

Wealth, Wages, and Employment

Preliminary

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Joint European Central Bank/Centre for Economic Policy Research Workshop
Household heterogeneity in macroeconomics
February 1, 2019

INTRODUCTION

- We want a real-world theory of the joint distribution of employment, wages, and wealth.
 - Workers are risk averse, only use self-insurance.
 - The employment and wage risk is endogenous.
 - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
 - Business cycles can be studied.
- Such a framework does not exist in the literature.
 1. Requires heterogeneous agents.
 2. No (search-matching) closed form solutions possible.
 3. Wage formation? Nash bargaining not very promising:
 - A bargaining problem where wages become a(n increasing) function of worker wealth.
 - Not time-consistent and bargaining with commitment makes no sense.
 - Not numerically well-behaved.
- We offer an alternative: competitive job search with commitment to a wage while the job lasts.

- At its core is [Aiyagari \(1994\)](#) meets [Moen \(1997\)](#).
- Developing empirically sound versions of these ideas compels us to
 - Add extreme value shocks to transform decision rules from functions into densities to weaken the correlation between states and choices.
 - Pose quits, on the job search, and explicit role for leisure so quitting is not only to search for better jobs
 - Use new potent tools to address the study of fluctuations in complicated economies [Boppart, Krusell, and Mitman \(2018\)](#)
- Related to [Lise \(2013\)](#), [Hornstein, Krusell, and Violante \(2011\)](#), [Krusell, Mukoyama, and Şahin \(2010\)](#), [Ravn and Sterk \(2016, 2017\)](#), [Den Haan, Rendahl, and Riegler \(2015\)](#).
- Especially, [Eeckhout and Sepahsalari \(2015\)](#), [Chaumont and Shi \(2017\)](#), [Griffy \(2017\)](#).

WHAT ARE THE USES?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.

TODAY: DISCUSS VARIOUS MODEL INGREDIENTS & FLUCTUATIONS

1. **Basic:** Exogenous Destruction, no Quits. Built on top of Growth Model. (GE versin of [Eeckhout and Sepahsalari \(2015\)](#)): But No outside the labor force, not a lot of wage dispersion.
2. **Quits:** Higher wage dispersion may arise to keep workers longer. (Endogenous quits via extreme value shocks). But Wealth trumps wages and wage dispersion collapses.
3. **Aiming Shocks** Weakens but does not destroy the dependency of wages on wealth. Larger Wage Dispersion.
4. **Aiming and Quitting Shocks** Stronger Wage dispersion. **Fluctuations.** Proccyclical quits.
5. **On the Job Search** workers may get outside offers and take them. (Some in [Chaumont and Shi \(2017\)](#)). **Fluctuations.** Excessive Quitting.
6. **Multiple types** Workers differ in the value of leisure, i.e. attachment to the labor market. Explicit role of Outside Labor Force. Under development.

BASIC: PRECAUTIONARY SAVINGS, COMPETITIVE SEARCH

- Jobs are created by firms (plants). A plant with capital plus a worker produce one (z) unit of the good.
 - Firms pay flow cost \bar{c} to post a vacancy in market $\{w, \theta\}$. Cannot change wage afterwards.
 - Plants (and their capital) are destroyed at rate δ . Workers will not want to quit (for now).
- Households differ in wealth and wages (if working). There are no state contingent claims, nor borrowing.
 - If employed, workers get w and save.
 - If unemployed, workers produce b and search in some $\{w, \theta\}$.
- General equilibrium: Workers own firms.

ORDER OF EVENTS OF BASIC MODEL

1. Households enter the period with or without a job: $\{e, u\}$.
2. **Production & Consumption**: Employed produce z on the job. Unemployed produce b at home. They choose savings.
3. **Job Separation**: Some employed workers receive exogenous job destruction shocks at rate δ . They cannot search this period.
4. **Search**: Potential entrants and the unemployed choose wage w and market tightness θ .
5. **Job Matching** : Some vacancies meet some unemployed job searchers. A match becomes operational the following period. Job finding and job filling rates $\psi^h(\theta), \psi^f(\theta)$.

BASIC MODEL: HOUSEHOLD PROBLEM

- Individual state: wealth and wage
 - If employed: (a, w)
 - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a, w) = \max_{c, a'} u(c) + \beta [(1 - \delta)V^e(a', w) + \delta V^u(a)]$$

$$\text{s.t. } c + a' = a(1 + r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c, a', w} u(c) + \beta \{ \psi^h[\theta(w)] V^e(a', w) + [1 - \psi^h[\theta(w)]] V^u(a') \}$$

$$\text{s.t. } c + a' = a(1 + r) + b, \quad a \geq 0$$

$\theta(w)$ is an equilibrium object

FIRMS POST VACANCIES AT DIFFERENT WAGES & FILLING PROBABILITIES

- Value of a job with wage w : uses constant \bar{k} capital that depreciates

$$\Omega(w) = z - \bar{k}\delta_k - w + \frac{1 - \delta}{1 + r}\Omega(w)$$

- Affine in w : $\Omega(w) = (z - \bar{k}\delta_k - w)\frac{1+r}{r+\delta}$

Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm includes posting a vacancy: $\psi^f[\theta(w)] \Omega(w)$
- Free entry condition requires that for all offered wages

$$\bar{c} + \bar{k} = \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \frac{\bar{k}(1 - \delta_k)}{1+r},$$

BASIC MODEL: STATIONARY EQUILIBRIUM

- A stationary equilibrium is functions $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$, an interest rate r , and a stationary distribution x over (a, w) , s.t.
 1. $\{V^e, V^u, g'^e, g'^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.
 2. Zero profit condition holds for active markets

$$\bar{c} + \bar{k} = \psi^f[\theta(w)] \frac{\Omega(w)}{1+r}, \quad \forall w \text{ that are offered}$$

