{(1+\beta)^\rho}\right)^{\frac{1}{1-\rho}}\right) \tag{16}$$

$\bar{\kappa}$ and $\bar{\kappa}^m$ are the minimum renegotiation costs required for the survival of full-commitment contracts. These conditions tell us that, the greater the consumption levels in a full-commitment contract, the harder it will be to avoid renegotiation (i.e. the greater is the scope for rearranging consumption profitably in period 1.)

The value of c_1^0 in the expression will be different depending on whether the period 0 market for full-commitment contracts is competitive or monopolized. Under competition from 41 and 42 we know that $c_1^F = \frac{\beta^{\frac{1}{\rho}} y}{1+2\beta^{\frac{1}{\rho}}}$. Since this does not depend on autarky utility (given a fixed value of y), $\bar{\kappa}$ too does not depend on how close or far to optimal consumption smoothing the consumer is in autarky.

If the market for period 0 full-commitment contracts is monopolized we are led to a cutoff $\bar{\kappa}^m$ that now depends on autarky utility. The higher is $U_0^A(Y_0)$ the higher will be c_1^{mF}. Since $c_1^F \geq c_1^{mF}$ for any initial Y_0 it must be the case that $\bar{\kappa}^m \leq \bar{\kappa}$.

Together, this indicates that, if renegotiation costs lie above cutoff $\bar{\kappa}$, full commitment contracts will be credible; and if renegotiation costs lie below this cutoff, the survival of full-commitment will depend on market structure and autarky utility. Proposition 1 summarizes the results.

Proposition 1. *Consider renegotiation costs $\bar{\kappa}$ and $\bar{\kappa}^m$ as defined in Conditions 16 and* **??**.

(a) The competitive full commitment contract survives if and only if $\kappa \geq \bar{\kappa}$.

(b) The monopolistic full commitment contract survives if and only if $\kappa \geq \bar{\kappa}^m$. $\bar{\kappa}^m$ is strictly rising in the consumer's autarky utility.

(c) $\bar{\kappa}^m \leq \bar{\kappa}$.

Note that the monopolist's ability to commit at lower threatened renegotiation cost ($\bar{\kappa}^M \leq \bar{\kappa}$) comes from the fact that its commitment contract is less susceptible to renegotiation. That is because, having, at the outset, extracted surplus by offering the consumer a contract with the lowest possible consumption, there is relatively less for the firm to capture via renegotiation in period 1. This stands in contrast to the intuitive notion that a monopolist might offer better commitment because it faces greater costs of breaking promises than a competitive firm does. Our result shows that even in the absence of this mechanism, monopoly contracts are inherently harder to break.

A further implication, of statement (b) in the proposition, is that under monopoly, consumers with already relatively smooth consumption are less likely to get full-commitment contracts that can be sustained. A consumer with a higher autarky utility must be offered higher consumption by the monopolist. And the no-renegotiation condition is harder to satisfy at higher levels of consumption.

4 Imperfect Commitment Contracts

We turn now to situations where renegotiation costs κ are positive but not sufficiently large to sustain full-commitment contracts. So long as One self's consumption tradeoffs differ from Zero's renegotiation can only harm (or, at best, not improve) Zero-self's intertemporal welfare. In the case of naifs, banks will capitalize on the consumer's failure to anticipate harmful renegotiation (Section 4.4). In the case of sophisticates, the Zero-self consumer will seek to avoid entering into multi-period contracts that are vulnerable to such harmful renegotiations. They will limit their choice to contracts that are renegotiation-proof or subgame-perfect (Sections 4.2 and 4.3). If renegotiation-proof constraint **??** binds at an optimum the resulting optimal contract will offer less consumption smoothing than the full-commitment contract. The absence of a sufficiently tough external deterrent to renegotiation leads the parties to build what commitment they can by tilting the terms of the period 1 continuation contract closer to One-self's preferences so as a way to reduce the scope for renegotiation. We call these 'imperfect commitment' contracts. These are still commitment contracts – renegotiation is avoided in equilibrium– but at the cost of now less than perfect consumption smoothing for Zero.

5 Renegotiation-proof contracts with $(0 \leq \kappa < \bar{\kappa})$

We turn now to the multi-period contract design problem between a sophisticated consumer and a bank in period 0 when bank renegotiation costs are not sufficient to sustain full-commitment contracts. We take renegotiation costs to be exogenously given.[15] There are two cases to consider: when the market for period 0 contracts is competitive and when it is monopolized.

5.1 Competitive renegotiation-proof contracts

If the period 0 market is competitive and the market for period 1 renegotiated contracts is expected to be a monopoly (i.e. an enforceably exclusive contract) the optimum contract will solve the following problem:

$$\max_{C_0} \; U_0 \left(C_0\right) \tag{4}$$

$$\Pi_0 \left(C_0; Y_0\right) \geq 0 \tag{5}$$

$$\Pi_1 \left(C_1^1 \left(C_1^0\right); C_1^0\right) \leq \kappa \tag{13}$$

If we instead the Zero-self consumer had expected the period 1 market for renegotiated contracts to be competitive then we would have had the same objective and period 0 bank participation constraint but we would have replaced the period 1 bank no-renegotiation constraint 13 with a no-renegotiation constraint on One-self:

$$U_1(C_1^1(C_1^0)) \leq U_1(C_1^0) \tag{12}$$

[15]In later sections we explore costly endogenous strategies to shape perceptions of renegotiation costs and competition across banks with differing levels.

However, as we have alluded to already and explain again below in either case – whether the period 1 market for renegotiated contracts is a monopoly or competitive – both constraints 12 and 13 will bind at an optimum.

Similar to a Stackelberg leader choosing the most favorable of possible market outcomes along their opponents reaction function in a Cournot duopoly game, Zero-self will choose the contract C_0^0 that offers the highest welfare considering the strategic game play between the bank and One-self in the period 1 subgame $\zeta(C_1^0, \kappa)$.

Figure 3 is useful for understanding expected game play in for different subgames $\zeta(C_1^0, \kappa)$ in period 1. The figure is zoomed in around the contract space of interest to make the relationships clearer. Suppose Zero-self has chosen a candidate level of period 0 consumption c_1^0. For this to be part of an optimum contract Zero must insure that its continuation contract $C_1^0 = (c_1^0, c_2^0)$ lies on the bank's binding period 0 zero profit condition (or else Zero-self can raise her welfare could be made higher). This is equivalent to stating that the continuation contract will lie along the period 1 budget line $c_1^0 + c_2^0 = y - c_0^0$ drawn running through point $R(0)$ in the figure.

Note that a bank will only accept to renegotiate away from a contract on this budget line if the renegotiated contract (c_1, c_2) lies on or below iso-cost line $c_1 + c_2 = y - c_0^0 - \kappa$ because by design any contract above this line cannot offer sufficient cost savings (i.e. profit gains) to cover renegotiation cost κ. We also know from our earlier analysis of best renegotiation offers (recall our derivation of $C_1^1(C_1^0)$ and $C_1^{m1}(C_1^0)$) that candidate renegotiation contracts must satisfy One's first order condition $u'(c_1) = \beta u'(c_2)$ – otherwise the gains to trade from renegotiation could be increased to make these renegotiation more attractive. The unique contract that just satisfies these two conditions – the most attractive renegotiation that is just rejected – is represented by $R(\kappa)$.[16]

Now draw an indifference curve for One-self's preferences through this point. All points on the period 1 budget line $c_1^0 + c_2^0 = y - c_0^0$ that also lie above this indifference curve will be renegotiation-proof contracts. To see this consider any contract P' on this segment. One-self will only accept renegotiation away from this contract if it moves One-self to an indifference curve just above the one running through P' but, by construction, any such contracts clearly lie above the bank's no-renegotiation and hence would be rejected. By similar reasoning any contract along the budget line outside of this segment would be renegotiated. Zero-self's welfare rises as we move to more balanced consumption bundles along this line so Zero's preferred renegotiation-proof contract will be the contract labeled P in the diagram. For all $\kappa < \bar{\kappa}$ the so chosen optimal renegotiation-proof contract will offer less consumption and hence lower welfare compared to the full commitment contract

[16] With period 0 is competitive we'll have $c_1^0 + c_2^0 = y - c_0^0$ and can therefore identify this point from $u'(c_1) = \beta u'(y - c_0^0 - \kappa - c_1)$. For the CRRA case $c_1 = (y - c_0^0 - \kappa)/(1 + \beta^{\frac{1}{\rho}})$

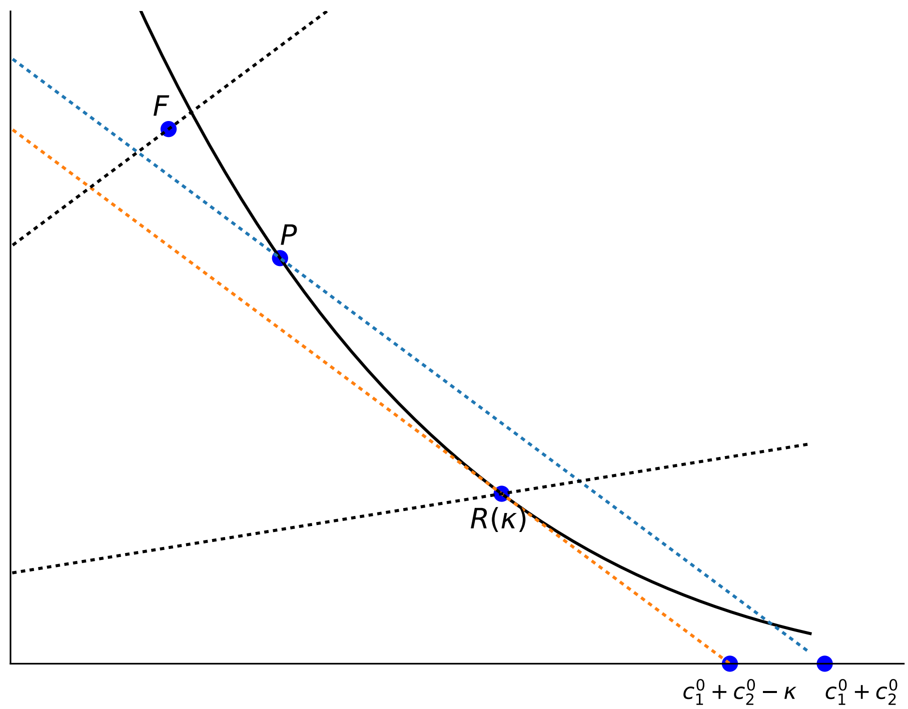


Figure 3: Competitive commitment contract with $\kappa < \bar{\kappa}$

(reached at $\kappa = \bar{\kappa}$).

Mathematically, for c_0^0 given the best continuation contract is found using three equations. Bank no-renegotiation condition $c_1^1 + c_2^1 = y - c_0^0 - \kappa$ and first order condition $u(c_1^1) = \beta u(c_1^2)$ to solve for best renegotiation offers c_1^0 and c_2^0 as a function of pre-determined y, c_0^0 and κ, and One-self no-renegotiation condition $u(c_1^0) + \beta u(c_2^0) = u(c_1^1) + \beta u(c_2^1)$ and One's first-order condition to then solve for the optimal continuation c_1^0 and c_2^0 as a function of those same pre-determined variables. Using this method to find the best continuation contract for candidate c_0^0 Zero-self can now search over all candidate c_0^0 to find the welfare maximizing three-period contract.

However, except for the special case where $\kappa = 0$ described below, there will be no simple closed form solution for the optimal contract even in the CRRA case. This can be seen from the fact that there are in fact two points where the no-renegotiation condition indifference curve crosses the period 1 zero-profit line. As described in the appendix for the CRRA case the c_1^0 coordinates of these two roots are given by the following non-linear equation:

$$u(c_1^0) + \beta u((y - c_0^0) - c_1^0) = (1 + \beta^{\frac{1}{\rho}})u\left(\frac{y - c_0^0 - \kappa}{1 + \beta^{\frac{1}{\rho}}}\right) \tag{17}$$

5.2 The competitive case when $\kappa = 0$

In the special case of perfect competition with costless renegotiation ($\kappa = 0$) the above equation has just one solution. In terms of figure 3 think of how P would slide down the period 1 budget line as κ shrinks until we get to a point where One's indifference curve is just tangent to the period 1 budget line. This continuation contract is 'renegotiation-proof' but only in the very narrow sense that it won't be renegotiated because it already delivers One-self's preferred consumption choice.