3. An interest rate r clears the asset market

$$\int a \, dx = \int \Omega(w) \, dx.$$

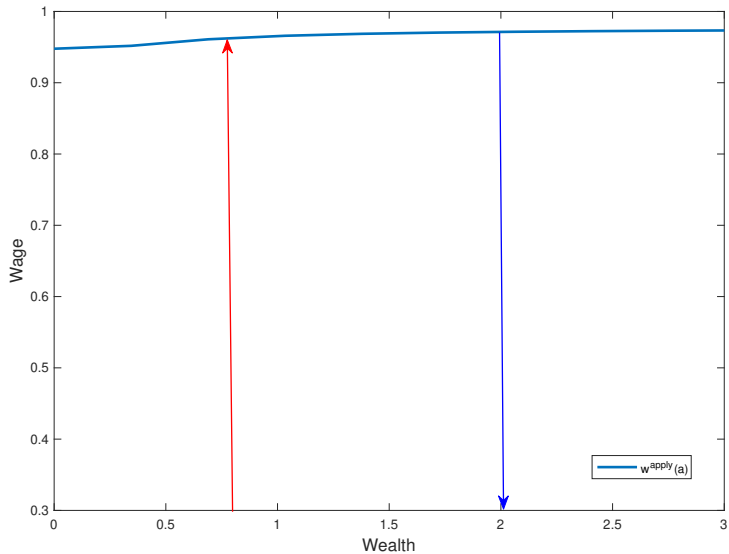
CHARACTERIZATION OF A WORKER'S DECISIONS

- Standard Euler equation for savings
- A F.O.C for wage applicants

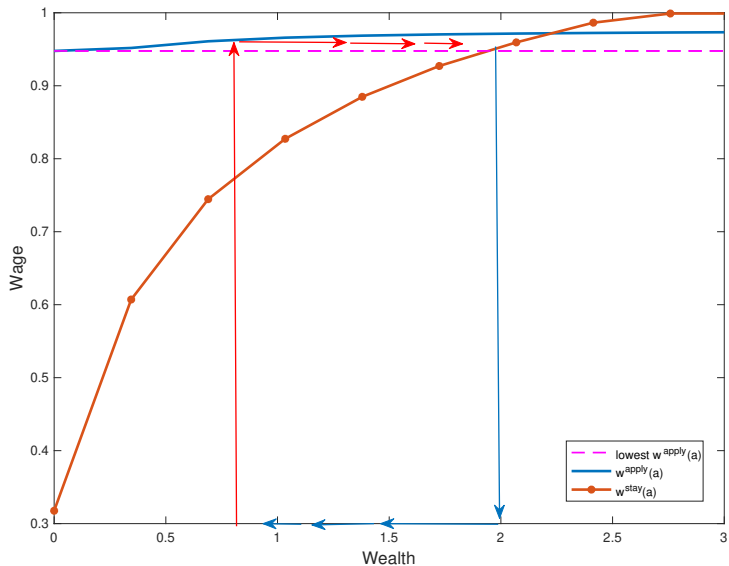
$$\psi^h[\theta(w)] V_w^e(a', w) = \psi_\theta^h[\theta(w)] \theta_w(w) [V^u(a') - V^e(a', w)]$$

- Households with more wealth are able to insure better against unemployment risk.
- As a result they apply for higher wage jobs and we have dispersion

HOW DOES THE MODEL WORK



HOW DOES THE MODEL WORK



1. Very Easy to Compute Steady-State with key Properties
 - i Risk-averse, only partially insured workers, endogenous unemployment
 - ii Can be solved with aggregate shocks too
 - iii Policy such as UI would both have insurance and incentive effects
 - iv Wage dispersion small—wealth doesn't matter too much
 - v ... so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings
2. In the following we will examine whether more wage dispersion obtains under additional assumptions –given that frictional wage dispersion is considered large in the data

1. Temporary Shocks to the utility of working or not working: Some workers quit.
2. Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.
3. Conditional on wealth, high wage workers quit less often.
4. But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and higher wages imply quit less often: Wage inequality collapses due to firms profit maximization.

QUITTING MODEL: TIME-LINE

1. Workers enters period with or without a job: $\{e, u\}$.
2. Production occurs and consumption/saving choice ensues:
3. Exogenous job/firm destruction happens.
4. **Quitting**: e draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision. Job losers cannot search this period.
5. **Search**: New or **Idle** firms post vacancies. Choose $\{w, \theta\}$. Wealth is not observable. (Unlike **Chaumont and Shi (2017)**). Not **Block Recursive** (It does not matter yet).
6. Matches occur

QUITTING MODEL: WORKERS

- Workers receive i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ to the utility of working or not the following period
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\} dF^\epsilon$$

V^e and V^u are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then \widehat{V} has a closed form and the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a, w) - V^u(a)]}}$$

higher $\alpha \rightarrow$ lower chance of quitting.

- Hence higher wages imply longer job durations. Firms could pay more to keep workers longer.

- Problem of the employed: just change \widehat{V}^e for V^e

$$V^e(a, w) = \max_{c, a'} u(c) + \beta \left[(1 - \delta) \widehat{V}^e(a', w) + \delta V^u(a) \right]$$

s.t. $c + a' = a(1 + r) + w, \quad a \geq 0$

- Problem of the unemployed is like before

QUITTING MODEL: VALUE OF THE FIRM

- Ω : Value of an idle firm, $\Omega^j(w)$: Value with with j -old worker. Free entry condition requires that for all offered wages

$$\bar{c} + \bar{k} = \frac{1}{1+r} \{ \psi^f[\theta(w)]\Omega^0(w) + [1 - \psi^f[\theta(w)]]\Omega \},$$

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$.
(One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q^e[g^{e,j}(a, w), w]$$

$g^{e,j}(a, w)$ savings rule of a j – tenured worker that was hired with wealth a

- Firm's value

$$\Omega^j(w) = z - \bar{k}\delta^k - w + \frac{1-\delta}{1+r} \{ \ell^j(w)\Omega^{j+1}(w) + [1 - \ell^j(w)] \Omega \}$$

QUITTING MODEL: SOLVING FORWARD FOR THE VALUE OF THE FIRM

$$\Omega^0(w) = (z - w - \delta_k k) Q^1(w) + (1 - \delta - \delta_k)k Q^0(w),$$

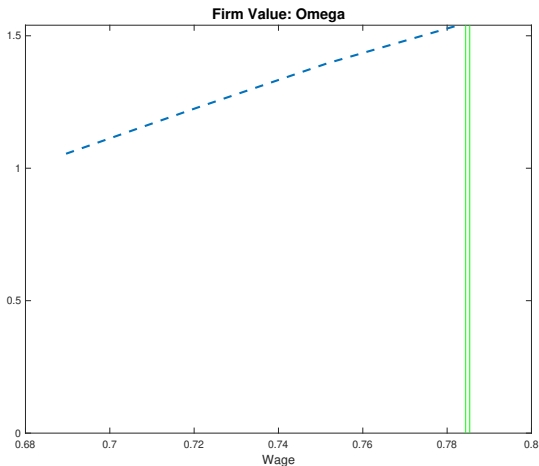
$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1 - \delta}{1 + r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1 - \delta}{1 + r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- New equilibrium objects $\{Q^0(w), Q^1(w)\}$. Rest is unchanged.
- It is Block Recursive, (even if contracts are not indexed by wealth (as in [Chaumont and Shi \(2017\)](#)) because wealth can be inferred from w).