The left panel the lower Figure 1b has figure 3 rotated counter-clockwise 90 degrees in order to show how Zero self's strategic choices will be made in $c_0 - c_1$ space in the right panel. In contrast to Figure 1a where $\kappa > \bar{\kappa}$ and where One-self can be credibly committed to dividing consumption resources equally between periods 1 and 2, when $\kappa = 0$ that division would always be renegotiated in ways harmful to Zero. Being a sophisticate, Zero understands that the only way to avoid renegotiation is to satisfy One's period 1 first-order condition $u'(c_1^0) = u'(c_1^0)$. For the CRRA case depicted these FOCs are indicated by the dashed line though OP' (or $c_2 = \beta^{\frac{1}{\rho}} c_1$) in the left panel.

One's best response function to Zero's period 0 consumption choice is given by the tangency between One-self's indifference curve and the period 1 budget line $y - c_0^0$ which can be easily solved to give:

$$c_1^1(c_0) = \frac{y - c_0}{1 + \beta^{\frac{1}{\rho}}} \tag{18}$$

$$c_2^1(c_0) = \beta^{\frac{1}{\rho}} \cdot c_1^1(c_0) \tag{19}$$

This subgame reaction function can be visualized as the line passing through yP in the right panel of Figure 1 (from expression 18) and the line passing through OP' in the left (from 19).

Being sophisticated, Zero anticipates her future self's reaction and strategically chooses c_0 to solve:

$$\max_{c_0} u(c_0) + \beta[u(c_1^1(c_0) + u(c_2^1(c_0))] \tag{20}$$

subject to the bank's period 0 participation (or zero-profit) constraint:

$$c_0 + c_1^1(c_0) + c_2^1(c_0) = y \tag{21}$$

From the fact that One wants to set $c_2^1 = \beta^{\frac{1}{\rho}} c_1^1$ and other substitutions the first-order condition for this problem can be written:

$$u'(c_0) = \Lambda u'(c_1)$$

$$\text{or } c_1 = \Lambda^{\frac{1}{\rho}} c_0 \tag{22}$$

where

$$\Lambda = \frac{(\beta + \beta^{\frac{1}{\rho}})}{1 + \beta^{\frac{1}{\rho}}} \tag{23}$$

Equation 22 defines the line passing through OP.

Zero self chooses the consumption contract along subgame reaction function that places her on the highest possible iso-utility surface. The equilibrium contract P satisfies both equation 18 (the line through yP) and equation 22 (the line through OP). These equations can be solved to yield the closed-form solution to the optimal contract

$$c_0^P = \frac{y}{1 + \Lambda^{\frac{1}{\rho}}(1 + \beta^{\frac{1}{\rho}})} \tag{24}$$

with $c_1^P = \Lambda^{\frac{1}{\rho}} c_0^P$ and $c_2^P = \beta^{\frac{1}{\rho}} \Lambda^{\frac{1}{\rho}} c_0^P$.

It is easy to check that $\Lambda > \beta$ for all $0 < \beta < 1$ and hence line OP is everywhere steeper than line OF. Combined with our earlier observation that line yP is everywhere steeper than line yF we conclude that $c_1^P > c_1^F$ over the same parameter range. Hence, the inability to enforce commitment leads to higher consumption in period one compared to a situation where it can be enforced. The subgame-perfect 'renegotiation-proof' contract will involve less net saving (or equivalently, more net debt) in period 1 than Zero self would like (as can be seen by $u'(c_1^P) < u'(c_2^P)$) and lower Zero self welfare compared to the full-commitment contract.

To illustrate, at our earlier parameterization $\beta = 0.5$ and $\rho = 1$ the full commitment contract will be $C_0^P = (150, 100, 50)$ which offers considerably less consumption smoothing in later periods compared to the contract with self-control $C_0^F = (150, 75, 75)$. If the consumer's initial income stream were $Y_0 = (100, 100, 100)$ then we could think of the consumer with self-control as sticking to a balanced repayment program to keep consumption steady in the last two periods. Compared to this the consumer without self-control rolls over debt rather than repay it in period one. The entire burden of repayment of the debt that Zero took out in period 0 now falls in period 2, whereas Zero would have preferred the burden to be shared equally between periods 1 and 2. If the income stream had instead been $Y_0 = (200, 50, 50)$ then a consumer without self-control would be viewed as raiding savings in period 1 that an otherwise identical consumer with self-control would have earmarked for period 2 consumption. In short, consumers that can obtain full-commitment contracts will save/repay more or borrow less in period 1 and consume more in period 2 and the inability to do so leads to lower welfare for Zero in this competitive setting.

More generally, if κ is small but non-zero, then Zero-self is not entirely bound to One-self's preferences. Credible division rules between periods 1 and 2 depend on the no-renegotiation constraint in some subtle ways. In the following sections, starting with monopoly, we show how the contract terms change in such cases.

5.3 Monopoly renegotiation-proof contracts

When the market for period 0 contracts is monopolized the analysis is broadly similar to the competitive case above, with some wrinkles. The monopoly bank will want to maximize multi-period profits (objective 8) subject to Zero-self's participation (constraint 9) and the constraint that continuation contracts be renegotiation proof (i.e. no-renegotiation constraints 12 and 13 both hold).

A sophisticated consumer can rationally anticipate how her later self may be tempted to renegotiate. Zero-self would agree to a contract that will be renegotiated but only if the bank adjusts the terms of the offered period 0 contract to compensate for the reduced consumption smoothing that would result. This however can only harm bank profits as the cheapest to satisfy Zero's participation constraint is by offering consumption smoothing. Hence the monopoly bank will itself insist on renegotiation-proof contracts.

The bank will be looking for the most profitable renegotiation-proof contract that lies

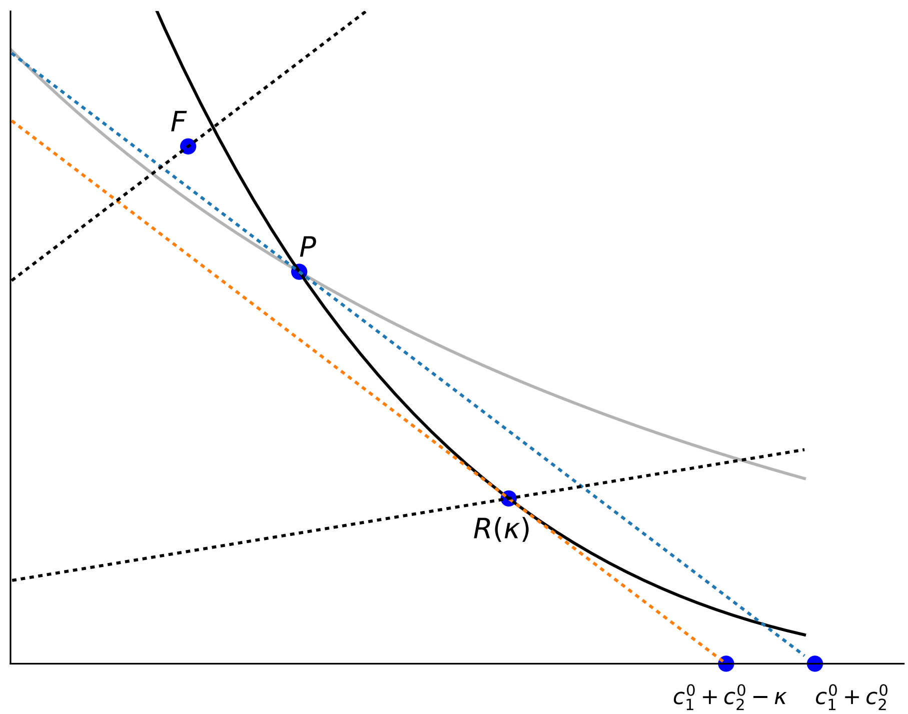


Figure 4: Monopoly commitment contract with $\kappa < \bar{\kappa}$

on Zero's participation constraint 9. If the bank has chosen candidate contract C_0^0 then the bank an optimum continuation contact $C_1^0 = (c_1^0, c_2^0)$ must lie along Zero's autarky utility surface which can be projected as indifference curve $\beta\left[u(c_1^0) + u(c_2^0)\right] = U_0^A - u(c_0^0)$ in $c_1 - c_2$ space. The optimal contract will be the most profitable renegotiation proof contract along this surface. The contract will be renegotiation proof when $c_1^1 + c_2^1 + \kappa \geq c_1^0 + c_2^0$ and $u(c_1^1) + \beta u(c_2^1) \leq u(c_1^0) + \beta u(c_2^0)$. Many points are renegotiation proof but the one that is most profitable amongst these will satisfy the two conditions exactly depicted in the figure by point P.

The renegotiation-proof contract can be explicitly derived for the CRRA case of $\kappa = 0$ (Equation 68). For, $\kappa > 0$, the contract cannot be explicitly derived in closed form, but its key properties can be established.

Proposition 2. *Suppose $\kappa < \bar{\kappa}$. If $U_0^A \leq U^m$, the bank offers the full-commitment contract C_0^{mF}. If $U_0^A > U^m$, the bank offers the renegotiation-proof contract C_0^{mP}, and:*

(a) $\Pi_0\left(C_0^{mP}; Y_0\right) < \Pi_0\left(C_0^{mF}; Y_0\right)$

(b) There is some $\bar{U}\epsilon\left(U^m, U_0^F\right)$ such that, if $U_0^A > \bar{U}$, the bank would earn negative profits and therefore will not offer a contract.

(c) $c_0^{mP} > c_0^{mF}$.

Proposition 2 compares the renegotiation-proof contract to the full-commitment contract when the renegotiation-proofness constraint binds. Part (a) states that bank profits will be lower than under full-commitment. The bank wishes it could promise to not renegotiate but it cannot make such a promise credible without giving up some profits. The problem here is not one of cheating or contract failure (as examined for example by Hansmann (1980) and Glaeser-Shleifer (2001)), it is the possibility of a legitimate renegotiation (a voluntary agreement to tear up the old contract) between the consumer and the firm. The monopolist would have gained from having higher renegotiation costs since in equilibrium renegotiation does not take place.

Part (b) follows from part (a). If the bank were able to provide full commitment, it could offer a profit-making contract to any individual with even minimal smoothing needs. Now however, for individuals whose autarky utility is close enough to U_0^F, the bank would make negative profits and therefore no contract is offered to them. This is because the renegotiation-proofness constraint may require even greater imbalance in consumption across periods 1 and 2 than under autarky.

Part (c) is about the terms of the contract itself–when full-commitment is not feasible, the renegotiation-proof contract will involve higher consumption in period 0 (i.e. either a smaller loan or less savings) compared to full commitment. The following is a sketch of the argument. Let the full-commitment contract be described by some $c_0 = c_0^{mF}$ (period 0 consumption) and $s = s^{mF}$ (sum of c_1 and c_2, which are equal in size). Since the full-commitment contract lies at an optimum, it must be true that at the levels of consumption specified by the contract, the marginal utility of present consumption is equal to the discounted marginal utility of future consumption:

$$\frac{du\left(c_0^{mF}\right)}{dc_0} = \frac{d\left(\beta u\left(\frac{s^{mF}}{2}\right) + \beta u\left(\frac{s^{mF}}{2}\right)\right)}{ds} \tag{25}$$

Now suppose $c_0^{mP} = c_0^{mF}$, so that period 0 consumption in the renegotiation-proof contract is held the same. Since any future consumption will be split unevenly, in order to continue to satisfy the consumer's period 0 participation constraint, it must be true that $s^{mP} > s^{mF}$. We show in the appendix that s^{mP} will be large enough that, at these values,

$$\frac{du\left(c_0\right)}{dc_0} > \frac{d\left(\beta u\left(c_1\left(s\right)\right) + \beta u\left(c_2\left(s\right)\right)\right)}{ds} \tag{26}$$

So the bank can do better by raising period 0 consumption at the expense of future consumption. The bank limits renegotiation possibilities by transferring consumption away from the future (when renegotiation is a temptation) to the present.

To summarize: the requirement that contracts be renegotiation-proof results in higher period 0 consumption and lower bank profits, and the denial of service (rationing) of consumers whose smoothing needs are relatively small.

5.4 Competition with $\kappa > 0$

As mentioned earlier, under competition, the terms of a renegotiated contract depend on exclusivity but the no-renegotiation constraint does not. So, regardless of exclusivity, the equilibrium contract under competition is given by:[17]

$$\max_{C_0}\ U_0\left(C_0\right)$$

$$s.t.\ \Pi_0\left(C_0; Y_0\right) \geq 0 \tag{27}$$

$$\Pi_1\left(C_1^{m1}\left(C_1\right); C_1\right) \leq \kappa \tag{28}$$

This yields a contract C_0^P. Given our assumption that $\kappa < \bar{\kappa}$, the renegotiation-proofness constraint (28) binds at any autarky utility. The properties of the equilibrium contract are summarized in the next proposition, which is structured like Proposition 2.

[17]A similar analysis could be conducted even if the costs of renegotiating another bank's contract are different from κ, the costs of renegotiating one's own contract.

Proposition 3. *Suppose $\kappa < \bar{\kappa}$. Then:*

(a) At any autarky utility, full commitment is infeasible, so $U_0\left(C_0^P\right) < U_0\left(C_0^F\right)$.

(b) If $U_0^A > \bar{U}$ (as defined in Proposition 2), the period 0 consumer would do worse than in autarky and therefore will not accept the contract.

(c) The relationship between c_0^P and c_0^F is ambiguous. There is some $\hat{\rho}$ such that: if $\rho \leq \hat{\rho}$, then $c_0^P > c_0^F$; if $\rho > \hat{\rho}$, then there are parameter values under which $c_0^P < c_0^F$.

The first two parts of the proposition are intuitively similar to the case of monopoly. First, the additional constraint results in a contract that cannot deliver the optimal utility to the period 0 consumer. Second, if the consumer's autarky utility is high, there cannot be renegotiation-proof contracts in equilibrium since the consumer could do better on her own. In fact, the cutoff autarky utility above which contracts are not offered is the same as under monopoly–this is the parameter region where it is impossible to simultaneously satisfy the consumer's participation constraint and the banks' zero-profit constraint.

Part (c) is a deviation from the results under monopoly. Under competition, we find that period 0 consumption under renegotiation-proof contracts may be bigger or smaller than under full-commitment. Again, we provide the intuition here (the actual proof involves a few additional steps). The competitive full-commitment contract must satisfy: $\frac{du\left(c_0^F\right)}{dc_0} = \frac{d\left(\beta u\left(\frac{s^F}{2}\right)+\beta u\left(\frac{s^F}{2}\right)\right)}{ds}$. If the renegotiation-proof contract were to have the same c_0, it must also have the same s (to continue satisfying the zero-profit constraint), but consumption will be split in period 1's favor. So, the marginal utility of future consumption becomes: $\frac{d\left(\beta u\left(c_1\left(s^F\right)\right)+\beta u\left(c_2\left(s^F\right)\right)\right)}{ds}$. If the utility function is relatively linear (low ρ), then an imbalanced split of s results in a lower marginal utility than from a balanced split. So:

$$\frac{d\left(\beta u\left(c_1\left(s^F\right)\right)+\beta u\left(c_2\left(s^F\right)\right)\right)}{ds} < \frac{d\left(\beta u\left(\frac{s^F}{2}\right)+\beta u\left(\frac{s^F}{2}\right)\right)}{ds} = \frac{du\left(c_0^F\right)}{dc_0} \tag{29}$$

In such a case, the renegotiation-proof contract must involve higher period 0 consumption than the full-commitment contract. If, on the other hand, the utility function is highly convex (high ρ), then an imbalanced split results in higher marginal utility, so the renegotiation-proof contract will have lower period 0 consumption than under full-commitment.[18] This can be seen more explicitly in the case of $\kappa = 0$ (Equation 69).

So, under competition, the renegotiation-proofness constraint could change the contract in either direction: a larger loan (less saved) or a smaller loan (more saved). The key reason that the latter possibility does not exist under monopoly is the following: under monopoly, a switch from full-commitment to renegotiation-proofness while maintaining the

[18]The actual argument, in the appendix, is a little more complicated since we must consider the effect of a change in s not just on utilities, but also on the relative ratios of c_1 and c_2.

same c_0 would require such a large jump in future total consumption (to maintain the same discounted utility under imbalanced consumption) that the marginal utility would necessarily fall. The contrast between monopoly and competition can also be explained using the intuition of income and substitution effects. Under monopoly, since the consumer is always left at her autarky utility, there are no income effects. When the renegotiation-proofness constraint binds, the price of future utility effectively rises, as a result of which substitution effects lead to greater period 0 consumption. Under competition, income and substitution effects counter each other; the net result depends on the consumer's coefficient of relative risk aversion (ρ).

Period 2 consumption however always falls relative to the full commitment case, even in the cases when Zero saves more/borrows less. In fact for CRRA utility the adjustment of period 0 consumption (in the absence of commitment compared to with commitment) is always relatively small while the adjustment to period 1 and period 2 consumption is relatively much larger.[19] In other words despite having a first-mover advantage, Zero can do little other than to partially accommodate to the consumption pattern that One-self wants to impose.

5.5 Contracting with Naive Hyperbolic Discounters

For naive agents, the problem of renegotiation does not lead to a renegotiation-proof contract. The naif believes she will not be tempted to renegotiate. Banks therefore offer contracts that take into account the potential renegotiation. Under monopoly, the bank adds to its profits by engaging in renegotiation that was not anticipated by the consumer in period 0. Under competition, banks return the potential surplus from renegotiation to the Zero-self.[20]

5.5.1 Monopoly

Relative to a sophisticated consumer, with a naive consumer the monopolist bank can make additional profits on two margins. First, since there is no perceived renegotiation problem, the consumer is willing to accept a contract that is more profitable for the bank up-front; subsequently, renegotiation generates additional profits for the bank. Notice that, unlike with sophisticates, service is not denied to any naif since the consumer would, at the very

[19] To illustrate, with $\kappa = 0$ at no point does period 0 consumption rise or fall by more than six percent for any value $\rho \in (0, \infty)$ and $\beta \in (0, 1)$ but at reasonable parameter values such as $\rho = 0.5$ and $\beta = 0.5$ in the absence of commitment period 1 consumption rises to 149 percent of the level it would be with commitment, and period 2 consumption falls to just 37 percent of what it would be.

[20] A similar analysis could be carried out if consumers were misinformed not about their own preferences but about κ.

least, be willing to accept the full-commitment contract (since she would not anticipate renegotiation).

With a naive hyperbolic discounter, the bank must choose between a renegotiation-proof contract and one that will be renegotiated upon. If the consumer's autarky utility is very low, then the initial contract can extract so much surplus that there is little to gain from renegotiation. But when autarky utility is high, the consumer must be offered a contract with high consumption in each period. It is such consumers, the ones who have relatively less need for banking, who will find their contracts renegotiated. In such cases, the bank solves the following problem:[21]

$$\begin{aligned} \underset{C_0}{max} \quad & \Pi_0\left(C_0;Y_0\right)+\Pi_1\left(C_1^{m1}\left(C_1\right);C_1\right)-\kappa \\ s.t. \quad & U_0\left(C_0\right)\geq U_0^A \end{aligned} \tag{30}$$

Let the solution be denoted C_0^{mN}. This is explicitly derived in the appendix (75, 76). The bank maximizes profits by offering a contract that divides future consumption as much in favor of period 2 as possible. We show that if $\rho<1$, the contract is at a corner solution where $c_1=0$. If $\rho>1$, an explicit solution does not exist, but maximization pushes the contract to a point where c_2 approaches infinity.[22] In each case, the greater the imbalance between the contracted c_1 and c_2, the greater the bank's profits from renegotiation.

This contract can be compared to the full-commitment contract and to the renegotiation-proof contract for sophisticates. In particular, it will involve lower period 0 consumption than under full-commitment or renegotiation-proofness. This result appears counter-intuitive. In the case of lending, it does not reinforce the narrative of banks preying on naive consumers by offering them relatively large loans with steep repayments. Indeed, there are other considerations beyond the scope of this model, such as the possibility of collateral seizure, that could generate large loans. But our limited model helps to highlight a particular aspect of contracting with naive hyperbolic discounters: here, the bank offers them relatively *small* loans because its gains from renegotiation depend on the surplus that the initial contract delivers to periods 1 and 2. In order to fully take advantage of the consumer's naivete, the consumer must start out with sufficiently small repayments that the bank could profit from rearranging them.

The next proposition summarizes the above discussion.

Proposition 4. *Suppose the consumer is naive. Then:*

(a) A monopoly contract will be accepted at any autarky utility.

[21] We do not need to worry about a renegotiation-proofness constraint here. Since period 0 believes her period 1 preferences are consistent with her own, she expects any renegotiation of the period 0 contract to yield the same discounted utility as the contract itself.

[22] This can be dealt with by a reasonable assumption of an upper bound on contract terms.

(b) There is some $U^N < U^m$ (U^m as defined in Proposition 1) such that, if $U_0^A \leq U^N$, the naive agent will receive the monopoly full commitment contract and it will not be renegotiated.

(c) If $U_0^A > U^N$, the monopoly contract will satisfy $c_0^{mN} < c_0^{mF} < c_0^{mP}$ (either explicitly or in the limit), and will be renegotiated in period 1.

5.5.2 Competition

Under competition too, contracts will be renegotiated and firms must account for renegotiation. First, note that if contracts are not exclusive, the equilibrium contract must be identical to the full-commitment contract. This is because the firm offering the contract in period 0 does not expect to benefit from renegotiation.

Under exclusive contracts, anticipated profits from future renegotiation will be returned to the consumer through more favorable initial contracts. The equilibrium contract satisfies:

$$\begin{aligned} &\underset{C_0}{max} U_0 \left(C_0\right) \\ &\quad s.t. \Pi_0 \left(C_0; Y_0\right) + \Pi_1 \left(C_1^{m1} \left(C_1\right); C_1\right) \geq \kappa \end{aligned} \tag{31}$$

Let the solution be denoted C^N. As under monopoly, first-order conditions lead to a corner solution where contracts favor period 2 relative to period 1. This maximizes the potential gains from renegotiation.

Unlike under monopoly, these anticipated gains must be returned to the consumer. Some of these gains are returned to the Zero-self, so there is no clear prediction about whether period 0 consumption will be lower or higher than under full commitment. This is formalized in Proposition 5.

Proposition 5. *Suppose the consumer is naive. Then:*

(a) The competitive contract will be accepted at any autarky utility and will be renegotiated in period 1.

(b) The non-exclusive competitive contract will be identical to the full-commitment contract, c_0^F.

(c) Under exclusive contracts, the relationship between c_0^N and c_0^F is ambiguous. If $\rho < 1$, $c_0^N < c_0^F$. If $\rho > 1$, then there are parameter values under which $c_0^N > c_0^F$.

6 Nonprofits

Suppose a firm has the possibility of operating as a legal non-profit. Narrowly stated, non-profit status means that the firm cannot tie managers' compensation or outside shareholders dividends to firm profits because, technically speaking, a non-profit firm has no shareholders. As described in the introduction we prefer a more elastic and encompassing definition of the non-profit term to include firms that may have shareholders but who have adopted ownership or governance structures that place credible and visible constraints on the distribution of profits to managers or investor shareholders. The principals of such firms may also directly care about social objectives such as the welfare of their customers, and not just about profits. This broader interpretation allows the non-profit firm category to include cooperatives as well as social enterprises and 'hybrid' firms which might be incorporated as for-profit firms but are owned and controlled by social investors.[23]

To model these ideas in a simple yet still rich manner, assume that we can classify a firms' nonprofit orientation by a simple parameter $\alpha \epsilon (0, 1]$, which can be viewed as the degree of 'non-profitness'. A lower α indicates a firm that because of its ownership and governance structure places the welfare of its clients ahead of that of its 'owners' and makes the capture of profits by principals (managers, outside investors) more difficult.