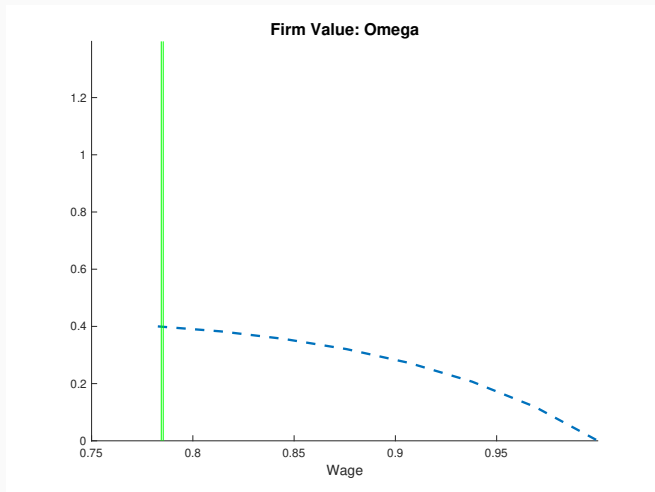
VALUE OF THE FIRM AS WAGE VARIES: THE POOR

- For the poorest, employment duration increases when wage goes up.
- Despite wage increases while output is fixed, firm value increases



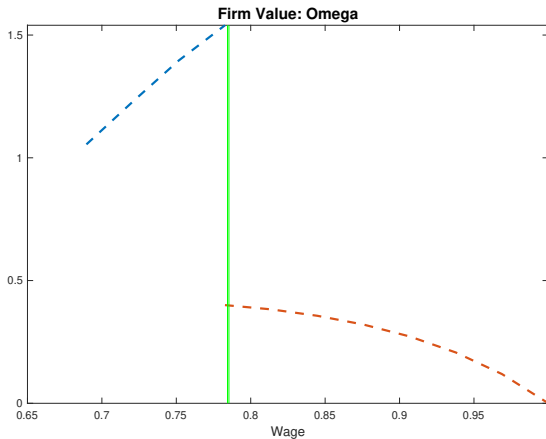
VALUE OF THE FIRM AS WAGE VARIES: THE RICH

- For the richest, employment duration increases but not fast enough.
- Firm value is decreasing in wages.



VALUE OF THE FIRM: ACCOUNTING FOR WORKER SELECTION

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion **COLLAPSES** due to selection.



EFFECT OF QUITTING: THE MECHANISM

- Two forces shape the dispersion of wages
 - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
 - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow and the effect of wealth dominates.
- Need to weaken link between wages and wealth

AIMING AND QUITTING SHOCKS MODEL: TIME-LINE

1. Workers enter period with or without a job: $\{e,u\}$. V^e, V^u defined here.
2. Production & Consumption:
3. Exogenous Separation.
4. Quitting $\widehat{V}^e(a', w)$, determined here.
5. **Search**: Firms choose $\{w, \theta\}$. The unemployed assesses the value of all wage applying options, receive match specific **aiming** shocks $\{\epsilon^{w'}\}$ and choose the wage level w' to apply. Those who successfully find jobs become e' , otherwise become u' .
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Matching

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^u(a') + \epsilon^{w'} \right] dF^\epsilon$$

- $h(w'; a')$ is now the logit choice density of wage for wealth level a'

$$h(w'; a') = \frac{\exp \{ \alpha^w [\psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a')] \}}{\int \exp \{ \alpha^w [\psi^h(\tilde{w}) V^e(a, \tilde{w}) + (1 - \psi^h(\tilde{w})) V^u(a')] \} d\tilde{w}}$$

no longer FOC for which wage to apply.

- After saving, the employed choose whether to quit as before

$$\widehat{V}^e(a', w) = \int \max \{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u \} dF^\epsilon$$

$V^e(a, w)$ and $V^u(a)$ are as before beginning of period values.

- The employed solve

$$V^e(a, w) = \max_{c, a' \geq 0} u(c) + \beta \left[(1 - \delta) \widehat{V}^e(a', w) + \delta V^u(a') \right]$$

s.t. $c + a' = a(1 + r) + w$

- The unemployed face the problem

$$V^u(a) = \max_{c, a' \geq 0} u(c) + \beta \widehat{V}^u(a')$$

s.t. $c + a' = a(1 + r) + b$

AIMING AND QUITTING SHOCKS MODEL: VALUE OF THE FIRM

- The value of the firm is again given like in the **Quitting** Model

$$\Omega^0(w) = (z - w - \delta_k k) Q^1(w) + (1 - \delta - \delta_k)k Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$
$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

$$\ell^j(w) = \int \{1 - q[g^{e,j}(a, w), w]\} h(w; a) dx^u(a)$$

- Explicitly **Not Block Recursive** unless contracts were indexed by wealth which is illegal.

- Higher wage dispersion
- Weaker but positive correlation between wage and wealth when hired
- Smooth firm problem: Firm value $\Omega^0(w)$ has no sharp drop due to composition
- Rich unemployed apply for higher wages (on average)
- But have more dispersion in its applications as utility differentials are lower

On the Job Search

ON THE JOB SEARCH MODEL: TIME-LINE

1. Workers enter period with or without a job: V^e, V^u .
2. Production & Consumption:
3. Exogenous Separation
4. **Quitting? Searching? Neither?:** Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become u' , those who search join the u , in case of finding a job become $\{e', w'\}$ but in case of no job finding remain e' with the same wage w and those who neither become e' with w . $\widehat{V}^E(a', w)$, is determined with respect to this stage.
5. **Search :** Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy; u and s assess the value of all wage applying options, receive match specific shocks $\{\epsilon^{w'}\}$ and choose the wage level w' to apply. Those who successfully find jobs become e' , otherwise become u' .
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Match

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a') + \epsilon^{w'} \right] dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a', w) = \int \max \{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s \} dF^\epsilon$$

- The value of searching is

$$V^s(a', w) = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^e(a', w) + \epsilon^{w'} \right] dF^\epsilon$$

- The probabilities of quitting and of searching

$$q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},$$

$$s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.$$

$\mu^s < 0$ is the mode of the shock ϵ^s which reflects the search cost.

- Households solve

$$V^e(a, w) = \max_{a' \geq 0} u[a(1+r) + w - a'] + \beta \left[\delta V^u(a') + (1-\delta) \widehat{V}^e(a', w) \right]$$

$$V^u(a) = \max_{c, a' \geq 0} u[a(1+r) + b - a'] + \beta \widehat{V}^u(a')$$

- The value of the firm is again given like in the **Quitting** Model

$$\Omega^0(w) = (z - w - \delta_k k) Q^1(w) + (1 - \delta - \delta_k) k Q^0(w),$$

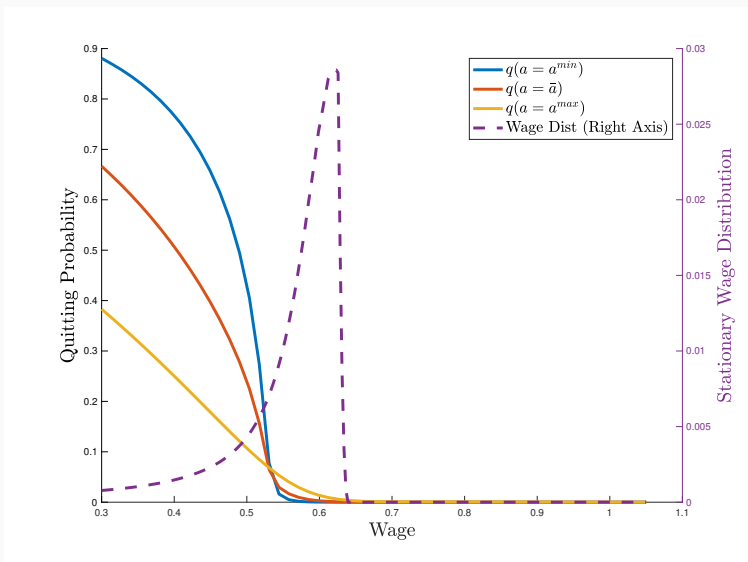
$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

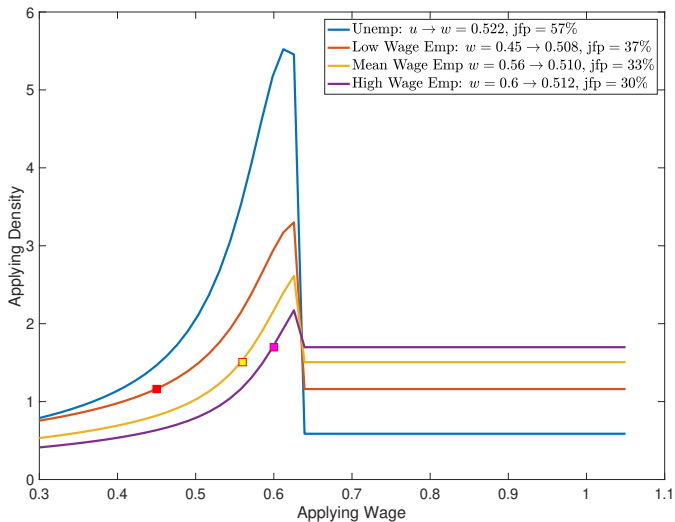
$$\ell^j(w) = 1 - \int h(w; a) q[g^{e \cdot j}(a, w), w] dx^u(a) - \int h(w; a) s[w; g^{e \cdot j}(a, w)] \left[\int \hat{h}[\tilde{w}; g^{e \cdot j}(a, w), w] \xi \phi^h(\tilde{w}) d(\tilde{w}) \right] dx^u(a)$$

OJS QUITTING PROBABILITIES, VARIOUS WEALTHS & WAGE DENSITY



- The rich pursue often other activities (leisure?)

OJS WHICH JOBS TO MOVE TO?



- The rich pursue often other activities (leisure?)
- Unemployed get jobs faster than searchers
- But ... to higher wages
- Higher wage guys move more and to higher wages than lower wage
- But to lower wages than their own
- Excessive quitting in expansions: Easy to come back. Quit to take advantage of a *vacation* a temporary non working opportunity.
- We are redefining the role of extreme value shocks so that searching for almost impossible to find jobs is not rewarding (t)
- Extend to types differ in value of leisure: Outside labor force.

ON THE JOB SEARCH MODEL: EQUILIBRIUM PROPERTIES

- Some good Properties
 - Low wage workers move more often than high wage workers
 - Low wage workers move to lower wages than high wage workers
- Still some unattractive properties
 - Unemployed apply to higher wages than employed.
 - We think that it is an artifact of the way aiming shocks enter: too much wait in the application process and not in the outcome. We are now changing the process of how to implement these shocks.
 - There is excessive quitting in expansions because it is easy to come back. All quitting is to take advantage of a *vacation* a temporary non working opportunit.
 - We propose an extension where some quitting is due to a more permanent switch into a low attachment stage (retirement, schooling, parenting). Business cycles are less tempting to quite: [A model of multiple types that differ in leisure valuation](#). Gives an explicit role to outside the labor force that is not purely temporary.

On the Job Search with Multiple Types

ON THE JOB SEARCH WITH TYPES η MODEL: TIME-LINE

1. Household enters period t with or without a job: $V^{e,\eta}, V^{u,\eta}$.
2. **Production & Consumption**: u produce b at home, e produce z on the market; they then choose consumption today and wealth level tomorrow $\{c, a'\}$. Types differ in $u^{e,\eta}(c) = u(c) - \chi^\eta$
3. **Separation**:
4. **Workers Change their type η according to $\Gamma_{\eta,\eta'}$** .
5. **Quitting? Searching? Neither?:** $\widehat{V}^{e,\eta'}(a', w)$, is determined here.
6. **Search**: Cannot condition on η' (it is illegal). Firms enter, job searchers apply
7. **Matching**: $\widehat{V}^{u,\eta'}(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.

- After saving, the unemployed problem is

$$\widehat{V}^{u,\eta'}(a') = \int \max_{w'} \left[\psi^h(w') V^{e,\eta'}(a', w') + (1 - \psi^h(w')) V^{u,\eta'}(a') + \epsilon^{w'} \right] dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^{e,\eta'}(a', w) = \int \max \{ V^{e,\eta'}(a', w) + \epsilon^e, V^{u,\eta'}(a') + \epsilon^u, V^{s,\eta'}(a', w) + \epsilon^s \} dF^\epsilon$$

- The value of searching is

$$V^{s,\eta'}(a', w) = \int \max_{w'} \left[\psi^h(w') V^{e,\eta'}(a', w') + [1 - \psi^h(w')] V^{e,\eta'}(a', w) + \epsilon^{w'} \right] dF^\epsilon$$

- The probabilities of quitting and of searching are

$$q^{\eta'}(a', w) = \frac{1}{1 + \exp(\alpha[V^{e,\eta'}(a', w) - V^{u,\eta'}(a')]) + \exp(\alpha[V^{s,\eta'}(a', w) - V^{u,\eta'}(a') + \mu^s])},$$

$$s^{\eta'}(a', w) = \frac{1}{1 + \exp(\alpha[V^{u,\eta'}(a') - V^{s,\eta'}(a', w)]) + \exp(\alpha[V^{e,\eta'}(a', w) - V^{s,\eta'}(a', w) - \mu^s])}.$$

- Household solves

$$V^{e,\eta}(a, w) = \max_{a' \geq 0} u[a(1+r) + w - a'] - \chi^\eta + \beta \sum_{\eta'} \Gamma_{\eta\eta'} \left[\delta V^{u,\eta'}(a') + (1-\delta) \widehat{V}^{e,\eta'}(a', w) \right]$$

$$V^{u,\eta}(a) = \max_{w, a' \geq 0} u[a(1+r) + b - a'] + \beta \sum_{\eta'} \Gamma_{\eta,\eta'} \widehat{V}^{u,\eta'}(a')$$

MULTIPLE TYPES MODEL: VALUE OF THE FIRM

- The value of the firm is again given like in the [Quitting](#) Model

$$\Omega^0(w) = (z - w - \delta_k k) Q^1(w) + (1 - \delta - \delta_k) k Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

$$\begin{aligned} \ell^j(w) &= 1 - \int \sum_{\eta} \{ h(w; a) q [g^{e,j}(a, w), w] x^u(\eta) \} dx^u(a) \\ &\quad - \int \sum_{\eta} \{ h(w; a) s(w; g^{e,j}(a, w)) H(w; a) x^u(\eta) \} dx^u(a) \end{aligned}$$

where $H(w; a) = \int \hat{h}(\tilde{w}; g^{e,j}(a, w), w) \xi \phi^h(\tilde{w}) d\tilde{w}$ and $x^u(\eta)$ is the stationary distribution of type η induced by $\Gamma_{\eta\eta'}$.

- Limited Comparable Results
- Right now we have five Economies
 1. **No Aiming and Not Quitting** The Benchmark
 2. **No Aiming and Quitting** Quitting is Small without Aiming
 3. **Aiming and Not Quitting** (Closed Economy) General Equilibrium
 4. An **Aiming and Quitting** Economy with the same interest rate
 5. An **Aiming-Quitting & On the Job Search** Economy (same r)
- Potential output is Normalized to 1.