Parameter α affects the firm's maximization problem in two ways. First, it reduces the firm's ability to capture its raw profits, so that if a firm produces profits Π, its principals only get to capture and enjoy $\alpha\Pi$. Glaeser & Shleifer (2001) adopt this approach and suggest that it describes the idea of how the principals of a nonprofit, though legally barred from paying themselves cash profits, might capture profits imperfectly via the consumption of perquisites or 'dividends in kind.' Second, α possibly alters the renegotiation costs that the firm will incur when it breaks its promises to customers (e.g. shame, regret, loss of social reputation). We now label this cost of renegotiation as $\eta(\alpha)$, and assume η falls weakly in α. In other words more profit-oriented firms feel lower non-pecuniary costs to renegotiating earlier commitments.

Our new assumptions lead to a modified no-renegotiation constraint (32). This new constraint states that the value of captured profits from not renegotiating the contract should exceed captured profits from renegotiation net of renegotiation costs.

$$\alpha\Pi_1(C_1^{m1}(C_1); C_1) \leq \eta(\alpha) \tag{32}$$

Notice that if we define $\kappa(\alpha) \equiv \frac{\eta(\alpha)}{\alpha}$, we can rewrite the no-renegotiation constraint as

$$\Pi_1(C_1^{m1}(C_1); C_1) \leq \kappa(\alpha) \tag{33}$$

This makes the no-renegotiation constraint look just like the earlier constraint (**??**)

[23]For example, most of the commercial firms that one finds in modern microfinance (including poster-child firms of the 'commercialization' revolution in microfinance such as Bancosol of Bolivia or Compartamos of Mexico) are incorporated as for-profit corporations but on close inspection turn out to be majority-owned and controlled by 'social investors' that are themselves non-profit foundations.

except that κ is now a function of α. Indeed the earlier analyzed renegotiation problems just become a special case with $\alpha = 1$.

Before proceeding, it should be noted that firms never have an incentive to switch to nonprofit status when consumers are naive. Since the consumer does not perceive a need for commitment, the nonprofit's promise of superior commitment is of no value to her.

Our analysis below focuses on firm principals that are purely self-interested. We derive conditions under which the pursuit of profits leads a firm to voluntarily switch to a form of governance with $\alpha < 1$. The rise in renegotiation costs ($\eta(\alpha)$) is obviously welcomed by firms facing sophististicated consumers, as this allows them to credibly offer better commitment upfront. But even in the absence of an explicit rise in these costs (i.e. if $\eta(\alpha) = \kappa$ for all α), there are conditions under which firms will opt for $\alpha < 1$.

6.1 Monopoly

In a pre-contract phase the firm now first establishes its type α via the adoption of legal non-profit status and/or by choosing credible and stable ownership and governance structures that commit it to those limitations. When facing a sophisticated hyperbolic discounter, a monopoly firm of type α designs a renegotiation-proof contract to solve

$$\underset{C_0}{max}\ \alpha\Pi_0(C_0; Y_0)$$

$$U_0(C_0) \geq U_0^A \tag{34}$$

$$\Pi_1\left(C_1^{m1}(C_1); C_1\right) \leq \kappa(\alpha) \tag{35}$$

Why might a profit-maximizing firm choose to operate as a nonprofit when that reduces its ability to capture profits? The answer lies in the loosening of the no-renegotiation constraint. Because the non-profit can more credibly commit to not renegotiate contracts that offer greater consumption smoothing in periods 1 and 2, period 0 becomes more willing to pay for this smoothing service.

The captured-profits maximizing solution gives a contract $C_0^{m\alpha}$. The no-renegotiation constraint is now relaxed compared to the earlier pure for-profit case. With a relaxed renegotiation-proof constraint $\Pi_0(C_0^{m\alpha}; Y) \geq \Pi_0(C_0^{mP}; Y)$ but whether or not it will be in the bank principals' best interest to strategically convert to non-profit status depends on whether the profits they can capture under non-profit status exceed the profits they could earn as a pure for-profit, in other words on whether $\alpha\Pi_0(C_0^{m\alpha}; Y) \geq \Pi_0(C_0^{mP}; Y)$. The monopolist faces a tradeoff in considering non-profit status: higher raw profits (as the commitment problem is partly solved) but a diminished capture of those raw profits.

Proposition 6 describes conditions under which a firm will operate as a non-profit.

Since the for-profit firm's no-renegotiation constraint binds, the nonprofit can offer greater commitment and thereby extract greater surplus from the consumer through the contract signed in period 0. The question is: does the rise in extracted surplus outweigh the fact that all profits are now discounted? To the extent that the loosening of the no-renegotiation constraint happens through the right-hand side (i.e. via term $\eta(\alpha)$, which represents the firm's motivation to honor the initial agreement), the firm benefits unambiguously–it is able to offer better commitment *and* fully retain the added profits.

If the right-hand side of the no-renegotiation constraint remains unchanged (as in the proposition), the firm is forced to address the tradeoff–a lower α means both better commitment and reduced ability to capture profits.

If the consumer's autarky consumption bundle is sufficiently close to optimal to start with, then even a nonprofit may be unable to offer sufficiently smoother consumption that allows it to cover costs. For intermediate levels of autarky utility, the nonprofit is able to more credibly promise it will not renegotiate, so it earns positive profits where the pure for-profit would have earned small or negative profits. Here, the gains that can be captured from nonprofit status are large relative to the profits that a for-profit would have made, so the firm prefers to operate as a nonprofit. As an example, consider an autarky consumption bundle at which the for-profit firm would earn zero profits. Now, the nonprofit firm can earn positive profits, so regardless of α nonprofit status dominates.

Finally, for autarky bundles far from the optimal, the for-profit firm would anyway be making substantial profits. In this case, the nonprofit's credibility advantages are not enough to outweigh the fact that it loses a significant amount of enjoyment of its profits due to legal restrictions.

Proposition 6. *Consider any $\bar{\alpha} < 1$ and a corresponding $\eta(\bar{\alpha}) = \kappa$. There is some $\underline{U}^{\alpha} \epsilon \left(U^m, \bar{U}\right)$ and $\bar{U}^{\alpha} \epsilon (\bar{U}, U_0^F]$ (with U^m as defined in Proposition 1 and $\bar{U}$ as defined in Proposition 2) such that, if $U_0^A \epsilon \left(\underline{U}^{\alpha}, \bar{U}^{\alpha}\right)$, the bank strictly prefers $\alpha = \bar{\alpha}$ over $\alpha = 1$.*

In Figure 5 we illustrate the case where non-pecuniary costs to breaking a promise not to renegotiate fall with α according to $\eta(\alpha) = 10(1-\alpha)$ and hence that the overall cost to renegotiation varies with α according to $\kappa(\alpha) = 10(1-\alpha)/\alpha$. The plots depict captured profits that would be achieved at different levels α starting from three different initial endowment streams. These three streams - (60, 120, 120), (90, 105, 105) and (120, 90, 90) – are equal in their present value of 300 but differ in terms of period 0 income (with remaining income allocated equally across period 1 and 2). The higher of the two curved lines represents 'raw' profits $\Pi_0(C_0^{m\alpha}; Y_0)$ and the lower curve captured profits $\alpha\Pi_0(C_0^{m\alpha}; Y_0)$. A horizontal line has been drawn in to indicate the level of profits $\Pi_0(C_0^{mP}; Y_0)$ captured by a pure for-profit ($\alpha = 1$). Consider the top panel where the customer has initial income $(60, 120, 120)$. As

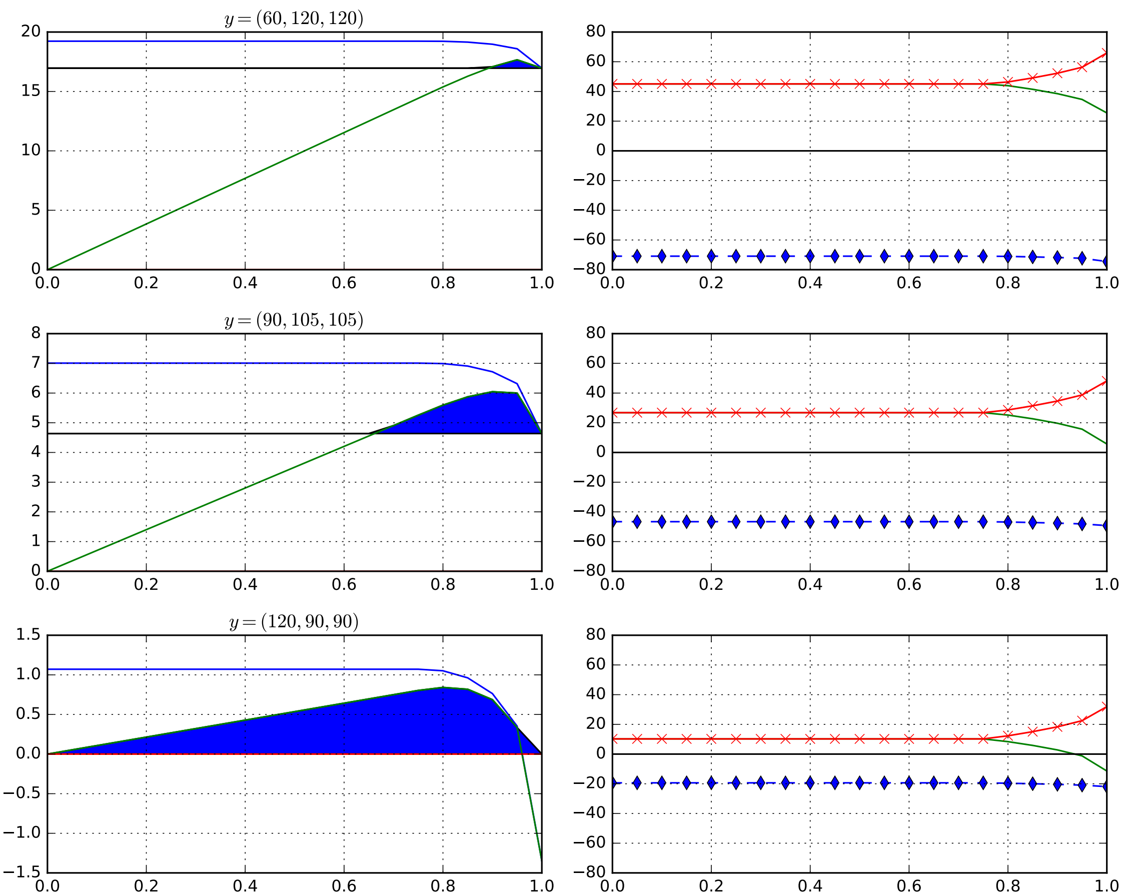


Figure 5: Choice of non-profit status and initial endowment income

this type of customer wants to borrow heavily in period 0 profits to the bank are large, even in the case of renegotiation-proof contracts. Adopting non-profit status by lowering α confers limited profit gain however: the cost of lowering alpha (giving up a share of already high profits) is not compensated for by the gains from being able to credibly commit to a smoother contract. However at $(90, 105, 105)$ the tradeoff is different and profits can be increased. In the picture any non-profit with an α between approximately 0.7 and less than one captures more profits than a pure for-profit. Finally for customers with an endowment $(120, 90, 90)$ are already fairly close to their preferred consumption stream so the profits to be captured even under full commitment are not that large. Indeed in this case a pure for-profit cannot earn positive profits. Here the cost of adopting non-profit status is low compared to the gains, and we the simulation reveal that any non-profit status firm captures more profits than a pure for-profit, and maximum captured profits are achieved at around $\alpha = 0.7$.

We conclude this section with a note on the nature of profit-capture restrictions faced by a nonprofit. Our assumption that the firm can capture a fixed fraction of raw profits was useful for exposition but is perhaps not realistic, and is not necessary for our results. We might imagine that, at low levels of profits, the nonprofit can capture most of the profits, and that as profits rise so do the restrictions on the firm's ability to capture them. In other words, a nonprofit captures $f(\Pi)$ of profits, where $f(0) = 0$, $0 < f'(\Pi) < 1$, and $f''(\Pi) < 0$. In such cases, we can again clearly see how non-profit status could be attractive to the firm: the concavity of f can leave the enjoyment of profits relatively unaffected while significantly loosening the no-renegotiation constraint (since renegotiation would raise profits further, and since f is concave, these additional profits would count for little).

6.2 Competition

6.2.1 Exclusive contracts

Consider what would happen in the competitive market situation now if contracts can be assumed to remain exclusive, so that any new surplus in the event of a renegotiation between the bank and the period 1 self goes to the bank (this grants the bank monopoly power in period 1). A firm of type α will be led to offer contract terms to solve

$$\begin{aligned} &\underset{C_0}{max} U_0(C_0) \\ &s.t. \alpha\Pi_0(C_0; Y_0) \geq 0 \qquad (36) \\ &\Pi_0(C_1^{m1}(C_1); C_1) \leq \kappa(\alpha) \qquad (37) \end{aligned}$$

Here, as before, $\kappa(\alpha) = \frac{\eta(\alpha)}{\alpha}$, captures the idea that the principals of a firm that adopts non-profit status commit themselves to capturing a smaller share of raw profits and may also suffer greater direct disutility from breaking promises to customers. Let the contract that solves this program be denoted $C_0^{e\alpha}$.

Consider first a field where all firms start as pure for-profits ($\alpha = 1$). If the no-renegotiation constraint binds, consumer welfare must be lower than that when the firms can commit to not renegotiate since an additional constraint is imposed. Starting from this situation consider now one firm's strategic choice of whether to adopt non-profit status (i.e. to change its ownership and governance structures to an $\alpha = \bar{\alpha} < 1$ relaxing the no-renegotiation constraint. One firm deviating into nonprofit status in this way can make positive profits. So, if the borrowers are sophisticated hyperbolics, in equilibrium all firms become nonprofit. As proven in the appendix, competition will ensure however that in equilibrium the principals of all firms are capturing zero profits.

6.2.2 Non-Exclusive Contracts

In the previous section, we had a setting with competition in period 0 but exclusive contracting and monopoly power in period 1. Now, assume that exclusivity and period 1 monopoly power disappears. Firms can compete to renegotiate each other's contracts in period 1.

If there were only nonprofits in equilibrium, any one firm could make positive profits by switching to for-profit status and undoing a rival bank's contract in period 1. The advantages of undercutting other firms' contracts outweigh the benefits of promising one's own clients it will not renegotiate. As a result, equilibrium contracts will be determined by for-profit firms, and consumers will be offered lower commitment than from non-profit firms alone.[24] Additionally, a smaller range of consumer types will be serviced relative to the case with exclusive contracts.

Proposition 7. *Consider any $\bar{\alpha} < 1$ and a corresponding $\eta(\bar{\alpha}) = \kappa$.*

(a) In a competitive banking market with exclusive contracts, all active firms will be nonprofits (contracts will be offered for $U_0^A \leq \bar{U}^\alpha$, with $\bar{U}^\alpha$ as defined in Proposition 6).

(b) In a competitive banking market with non-exclusive contracts, for-profits must exist in equilibrium (contracts will be offered for $U_0^A \leq \bar{U}$, with $\bar{U}$ as defined in Proposition 2).

7 Discussion and Extensions

The model above formalizes the renegotiation problem faced by banks that contract with hyperbolic discounters, and shows how the problem is addressed in equilibrium contracts.

[24] The same argument applies if banks can costlessly renegotiate other bank's contracts.

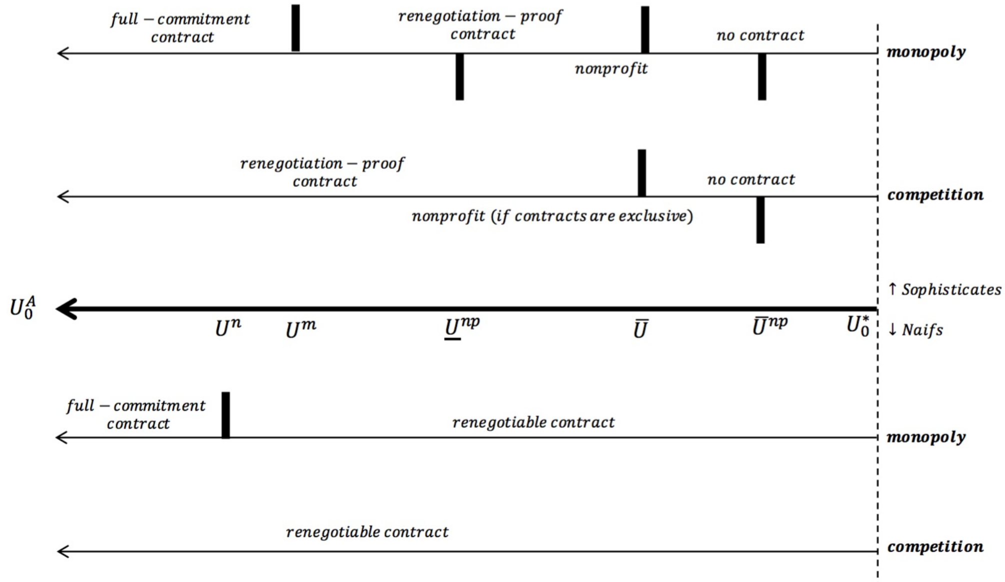


Figure 6: Summary of results

Figure 6 summarizes the key results of Sections 4-6. We show how contracts depend on relative distances from the optimal autarky utility, U_0^F. The results, taken together, generate some natural yet novel empirical predictions that are in principle testable.

We first discuss naive hyperbolic discounters. Under monopoly, contracts will be subject to renegotiation if the consumer is close enough to optimal autarky; if not, the full-commitment contract leaves the consumer with so little consumption that there is no point renegotiating. The parameter region in which contracts are renegotiated is relatively large, and even includes cases where the sophisticate would be offered a full-commitment contract. The renegotiable contract involves less period-0 consumption that the full-commitment contract as this allows the bank to exploit renegotiation possibilities most comprehensively.

Under competition, any contract will be renegotiated, even for consumers whose autarky outcomes are very poor. This is because competitive full-commitment contracts always leave consumers with relatively high levels of future consumption. The implications for contract terms are ambiguous.

Next, we turn to sophisticated hyperbolic discounters under monopoly. If the consumer's smoothing needs are large (i.e. autarky utility is low), full-commitment is feasible since the contract terms leave little that is susceptible to renegotiation. If smoothing needs are moderate, the consumer is offered a renegotiation-proof contract which has a larger period-0 consumption than the full-commitment contract (this serves to reduce the contract's susceptibility to renegotiation). If smoothing needs are small, the consumer is better off in autarky than in any contract that satisfies the renegotiation-proof constraint. As a result of the problem of renegotiation, such consumers will not be offered contracts.

Under competition, sophisticates will not be offered full-commitment contracts. Since full-commitment would entail high consumption levels in periods 1 and 2 regardless of autarky utility, the renegotiation-proofness constraint must always bind. As under naivete, the implications for contract terms are ambiguous.

Finally, we derive conditions under which banks will operate as nonprofits. A monopoly will switch to a nonprofit if, as a for-profit its profits were close to zero (above or below). In these cases, the bank is willing to forgo some enjoyment of its profits in exchange for a loosened renegotiation-proofness constraint. So, nonprofits will be able to serve some consumers relatively closer to optimal autarky utility, ones who would be unbanked under for-profit monopoly.

Under competition, banks should operate as nonprofits if contracts are exclusive, as they can capture the surplus from improved contracts possible under nonprofit status. If contracts are non-exclusive, no bank can benefit from operating as a nonprofit, and for-profit banks prevail.

7.1 Additional Considerations

Our model delivers predictions about how renegotiation concerns affect commitment contracts, and about parameter regions in which these concerns actually matter. In particular, we generate comparative statics over autarky utilities. Autarky utility is not informative in isolation, but in conjunction with total income serves as an indicator of the extent of smoothing that remains to be provided by a bank. As we show, contract terms depend in particular ways at different degrees of smoothing needs.

The sizes of relevant parameter regions discussed above will vary according to other parameter values such as the cost of renegotiation, κ, and total income, y. For example, as κ rises there will be an expansion of the parameter region in which full commitment survives. On the other hand, as y rises, commitment in general will be harder to sustain since contracts terms must allow for higher consumption in periods 1 and 2.

While the focus of our paper is on contracts, a few observations on welfare can be made. Under hyperbolic discounting, there is no obvious notion of welfare, and for our purpose we take it to the discounted utility of the Zero-self. Clearly, for sophisticated hyperbolic discounters under monopoly, welfare remains constant regardless of renegotiation concerns and bank governance–the consumer is always left with autarky utility. Under competition, welfare is lower under renegotiation-proof contracts relative to full-commitment (when the constraint binds), and nonprofits serve to raise welfare.

7.1.1 Equilibria allowing period 1 contracts

Our preceding analysis made the simplifying assumption that contracts between consumers and banks could only be initiated in period 0. The alternative to a period 0 contract was autarky for the consumer in the monopoly case and zero profits for the bank under competition. This served to streamline the analysis. We now discuss the interesting problem of how the contract space is enriched by allowing unbanked consumers to sign two-period contracts in period 1, possibly without contracting in period 0.[25] The main change will relate to the formulation of reservation values and their implications for the shape and feasibility of the period 0 contracts. The main qualitative results of the paper go through but the discussion raises interesting questions about circumstances where consumers could be better off under an autarky economy compared to one with a financial intermediary.

Consider the monopolist bank facing a sophisticated hyperbolic discounter. If a contract were not signed in period 0, they would meet again in period 1. In period 1, the contract must satisfy the One-self's participation constraint, which would be determined by some unbanked consumption path $C_1^{A'}$. This can be stated formally. Given some consumption path $C_1^{A'}$, the bank solves:

$$\begin{aligned} &\max_{C_1} \Pi_1\left(C_1; C_1^{A'}\right) \\ \text{s.t. } &U_1\left(C_1\right) \geq U_1\left(C_1^{A'}\right) \end{aligned}$$

Let the solution be denoted $C_1^{m'}$. Two observations can be made. First, the bank can always offer a period 1 contract that delivers nonnegative profits. This is because any contract will satisfy One-self's optimality condition, $u'\left(c_1^{m'}\right) = \beta u'\left(c_2^{m'}\right)$. So, except in the special case where the autarky consumption path satisfies this condition, the bank can make positive profits in period 1. Second, the autarky consumption path $C_1^{A'}$ might differ from C_1^A, the consumer's autarky utility in the absence of banking. In other words, C_1^A maximizes $U_0\left(C_0^A\right)$ while $C_1^{A'}$ maximizes $U_0\left(c_0^{A'}, c_1^{m'}, c_2^{m'}\right)$. In the latter case, period 0 anticipates that consumption across periods 1 and 2 is guaranteed to satisfy period 1's optimality condition. We can denote $C_0^B \equiv \left(c_0^{A'}, c_1^{m'}, c_2^{m'}\right)$, which corresponds to a Zero-self utility of U_0^B.

In period 0, any contract must meet the Zero-self's reservation utility, U_0^B:

$$\begin{aligned} &\underset{C_0}{max}\Pi_0\left(C_0; Y_0\right) \\ &\text{s.t.} U_0\left(C_0\right) \geq U_0^B \end{aligned}$$

[25] We thank Abhijit Banerjee for very helpful discussions about this point.

The maximization problem looks familiar, apart from the modified reservation utility. The Zero-self's discounted utility from such a contract is no longer monotonic in her Zero-self's full-autarky utility, U_0^A. For example, consider two hypothetical consumers who in autarky must consume their income streams, which deliver the same autarky utility but through different consumption paths: consumer X has $c_1^A = c_2^A$ while consumer Y has $c_1^A > c_2^A$ in a way that satisfies period 1's optimality condition. Then, for consumer X, $U_0^B < U_0^A$ while for consumer Y, $U_0^B = U_0^A$. It follows that, since period 0 contracts depend on the distribution of future consumption, a consumer who fares relatively better in the absence of a bank may fare relatively worse under a banking contract.

Given this benchmark full-commitment contract, the renegotiation-proof contract can be solved for by adding a no-renegotiation constraint to the above maximization problem. The constraint is the same as used previously, and again narrows the set of contracts that can be offered in period 0. As in Proposition 2 (parts a and c), the renegotiation-proof constraint results in lower profits and greater period 0 consumption relative to full-commitment. These results are independent of the period 0 reservation utility and therefore remain unchanged.