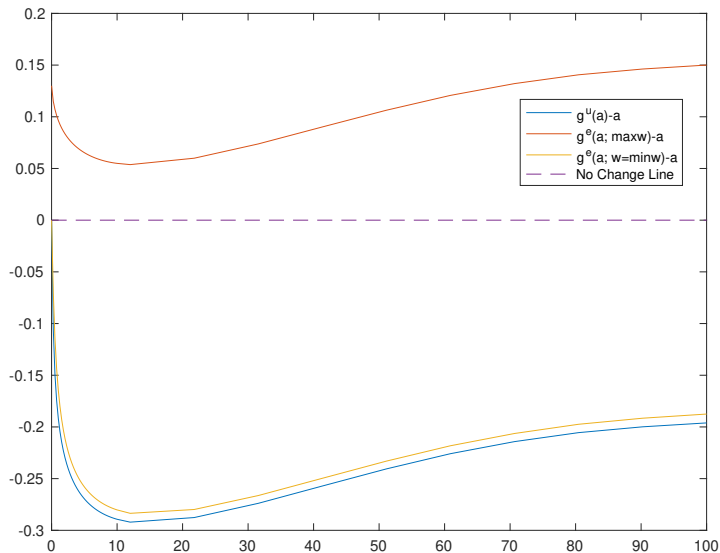
Steady State Statistics of Various Economies: $r = 1\%$ quarterly

	NANQ	QNA	ANQ	AQ	AQOJS	High b	High β
Market Output	0.947	0.851	0.956	0.854	0.870	0.886	0.866
Avg Consumption	0.650	0.614	0.639	0.624	0.608	0.615	0.613
Avg Wage	0.639	0.626	0.604	0.577	0.562	0.590	0.559
Wage of Newly hired Unemp	0.638	0.620	0.603	0.527	0.523	0.571	0.520
Wage of Newly Hired Emp	-	-	-	-	0.512	0.567	0.510
Std Wage	0.001	0.001	0.027	0.061	0.063	0.040	0.065
Mean-min Wage	1.005	1.000	1.385	1.925	1.872	1.311	1.862
Max-mean Wage	1.004	1.004	1.077	1.126	1.133	1.094	1.140
Avg Wealth	2.848	3.706	4.899	8.689	8.031	4.090	9.251
Stock Value	7.121	6.933	8.502	8.671	9.212	8.354	9.278
Consumption-Wealth Ratio	0.228	0.166	0.130	0.072	0.076	0.150	0.069
All Vacancies	0.055	0.085	0.121	0.185	0.193	0.137	0.198
Unemployment Rate	0.053	0.097	0.044	0.078	0.074	0.074	0.075
Non Emp Rate	0.053	0.149	0.044	0.146	0.130	0.114	0.134
Quitt Rate	0.000	0.053	0.000	0.068	0.056	0.040	0.059
Newly Hired Unemp	0.029	0.070	0.029	0.083	0.074	0.061	0.076
Avg Unemp Duration	1.805	2.128	1.368	1.702	1.695	1.797	1.694
Avg Emp Duration	33.33	12.34	33.33	10.37	11.84	14.63	11.44
Std Consumption	0.027	0.032	0.031	0.036	0.035	0.029	0.038
Std Wealth	0.656	1.440	2.142	4.543	4.044	2.021	4.486
Max-mean Wealth	1.479	3.408	4.298	4.179	4.063	4.375	4.016

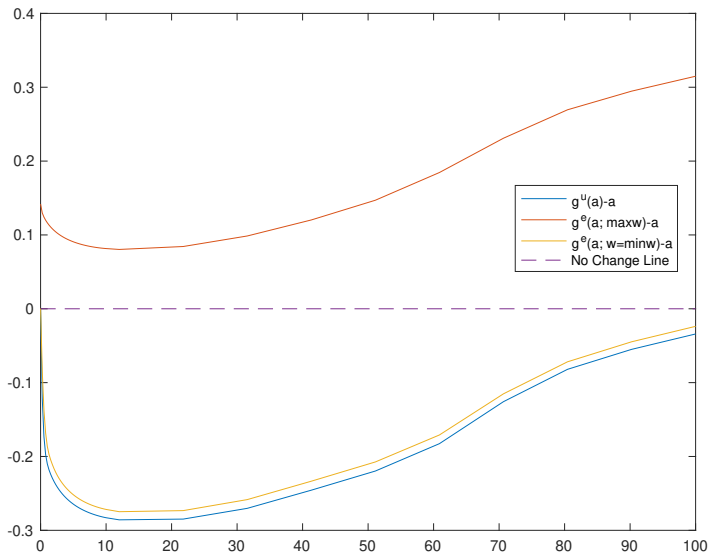
A SURPRISING ISSUE

- Models with Quitting seem to have *non-standard* properties with respect to asset holdings.
- $\beta \rightarrow (1 + r)^{-1}$ does not mean $\int_a a dx \rightarrow \infty$.
- In fact for β slightly bigger than $(1 + r)^{-1}$, wealth is too little.
- But if we increase β a tiny bit then wealth becomes unbounded
- This is because the net savings function is NOT monotonic
- This is due to the relevant density of the Gumbel distribution relative to marginal utility
- Need to kill people and create non-profit sector

NET SAVINGS FUNCTION NON-EXPLODING



NET SAVINGS FUNCTION EXPLODING



Aggregate Fluctuations

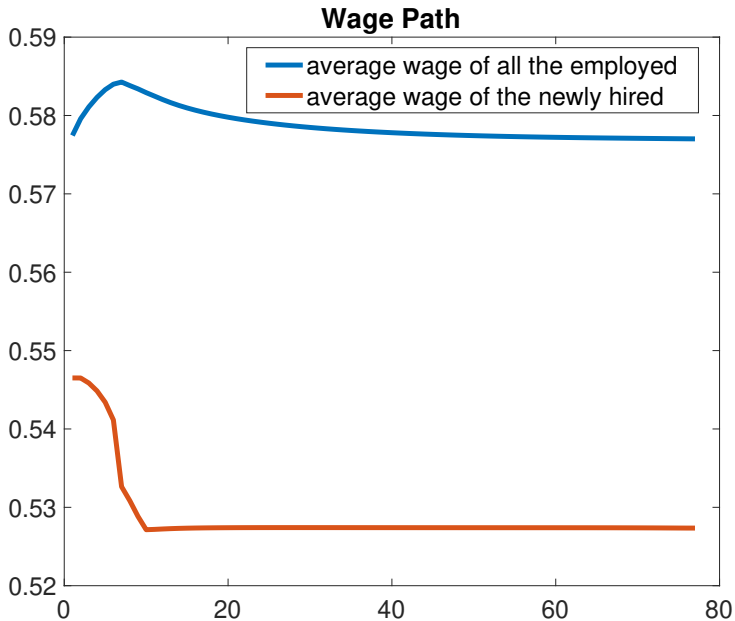
WHAT IS NEEDED?