A key difference here, however, is that a renegotiation-proof contract will be offered to all consumers (unlike before, where the bank was better off not contracting with consumers whose autarky utility left them close enough to the first-best). Intuitively, this is because the alternative to a period 0 contract is not autarky; rather, it is a period 1 contract that tilts consumption in period 1's favor. Since, in period 0, the bank can at least offer the consumer a consumption path of C_0^B, it ensures that a contract will be accepted.

By opening up the possibility of period 1 contracts, we introduce an additional consideration–the same bank that offers commitment itself creates a need for commitment. By threatening to fully indulge the One-self's preferences, the bank is always able to induce the Zero-self to accept an offer of partial commitment, no matter how weak.

Finally, observe that the bank's decision about whether to operate as a nonprofit is subject to the same tradeoff between improved commitment and reduced enjoyment of profits. However, the attractiveness of nonprofit status drops (relative to the case where period 1 contracts are disallowed) due to the fact that even the for-profit bank finds it profitable to offer contracts to consumers at all levels of autarky utility.

Next, we turn to competition. In most cases, the possibility of period 1 contracts leaves our previous analysis unaltered. This is because competitive contracts do not depend on autarky utility. The only modification to our previous results relates to Proposition 3 (part b). Now, the consumer will accept a period 0 renegotiation-proof contract at any autarky utility since not doing so exposes her to a period 1 contract that fully satisfies period 1's taste for imbalanced consumption.

8 Conclusion

The starting point for this paper is the observation that the solution to any commitment problem must also address a renegotiation problem. We show how the renegotiation problem affects different types of consumers and how it changes contract terms in sometimes unexpected ways. In this context, we also provide a rationalization of commercial nonprofits in the absence of asymmetric information.

We argue that the model sheds some light on trends in microfinance, payday lending, and mortgage lending. We hope this paper also offers a framework that can be built upon. The incorporation of additional 'real-world' factors could improve our understanding of particular institutions and generate empirically relevant comparative statics. Examples of these include nondeterministic incomes, private and heterogenous types, collateral and strategic default, and longer time horizons.

Finally, the differences between monopoly and competition open up some new, potentially interesting questions. How does market structure evolve and what are the implications for commitment? And through this evolution might there emerge third parties to contracts between consumers and banks that can more effectively enforce the commitment that is sought after on both sides of the market?

9 Appendix: CRRA Derivations and Proofs

9.1 Full-commitment

9.1.1 Competition

The first-order conditions are:

$$c_1 = \beta^{\frac{1}{\rho}} c_0 \tag{38}$$

$$c_2 = c_1 \tag{39}$$

The first-order conditions and budget constraint can be combined to give:

$$c_1 = \frac{y - c_0}{2} \tag{40}$$

Then, the competitive full commitment contract C_0^F is:

$$c_0^F = \frac{y}{1+2\beta^{\frac{1}{\rho}}} \tag{41}$$

$$c_1^F = c_2^F = \beta^{\frac{1}{\rho}} c_0^F \tag{42}$$

or compactly on a single line:

$$C_0^F = \left(1, \beta^{\frac{1}{\rho}}, \beta^{\frac{1}{\rho}}\right) \cdot \left(\frac{y}{1+2\beta^{\frac{1}{\rho}}}\right) \tag{43}$$

9.1.2 Monopoly

For the monopolist bank that offers full-commitment, the solution is determined by the first-order condition and the consumer's participation constraint to yield:

$$C_0^{mF} = \left(1, \beta^{\frac{1}{\rho}}, \beta^{\frac{1}{\rho}}\right) \cdot \left(\frac{U_0^A (1-\rho)}{1+2\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}} \tag{44}$$

$$\Pi_0 \left(C_0^{mF}; Y_0\right) = y - \left(U_0^A (1-\rho)\right)^{\frac{1}{1-\rho}} \left(1+2\beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}} \tag{45}$$

9.2 Renegotiation

9.2.1 Renegotiated contracts

Given an existing continuation contract C_1^0, assuming the contract is renegotiated, the competitively renegotiated contract will be:

$$C_1^1 \left(C_1^0\right) = \left(1, \beta^{\frac{1}{\rho}}\right) \cdot \left(\frac{c_1^0 + c_2^0 - \kappa}{1+\beta^{\frac{1}{\rho}}}\right) \tag{46}$$

And under monopoly, the renegotiated contract will be:

$$C_1^{m1} \left(C_1^0\right) = \left(1, \beta^{\frac{1}{\rho}}\right) \cdot \left(\frac{(c_1^0)^{1-\rho} + \beta (c_2^0)^{1-\rho}}{1+\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}} \tag{47}$$

The corresponding profit gains from renegotiation in the monopoly case are:

$$\Pi_1 \left(C_1^{m1}\left(C_1^0\right); C_1^0\right) = \left(c_1^0 + c_2^0\right) - \left((c_1^0)^{1-\rho} + \beta (c_2^0)^{1-\rho}\right)^{\frac{1}{1-\rho}} \left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}} \tag{48}$$

9.2.2 No-renegotiation constraint

Conditions 12 and 13 are equivalent to writing:

$$u(c_1^0) + \beta u(c_2^0) \geq u(c_1^1) + \beta u(c_2^1)$$

$$c_1^1 + c_2^1 - \kappa \geq c_1^0 + c_2^0$$

In this CRRA case the second condition can be rewritten

$$c_1^1 \geq \frac{c_1^0 + c_2^0 - \kappa}{1 + \beta^{\frac{1}{\rho}}}$$

Substituting this into the first yields the following no-renegotiation condition which will be identical in the period 1 monopoly and competitive case:

$$u(c_1^0) + \beta u(c_2^0) \geq (1 + \beta^{\frac{1}{\rho}}) u\left(\frac{c_1^0 + c_2^0 - \kappa}{1 + \beta^{\frac{1}{\rho}}}\right)$$

Alternatively, this condition can be written:

$$\left(c_1^0 + c_2^0\right) - \left((c_1^0)^{1-\rho} + \beta (c_2^0)^{1-\rho}\right)^{\frac{1}{1-\rho}} \left(1 + \beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}} \leq \kappa \tag{49}$$

− − − − − − − − −

9.3 Renegotiation

In both cases we are assuming that κ is not large enought to eliminate the gains to trade.

An alternate way to restate the above is the following: Let s and α be defined such that $c_1 = \alpha s$ and $c_2 = (1 - \alpha) s$. Then:

$$C_1^{m1}\left(C_1^0\right) = \left(s, s\beta^{\frac{1}{\rho}}\right) \cdot \left(\frac{\alpha^{1-\rho} + \beta (1-\alpha)^{1-\rho}}{1 + \beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}} \tag{50}$$

Profit gains from renegotiation become:

$$\Pi_1\left(C_1^{m1}(C_1); C_1\right) = (s)\left(1 - \left(\alpha^{1-\rho} + \beta (1-\alpha)^{1-\rho}\right)^{\frac{1}{1-\rho}}\right) \tag{51}$$

By construction, profits from renegotiation are strictly positive (and increasing in s), except in the special case where C_1 is optimal from period 1's perspective ($(1-\alpha) = \beta^{\frac{1}{\rho}}\alpha$), in which case they are 0. It can also easily be confirmed that profits from renegotiation fall in α as long as the allocation is such that period 1 would like a larger α than the current contract offers.

Proof of Proposition 1: (a) In any full-commitment contract, $\alpha = \frac{1}{2}$. Inserting this into (51), the following must be satisfied for a full-commitment contract to survive:

$$k \geq s\left(1 - \frac{1}{2}\left(1+\beta\right)^{\frac{1}{1-\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}}\right) \tag{52}$$

Substituting for $s = y - c_0^F$ from the competitive full-commitment contract, we can rewrite the above condition as:

$$k \geq \frac{2\beta^{\frac{1}{\rho}}y}{1+2\beta^{\frac{1}{\rho}}}\left(1 - \frac{1}{2}\left(1-\beta\right)^{\frac{1}{1-\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}}\right) \equiv \bar{\kappa} \tag{53}$$

Since in any full-commitment contract (monopoly and competition), s will be no larger than $\frac{2\beta^{\frac{1}{\rho}}y}{1+2\beta^{\frac{1}{\rho}}}$, no full-commitment contract will be renegotiated if $\kappa \geq \bar{\kappa}$.

(c) If $\kappa < \bar{\kappa}$, condition 53 fails, so the competitive full-commitment contract cannot survive.

(b) The monopolist full-commitment contract will not survive if s is sufficiently large. Since s^{mF} is exponentially (and therefore monotonically) increasing in U_0^A, there must be some U^m such that the contract will not survive if and only if $U_0^A > U^m$. Since the contract cannot survive at U_0^F (here, the contract is identical to the competitive contract), $U^m < U_0^F$.

□

9.4 Renegotiation-Proof Contracts

9.4.1 Sophisticated Hyperbolic Discounters

When the renegotiation-proofness constraint binds, consumption in periods 1 and 2 must satisfy:

$$(s)\left(1 - \left(\alpha^{1-\rho} - \beta\left(1-\alpha\right)^{1-\rho}\right)^{\frac{1}{1-\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}}\right) = \kappa \tag{54}$$

For any s, there may be two values of α that satisfy the constraint with equality–one with too little consumption relative to period 1's optimal, one with too much consumption relative to period 1's optimal. The relevant value for us is the first. This defines a continuous function $\alpha(s)$.

$$\alpha(s) = min\left\{\alpha : (s)\left(1 - \left(\alpha^{1-\rho} + \beta\left(1-\alpha\right)^{1-\rho}\right)^{\frac{1}{1-\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{-\rho}{1-\rho}}\right) = \kappa\right\} \tag{55}$$

As s rises, to continue satisfying the constraint we must have $\alpha(s)$ rising too (if fractions stayed constant, profits from renegotiation would rise).

We can also rewrite the first-order condition of the bank's maximization problem using the new notation. For any s and α, let $V(s,\alpha) = u(\alpha s) + u((1-\alpha)s)$. This is the discounted utility over periods 1 and 2, from period 0's perspective. The solution, $C_0 = (c_0, \alpha s, (1-\alpha)s)$, must satisfy:

$$\frac{du(c_0)}{dc_0} = \beta \frac{dV(s,\alpha)}{ds} \tag{56}$$

In other words, at the profit-maximizing contract the marginal dollar should be equally valuable whether consumed immediately or distributed across future periods.

Proof of Proposition 2: (a) Since the full-commitment profit-maximizing contract was uniquely determined, and since it does not satisfy the renegotiation-proofness constraint, the renegotiation-proof contract must yield lower profits than the full-commitment contract does.

(b) Clearly, $\Pi_0\left(C_0^{mP}; Y_0\right)$ falls strictly in U_0^A (if autarky utility falls, the bank can always do better, at least by simply lowering c_0). Since at $U_0^A = U^m$, $\Pi_0\left(C_0^{mP}; Y_0\right) = \Pi_0\left(C_0^{mF}; Y_0\right) > 0$ and at $U_0^A = U_0^F$, $\Pi_0\left(C_0^{mP}; Y_0\right) < \Pi_0\left(C_0^{mF}; Y_0\right) = 0$, there must be some intermediate autarky utility above which the bank's maximized profits will be negative.

(c) consider any $c_0 \leq c_0^{mF}$ and s such that $U_0\left(c_0, \frac{s}{2}, \frac{s}{2}\right) = U_0^A$. We can find the corresponding $\bar{s}$ that, while satisfying the participation constraint, gives the same utility from period 0's perspective:

$$V\left(s, \frac{1}{2}\right) = \bar{V}\left(\bar{s}, \alpha(\bar{s})\right) \tag{57}$$

$$\Rightarrow 2\frac{\left(\frac{1}{2}s\right)^{1-\rho}}{1-\rho} = \frac{\left(\alpha(\bar{s})\bar{s}\right)^{1-\rho}}{1-\rho} + \frac{\left((1-\alpha(\bar{s}))\bar{s}\right)^{1-\rho}}{1-\rho} \tag{58}$$

$$\Rightarrow \bar{s} = s\left(\frac{2\left(\frac{1}{2}\right)^{1-\rho}}{\alpha(\bar{s})^{1-\rho} + (1-\alpha(\bar{s}))^{1-\rho}}\right)^{\frac{1}{1-\rho}} \tag{59}$$

From this, we get the following inequality:

$$\frac{dV(\bar{s},\alpha(\bar{s}))}{ds} = \bar{s}^{-\rho}\left(\alpha(\bar{s})^{1-\rho}+(1-\alpha(s))^{1-\rho}\right)+\frac{d\alpha(\bar{s})}{ds}\bar{s}^{1-\rho}\left(\alpha(\bar{s})^{-\rho}-(1-\alpha)^{-\rho}\right) \quad (60)$$

$$< \bar{s}^{-\rho}\left(\alpha(\bar{s})^{1-\rho}+(1-\alpha(s))^{1-\rho}\right) \quad (61)$$

$$= s^{-\rho}\left(2\left(\frac{1}{2}\right)^{1-\rho}\right)\left(\frac{2\left(\frac{1}{2}\right)^{1-\rho}}{\alpha(\bar{s})^{1-\rho}+(1-\alpha(\bar{s}))^{1-\rho}}\right)^{\frac{-1}{1-\rho}} \quad (62)$$

$$< s^{-\rho}\left(2\left(\frac{1}{2}\right)^{1-\rho}\right) = \frac{dV\left(s,\frac{1}{2}\right)}{ds} \quad (63)$$

The first line above splits the effect of s on V into two–the first term represents the change in utility holding α constant, and the second term represents the (negative) effect of the further skewing of consumption that results from a rise in s. The final inequality follows from the fact that, since $\bar{s} > s^{mF}$, the renegotiation constraint must bind so that $\alpha(\bar{s}) > \frac{1}{2}$. Finally, the following inequality holds:

$$\frac{du(c_0)}{dc_0} \geq \frac{du\left(c_0^{mF}\right)}{dc_0} = \beta\frac{dV\left(s^{mF},\frac{1}{2}\right)}{ds} \quad (64)$$

$$\geq \beta\frac{dV\left(s,\frac{1}{2}\right)}{ds} > \beta\frac{dV(\bar{s},\alpha(\bar{s}))}{ds} \quad (65)$$

We have shown that at any $c_0 \leq c_0^{mF}$, for a contract that satisfies the renegotiation-proofness constraint, the marginal utility of period 0 consumption will be higher than the discounted marginal utility of future consumption, so the bank could earn strictly higher profits by raising c_0 and lowering s further. Therefore, in the renegotiation-proof contract, $c_0^{mP} > c_0^{mF}$. □

Proof of Proposition 3: (a) We know that $U_0\left(C_0^F\right) = U_0^F$. By assumption, since the renegotiation-proofness constraint is binding, the renegotiation-proof contract cannot offer the optimal consumption path. Therefore $U_0\left(C_0^P\right) < U_0\left(C_0^F\right)$.

(b) Consider $\bar{U}$, as constructed in Proposition 2. If $U_0^A > \bar{U}$, it is impossible to construct a contract that earns nonnegative profits and gives the consumer at least autarky utility. Therefore, any contract that earns zero profits would give the period 0 consumer less than autarky utility. (As an aside, observe that $\bar{U} = U_0\left(C_0^P\right)$.)

(c) At the full-commitment contract:

$$\frac{du\left(c_0^F\right)}{dc} = \beta\frac{dV\left(s^F,\frac{1}{2}\right)}{ds} = \left(s^F\right)^{-\rho}\left(2\left(\frac{1}{2}\right)^{1-\rho}\right) \quad (66)$$

Consider a renegotiation-proof contract with $c_0 = c_0^F$. To keep bank profits zero, this contract would also have $s = s^F$. But in the renegotiation-proof contract, s must be divided according to the fraction $\alpha\left(s^F\right)$. So:

$$\begin{aligned} \frac{dV\left(s^F, \alpha\left(s^F\right)\right)}{ds} &= \left(s^F\right)^{-\rho}\left(\alpha\left(s^F\right)^{1-\rho} + \left(1-\alpha\left(s^F\right)\right)^{1-\rho}\right) \\ &+ \frac{d\alpha\left(s^F\right)}{ds}\left(s^F\right)^{1-\rho}\left(\alpha\left(s^F\right)^{-\rho} - \left(1-\alpha\left(s^F\right)\right)^{-\rho}\right) \end{aligned} \tag{67}$$

The first term–the direct effect of a change in s–is weakly less than $\frac{dV\left(s^F, \frac{1}{2}\right)}{ds}$ if $\rho \leq 1$ and strictly greater if $\rho > 1$. The second term–the component of $\frac{dV}{ds}$ that is driven by the change in α–is strictly negative. Therefore, if $\rho < 1$, $\frac{dV\left(s^F, \alpha\left(s^F\right)\right)}{ds} < \frac{dV\left(s^F, \frac{1}{2}\right)}{ds} = \frac{du\left(c_0^F\right)}{dc}$, so the renegotiation-proof contract must satisfy $c_0^P > c_0^F$.

Next, we consider the case when $\rho > 1$. We can make the following observations about $\alpha(s)$. First, $\lim_{\kappa\to 0}\alpha(s) = \frac{\beta^{\frac{-1}{\rho}}}{1+\beta^{\frac{-1}{\rho}}}$ (this follows from the fact that at $\kappa = 0$, the contract must satisfy $u'(c_1) = \beta u'(c_2)$). Second, implicitly differentiating equation 55 with respect to s, and combining it with the previous limit result, we get $\lim_{\kappa\to 0}\frac{d\alpha(s)}{ds} = 0$. Therefore, if $\rho > 1$ and κ is small enough, the second term in Equation 67 will be sufficiently small in magnitude that $\frac{dV\left(s^F, \alpha\left(s^F\right)\right)}{ds} > \frac{dV\left(s^F, \frac{1}{2}\right)}{ds} = \frac{du\left(c_0^F\right)}{dc}$. In this case, the renegotiation-proof contract must satisfy $c_0^P < c_0^F$. $\square$

If $\kappa = 0$, the renegotiation-proof contracts can be explicitly derived since in any contract it must be true that $c_2 = \beta^{\frac{1}{\rho}} c_1$. Solving the respective maximization problems, we get the following equilibrium contracts for monopoly and competition, respectively:

$$C_0^{mP} = \left(\left(\frac{U_0^A(1-\rho)}{1+\beta^{\frac{1}{\rho}}\left(\frac{\left(1+\beta^{\frac{1-\rho}{\rho}}\right)^{\frac{1}{\rho}}}{\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{1-\rho}{\rho}}}\right)}\right)^{\frac{1}{1-\rho}}, \left(\frac{\beta+\beta^{\frac{1}{\rho}}}{1+\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}} c_0^{mP}, \beta^{\frac{1}{\rho}}\left(\frac{\beta+\beta^{\frac{1}{\rho}}}{1+\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{\rho}} c_0^{mP}\right) \tag{68}$$

$$C_0^P = \left(\frac{y}{1+\beta+\beta^{\frac{1}{\rho}}}, \left(\frac{\beta+\beta^{\frac{1}{\rho}}}{1+\beta^{\frac{1}{\rho}}}\right) c_0^P, \beta^{\frac{1}{\rho}}\left(\frac{\beta+\beta^{\frac{1}{\rho}}}{1+\beta^{\frac{1}{\rho}}}\right) c_0^P\right) \tag{69}$$

It can easily be established that $c_0^{mP} > c_0^{mF}$, $c_0^P > c_0^{mF}$ if $\rho > 1$, and $c_0^P < c_0^{mF}$ if $\rho < 1$.

9.4.2 Naive Hyperbolic Discounters

Suppose the monopolist intends to renegotiate the contract. The maximization problem, combined with the expression for $C_1^{m1}(C_1)$ (50), simplifies to:

$$\underset{c_0,c_1,c_2}{max}\, y - c_0 - \frac{\left(c_1^{1-\rho} + \beta c_2^{1-\rho}\right)^{\frac{1}{1-\rho}}}{\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{\rho}{1-\rho}}} - \kappa \tag{70}$$

$$s.t. \frac{c_0^{1-\rho}}{1-\rho} + \beta\frac{c_1^{1-\rho}}{1-\rho} + \beta\frac{c_2^{1-\rho}}{1-\rho} \geq U_0^A \tag{71}$$

The partial derivatives of the resulting Lagrangian are:

$$\frac{\partial\mathcal{L}}{\partial c_0} = -1 - \lambda c_0^{-\rho} \tag{72}$$

$$\frac{\partial\mathcal{L}}{\partial c_1} = c_1^{-\rho}\left[-\left(\frac{c_1^{1-\rho}+\beta c_2^{1-\rho}}{1+\beta^{\frac{1}{\rho}}}\right)^{\frac{\rho}{1-\rho}} - \lambda\beta\right] \tag{73}$$

$$\frac{\partial\mathcal{L}}{\partial c_2} = c_2^{-\rho}\left[-\beta\left(\frac{c_1^{1-\rho}+\beta c_2^{1-\rho}}{1+\beta^{\frac{1}{\rho}}}\right)^{\frac{\rho}{1-\rho}} - \lambda\beta\right] \tag{74}$$

An interior solution, with $\frac{\partial\mathcal{L}}{\partial c_1} = 0$ and $\frac{\partial\mathcal{L}}{\partial c_2} = 0$ does not exist (on a $c_1 - c_2$ plot, the two first-order conditions do not intersect). If $\rho < 1$, the Lagrangian is maximized at a corner solution with $c_1 = 0$. If $\rho > 1$, the Lagrangian is maximized at the limit as c_2 approaches infinity. Using this, the maximization problem can be re-solved. If $\rho < 1$:

$$C_0^{mN} = \left(\left(\frac{U_0^A(1-\rho)}{2+\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}}, 0, \left(\frac{1+\beta^{\frac{1}{\rho}}}{\beta}\right)^{\frac{1}{1-\rho}}\left(\frac{U_0^A(1-\rho)}{2+\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}}\right) \tag{75}$$

If $\rho > 1$, the solution is undefined, but in the limit is given by:

$$C_0^{mN} = \left(\left(\frac{U_0^A(1-\rho)}{1+\left(1+\beta^{\frac{1}{\rho}}\right)\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}}, \beta^{\frac{1}{\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{1}{1-\rho}}\left(\frac{U_0^A(1-\rho)}{1+\left(1+\beta^{\frac{1}{\rho}}\right)\beta^{\frac{1}{\rho}}}\right)^{\frac{1}{1-\rho}}, \infty\right) \tag{76}$$

Proof of Proposition 4: (a) At any autarky utility, the monopolist could at least offer the full-commitment contract.

(b) The bank must choose between a renegotiation-proof contract and a renegotiable contract (75, 76). By construction of U^m, the following must be true at any $U_0^A \geq U^m$:

$$\Pi_0\left(C_0^{mP}; Y_0\right) \leq \Pi_0\left(C_0^{mF}; Y_0\right) \leq \Pi_0\left(C_0^{mF}; Y_0\right) + \Pi_1\left(C_1^{m1}\left(C_1^{mF}\right); C_1^{mF}\right) - \kappa \tag{77}$$

Since C_0^{mN} is uniquely determined and $C_0^{mN} \neq C_0^{mF}$, profits from the best renegotiable contract must be strictly higher than profits from the renegotiation-proof contract at any $U_0^A \geq U^m$.

The following can be verified from the explicit derivations of C_0^{mF} and C_0^{mN}. First, if U_0^A is sufficiently small, $\Pi_0\left(C_0^{mF}; Y_0\right) > \Pi\left(C_0^{mN}; Y_0\right) + \Pi_1\left(C_1^{m1}\left(C_1^{mN}\right); C_1^{mN}\right) - \kappa$. Second,

$$\frac{d}{dU_0^A}\Pi_0\left(C_0^{mF}; Y_0\right) > \frac{d}{dU_0^A}\left[\Pi\left(C_0^{mN}; Y_0\right) + \Pi_1\left(C_1^{m1}\left(C_1^{mN}\right); C_1^{mN}\right) - \kappa\right] \tag{78}$$

It follows that there is some $U^N < U^m$ such that if $U_0^A \leq U^N$, the naive agent will receive the monopoly full commitment contract, which will be renegotiation-proof.