- Two steps
 1. Compute the TRUE impulse response to an MIT Shock
 2. Use this path as a dynamic linear approximation to generate fluctuations ([Boppart, Krusell, and Mitman \(2018\)](#))
- The transition is a large but doable problem:
 - Firms need to know functions $\{Q_t^0(w), Q_t^1(w), \psi^f(w)\}$ at each stage (no block recursivity)
 - Households need to know $\phi_t^h(w)$ job finding probabilities every period.
 - Also need to know sequence of interest rates (not today)
- So it is a second order difference functional equation.

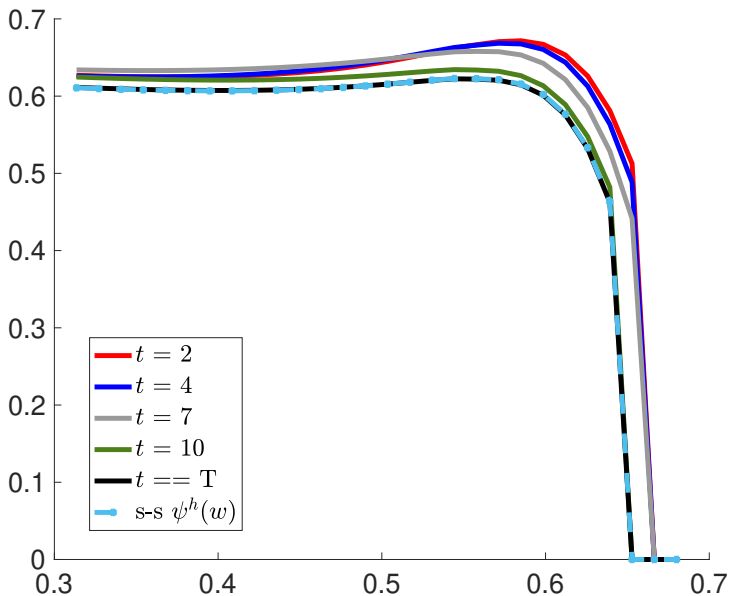
AIMING AND QUITTING MODEL.

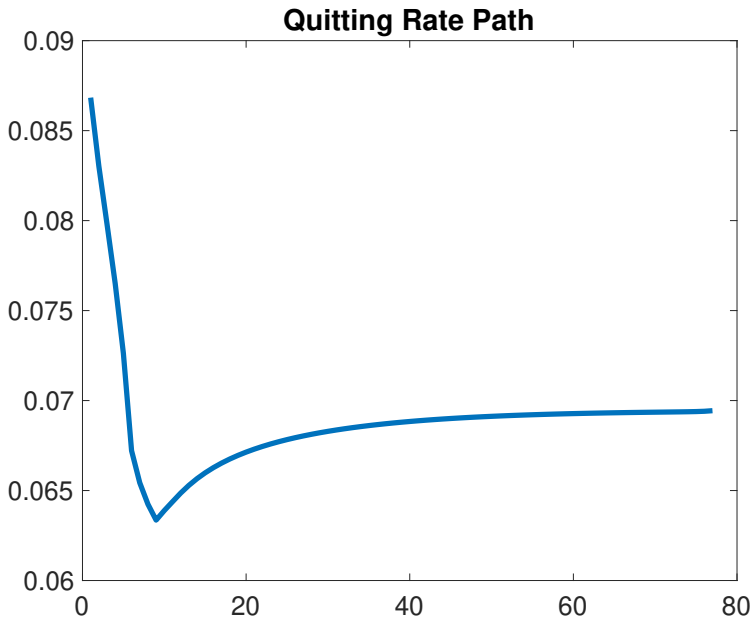
5% PRODUCTIVITY SHOCK ($\rho = .99$) FOR 10 PERIODS

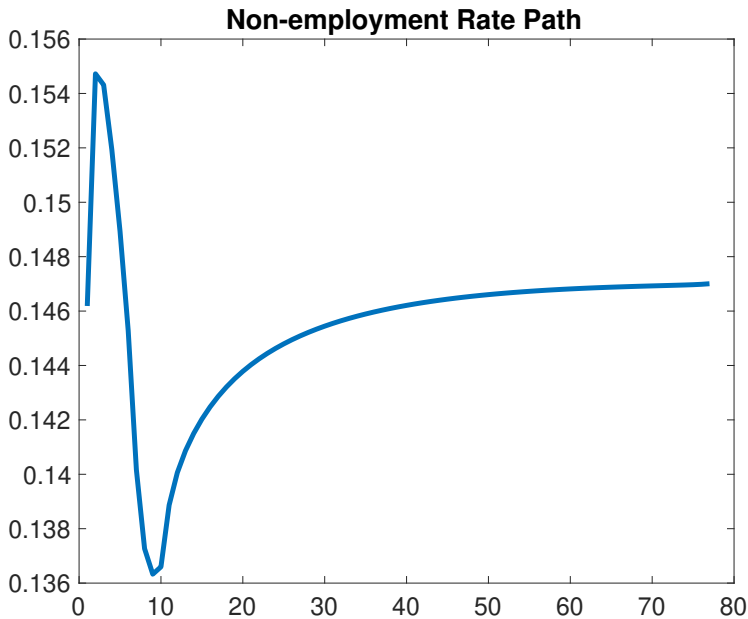
- Average wages don't move much but wages of new workers do!
- Newly hired Wage Distribution Shifts upward
- Quits are procyclical but excessive
- Employment moves more (not so much of Shimer puzzle)



AIMING AND QUITTING MODEL. 5% PRODUCTIVITY SHOCK ($\rho = .99$)