(c) This can be confirmed from the explicit formulations of C_0^{mF} (44) and C_0^{mN} (75, 76). □

We now derive equilibrium contracts for naive consumers under perfect competition. Suppose contracts are exclusive. Then, a contract that is renegotiated satisfies:

$$\max_{c_0, c_1, c_2} \frac{c_0^{1-\rho}}{1-\rho} + \beta\frac{c_1^{1-\rho}}{1-\rho} + \beta\frac{c_2^{1-\rho}}{1-\rho} \tag{79}$$

$$s.t. y - c_0 - \frac{\left(c_1^{1-\rho} + \beta c_2^{1-\rho}\right)^{\frac{1}{1-\rho}}}{\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{\rho}{1-\rho}}} - \kappa \geq 0 \tag{80}$$

The first-order conditions are the same as under monopoly (72, 73, 74). Combining these with the zero-profit constraint, we get the following solution. If $\rho < 1$:

$$C_0^N = \left(\frac{y-\kappa}{2+\beta^{\frac{1}{\rho}}}, 0, \left(\frac{1+\beta^{\frac{1}{\rho}}}{\beta}\right)^{\frac{1}{1-\rho}}\left(\frac{y-\kappa}{2+\beta^{\frac{1}{\rho}}}\right)\right) \tag{81}$$

If $\rho > 1$, the solution is undefined, but in the limit is given by:

$$C_0^N = \left(\frac{y-\kappa}{1+\beta^{\frac{1}{\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)}, \beta^{\frac{1}{\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)^{\frac{1}{1-\rho}}\left(\frac{y-\kappa}{1+\beta^{\frac{1}{\rho}}\left(1+\beta^{\frac{1}{\rho}}\right)}\right), \infty\right) \tag{82}$$

Proof of Proposition 5: (a) Banks can at least offer the consumer the full-commitment contract, so a contract is feasible at any autarky utility. Since, from the consumer's perspective, any renegotiation-proof contract is strictly dominated by the full-commitment contract (which will be renegotiated), in equilibrium she will be offered a contract that will be renegotiated.

(b) Under non-exclusive contracts, firms offering period 0 contracts do not benefit from renegotiation (profits from renegotiation will equal κ). So the equilibrium contract is identical to the full-commitment contract.

(c) Suppose $\rho < 1$. Comparing C_0^F (**??**) to C_0^N (81), it is clear that $c_0^N < c_0^F$. Suppose $\rho > 1$. If κ is small enough, $c_0^N > c_0^F$. $\square$

9.5 Nonprofits

Lemma 1. $\Pi_0\left(C_0^{mP};Y_0\right)$ *and* $\bar{\alpha}\Pi_0\left(C_0^{m\alpha};Y_0\right)$ *are continuously decreasing in* U_0^A.

Proof of Lemma 1: We prove the above for renegotiation-proof contracts of for-profit banks. The same argument applies to nonprofit banks. First, it is clear that profits are strictly decreasing in U_0^A: If autarky utility drops from $U_0^A = U$ to $U_0^A = \bar{U}$, at $\bar{U}$ the bank can always do better than offering the contract it offered at U.

Next, we prove right-continuity at any $U_0^A = U$. Let the maximized profits at U be $\Pi_0\left(C_0;Y_0\right)$, where $C_0 = \left(c_0,c_1,c_2\right)$. This contract must satisfy $U_0\left(C_0\right) = U$. For any $\bar{U} > U$, profits must be lower, and bounded below by $\Pi_0\left(\bar{C}_0;Y_0\right)$, with the contract defined as $\bar{C}_0 = \left(c_0+x,c_1,c_2\right)$ where x satisfies $U_0\left(\bar{C}_0\right) = \bar{U}$. Since $\underset{\bar{U}\to U^+}{lim}\Pi_0\left(\bar{C}_0;Y_0\right) = \Pi_0\left(C_0;Y_0\right)$, the profit function is right-continuous.

Finally, we prove left-continuity at at any $U_0^A = U$. For any $\bar{U} < U$, denote maximized profits $\Pi_0\left(\bar{C}_0;Y_0\right)$, where $\bar{C}_0 = \left(\bar{c}_0,\bar{c}_1,\bar{c}_2\right)$. These contracts must satisfy $U_0\left(\bar{C}_0\right) = \bar{U}$. At U, profits must be lower, and bounded below by $\Pi_0\left(C_0;Y_0\right)$, with the contract defined as $C_0 = \left(\bar{c}_0+x,\bar{c}_1,\bar{c}_2\right)$ where x satisfies $U_0\left(C_0\right) = U$. Since $\underset{\bar{U}\to U^-}{lim}\Pi_0\left(\bar{C}_0;Y_0\right) = \Pi_0\left(C_0;Y_0\right)$, the profit function is left-continuous. $\square$

Proof of Proposition 6: Let $\eta\left(\bar{\alpha}\right) = \kappa$, to minimize the attractiveness of the nonprofit. Consider $U_0^A = \bar{U}$. Since the for-profit's renegotiation-proofness binds and leaves the firm with zero profits, and since the non-profit's renegotiation-proofness constraint is looser than the for-profit's, we know that $\bar{\alpha}\Pi\left(C_0^{m\alpha};Y_0\right) > 0 = \Pi\left(C_0^{mP};Y_0\right)$. Since profits must be continuously decreasing in U_0^A, and since $\bar{\alpha}\Pi\left(C_0^{m\alpha};Y_0\right) < \Pi\left(C_0^{mP},;Y\right)$ at $U_0^A = U^m$ (where the for-profit's renegotiation-proofness constraint no longer binds) and $\bar{\alpha}\Pi\left(C_0^{m\alpha};Y_0\right) \leq 0$ at $U_0^A = U_0^F$, there must exist autarky utility values as described in the proposition statement such that, if $U_0^A > \underline{U}^\alpha$ the bank strictly prefers to operate as a nonprofit relative to a for-profit, and if $U_0^A \geq \bar{U}^\alpha$ it weakly prefers to not offer a contract. $\square$

Proof of Proposition 7: (a) Suppose all firms are for-profit. There is some ε_1 and ε_2 satisfying $0 < \varepsilon_2 < \varepsilon_1$ and a corresponding $\hat{C}_0 = \left(c_0^\alpha, c_1^\alpha - \varepsilon_1, c_2^\alpha + \varepsilon_2\right)$ such that $U_0\left(C_0^\alpha\right) =$

$U_0\left(\hat{C}_0\right)$ and $\Pi_0(C_1^{m1}\left(\hat{C}_1\right);\hat{C}_1) \leq \kappa(\bar{\alpha}) < \kappa(1)$. So, any firm can make positive profits by operating as a non-profit. Therefore, in equilibrium, consumers will borrow only from non-profit firms. Given the construction of $\bar{U}^{\alpha}$, firms can make nonnegative profits while satisfying the participation constraint only if $U_0^A \leq \bar{U}^{\alpha}$.

(b) If all firms are nonprofit, an individual firm has a strict incentive to switch to for-profit status, and make profits in period 1. Therefore, there must be for-profits in equilibrium, and equilibrium contracts will be constrained by their presence. □

References

Akerlof, George A., & Shiller, Robert J. 2015. *Phishing for phools: the economics of manipulation and deception*. Princeton: Princeton University Press.

Amador, Manuel, Werning, IvÃ¡n, & Angeletos, George-Marios. 2006. Commitment vs. Flexibility. *Econometrica*, **74**(2), 365–396.

Ariely, D., & Wertenbroch, K. 2002. Procrastination, Deadlines, and Performance: Self-Control by Precommitment. *Psychological Science*, **13**(3), 219–224.

Armstrong, Mark, & Vickers, John. 2012. Consumer Protection and Contingent Charges. *Journal of Economic Literature*, **50**(2), 477–493.

Arrow, Kenneth J. 1963. Uncertainty and the Welfare Economics of Medical Care. *American Economic Review*, **53**(5), 941–973.

Ashraf, N., Karlan, D., & Yin, W. 2006. Tying Odysseus to the Mast: Evidence From a Commitment Savings Product in the Philippines. *The Quarterly Journal of Economics*, **121**(2), 635–672.

Basu, Karna. 2014. Commitment savings in informal banking markets. *Journal of Development Economics*, **107**(Mar.), 97–111.

Bauer, Michal, ChytilovÃ¡, Julie, & Morduch, Jonathan. 2012. Behavioral Foundations of Microcredit: Experimental and Survey Evidence from Rural India. *American Economic Review*, **102**(2), 1118–1139.

Besley, Timothy, & Ghatak, Maitreesh. 2005. Competition and Incentives with Motivated Agents. *American Economic Review*, **95**(3), 616–636.

Bond, Philip, Musto, David K., & Yilmaz, Bilge. 2009. Predatory mortgage lending. *Journal of Financial Economics*, **94**(3), 412–427.

Bryan, Gharad, Karlan, Dean, & Nelson, Scott. 2010. Commitment Devices. *Annual Review of Economics*, **2**(1), 671–698.

Bubb, Ryan, & Kaufman, Alex. 2013. Consumer biases and mutual ownership. *Journal of Public Economics*, **105**(Sept.), 39–57.

Conning, Jonathan, & Morduch, Jonathan. 2011. Microfinance and Social Investment. *Annual Review of Financial Economics*, **3**(1), 407–434.

Cull, Robert, Demirguc-Kunt, Asli, & Morduch, Jonathan. 2009. Microfinance Meets the Market. *Journal of Economic Perspectives*, **23**(1), 167–192.

Demyanyk, Yuliya, & Van Hemert, Otto. 2011. Understanding the Subprime Mortgage Crisis. *Review of Financial Studies*, **24**(6), 1848–1880.

Dewatripont, Mathias, & Tirole, Jean. 1999. *The Prudential Regulation of Banks*. 2. print edn. The Walras-Pareto lectures, no. 1. Cambridge, Mass.: MIT Press.

DeYoung, et al. 2015. Reframing the Debate about Payday Lending. *Liberty Street Economics*, Oct.

Gabaix, X., & Laibson, D. 2006. Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets. *The Quarterly Journal of Economics*, **121**(2), 505–540.

Glaeser, Edward L., & Shleifer, Andrei. 2001. Not-for-profit entrepreneurs. *Journal of Public Economics*, **81**(1), 99–115.

Gottlieb, Daniel. 2008. Competition over Time-Inconsistent Consumers. *Journal of Public Economic Theory*, **10**(4), 673–684.

Gugerty, Mary Kay. 2007. You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya. *Economic Development and Cultural Change*, **55**(2), 251–282.

Guha, Brishti, & Roy Chowdhury, Prabal. 2013. Micro-finance competition: Motivated micro-lenders, double-dipping and default. *Journal of Development Economics*, **105**(Nov.), 86–102.

Hansmann, Henry. 1996. *The ownership of enterprise*. 1. harvard univ. press paperback ed edn. Cambridge, MA: Harvard University Press.

Heidhues, Paul, & Koszegi, Botond. 2010. Exploiting Naivete about Self-Control in the Credit Market. *American Economic Review*, **100**(5), 2279–2303.

Laibson, David, Repetto, Andrea, & Tobacman, Jeremy. 2003. A Debt Puzzle. *Pages 228–266 of:* Aghion, Philippe, Frydman, Roman, Stiglitz, Joseph, & Woodford, Michael (eds), *Knowledge, Information, and Expectations in Modern Economics: In Honor of Edmund S. Phelps.* Princeton, NJ: Princeton University Press.

McIntosh, Craig, & Wydick, Bruce. 2005. Competition and microfinance. *Journal of Development Economics*, **78**(2), 271–298.

Mendez, Rodrigue. 2012. Predatory Lending: A Model of Behavioral Pricing. *Paris School of Economics working paper.*

Oak, Mandar, & Swamy, Anand. 2010. Only Twice as Much: A Rule for Regulating Lenders. *Economic Development and Cultural Change*, **58**(4), 775–803.

O'Donoghue, T., & Rabin, M. 2001. Choice and Procrastination. *The Quarterly Journal of Economics*, **116**(1), 121–160.

Rhyne, Elisabeth. 2011. On Microfinance: Who's to Blame for the Crisis in Andhra Pradesh? *The Huffington Post*, May.

Tanaka, Tomomi, Camerer, Colin F, & Nguyen, Quang. 2010. Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam. *American Economic Review*, **100**(1), 557–571.

Thaler, Richard H., & Benartzi, Shlomo. 2004. Save more tomorrow: Using behavioral economics to increase employee saving. *Journal of Political Economy*, **112**(S1), S164–S187.