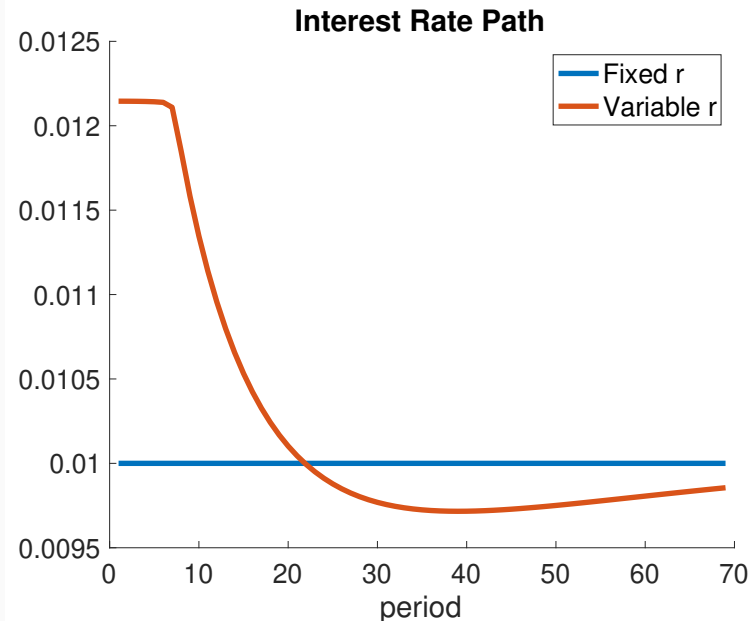


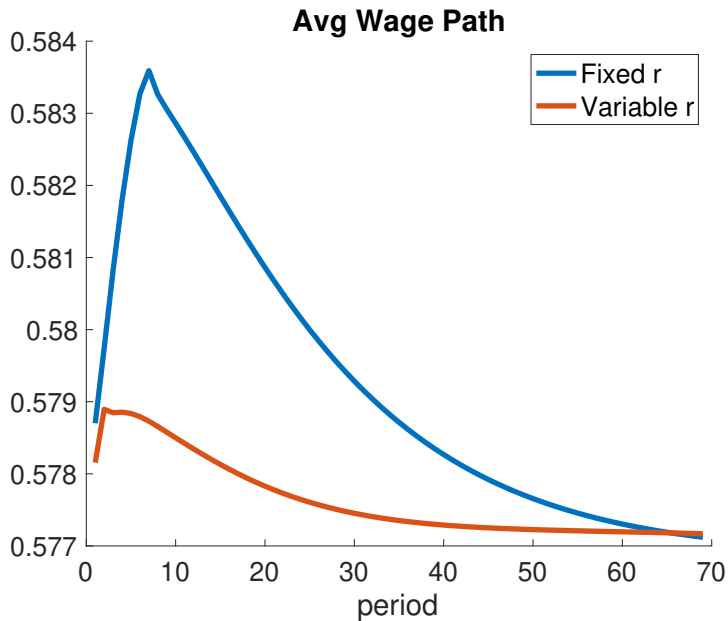


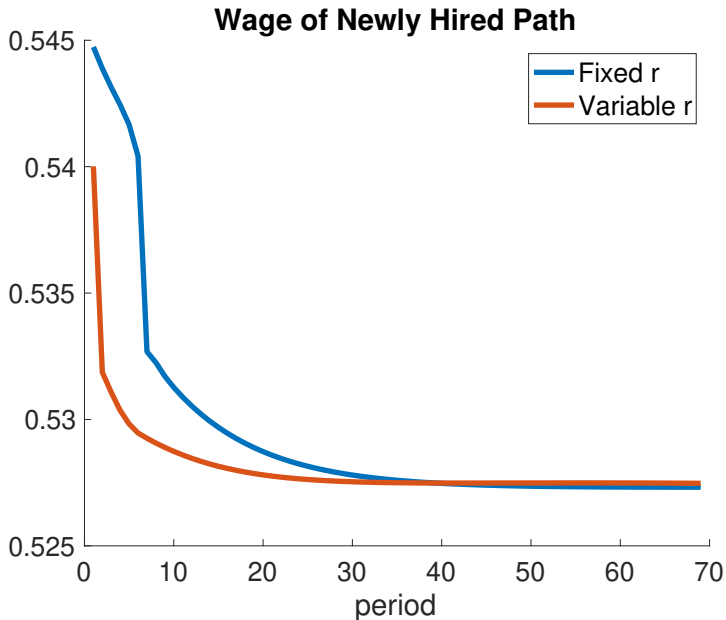
AIMING AND QUITTING MODEL (ENDOGENOUS r).

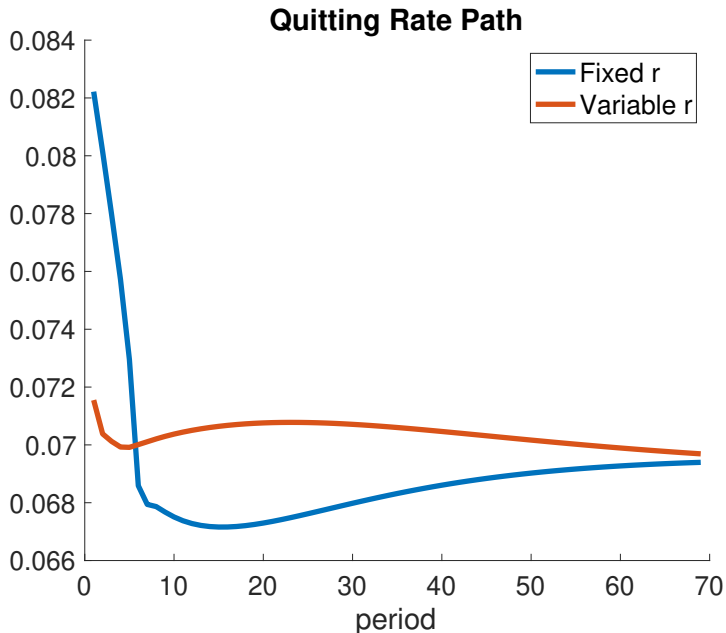
5% PRODUCTIVITY SHOCK ($\rho = .9$) NO TRUNCATION

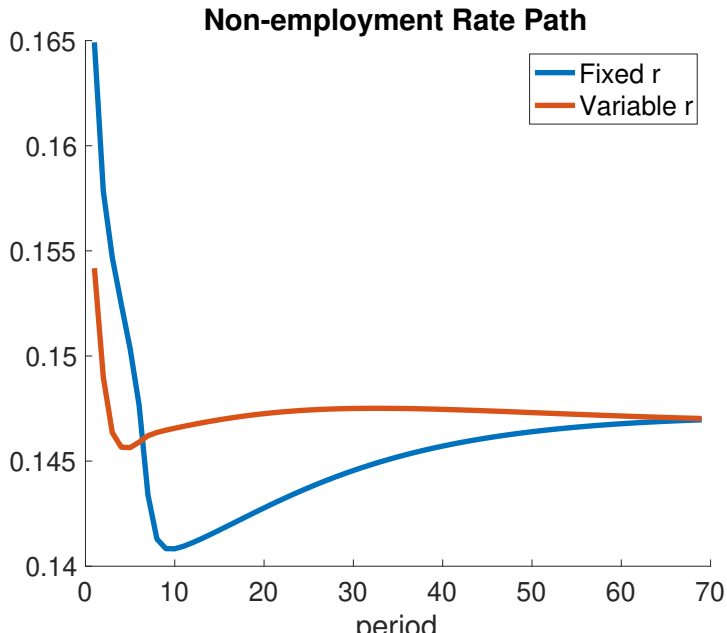
- Interest rate r goes up endogenously as a response of positive technology shocks
- As a consequence wages and employment move less
- Quits are still pro-cyclical but much less in magnitude
- Employment still moves more (compared to wages), but in the wrong direction (wealth effect)



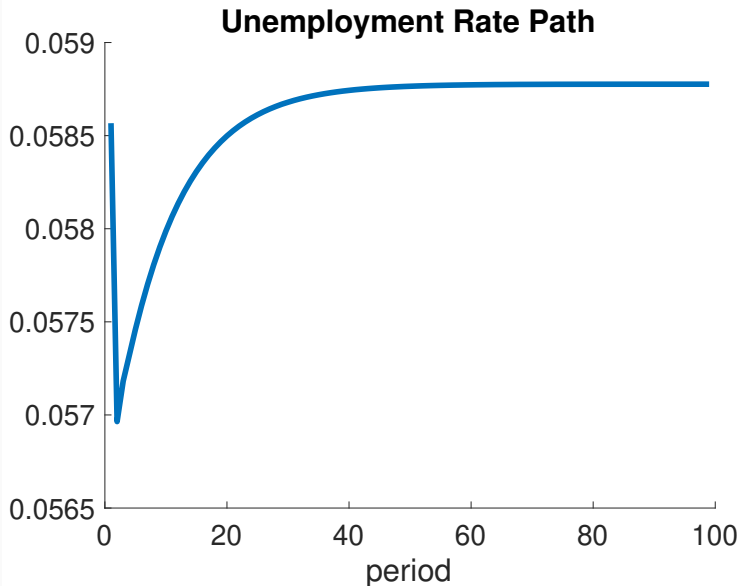






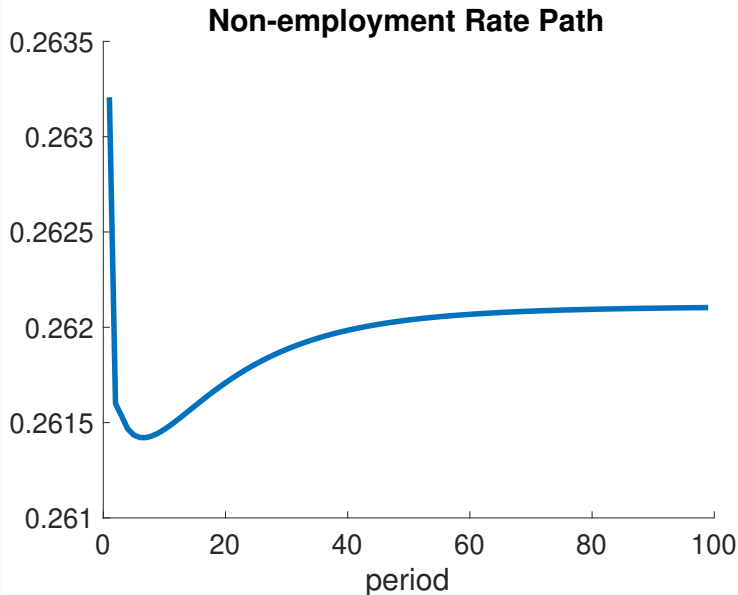


SMALL DROP UNEMP

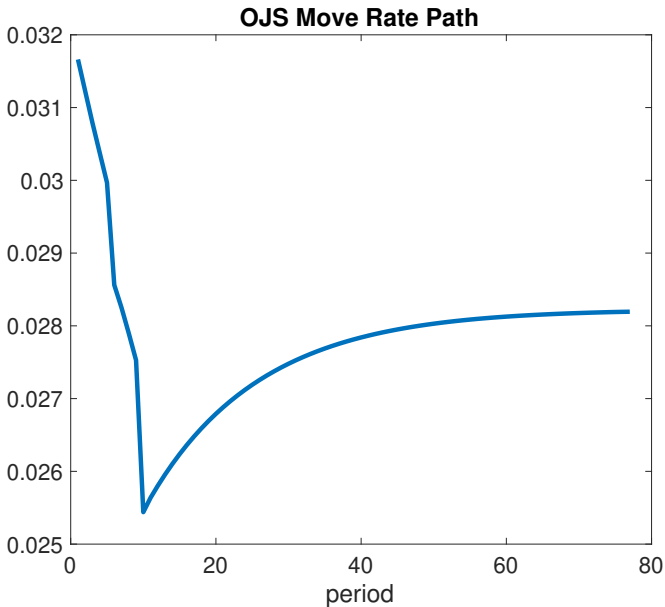


MULTI TYPES DIFF IN LABOR ATTACHMENT. SOME QUILTS MORE PERM

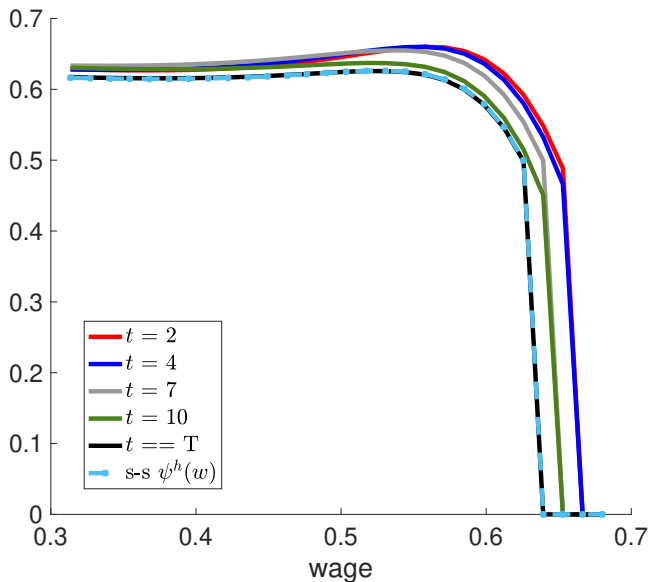
MUCH SMALLER HIKE

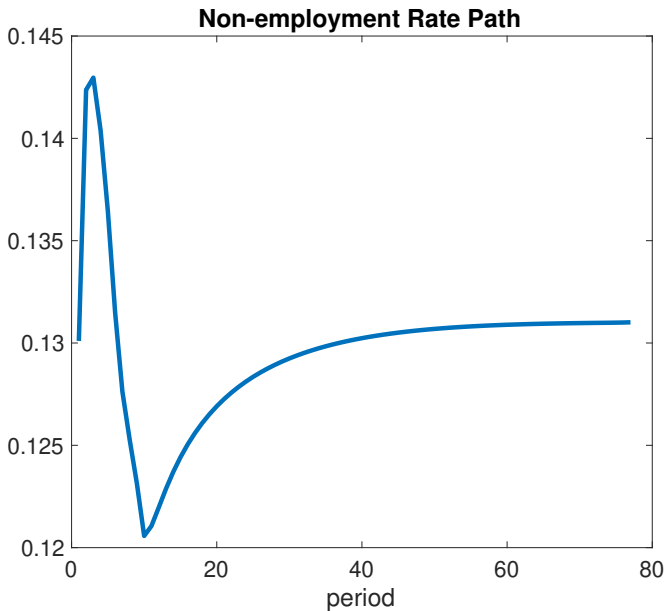


Business Cycle Behavior of On the Job Search



OJS 5% PRODUCTIVITY SHOCK ($\rho = .99$) WAGE OFFERS





- Switches are Procyclical
- Average wages don't move much but wages of new workers do!
- Newly hired Wage Distribution Shifts upward
- Quits are procyclical but excessive
- But Employment moves down!!! (Excessive Quitting)

CONCLUSIONS

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
 1. Aiyagari models (output, consumption, investment, interest rates)
 2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
 3. Add tools from Empirical Micro to soften wage-wealth correlations.
- Useful for business cycle analysis: We are getting procyclical
 - Quits
 - Employment after a lag
 - Investment and Consumption
 - But Perhaps Expansions and Recessions Should Arrive Slower
- Exciting set of continuation projects:
 - Efficiency Wages, Endogenous Productivity (firms use different technologies with different costs of idleness)
 - Move towards more sophisticated life cycle movements

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