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Abstract

With short-term interest rates at the zero lower bound, forward guidance has become a key tool for central bankers, and yet we know little about its effectiveness. Standard medium-scale DSGE models tend to grossly overestimate the impact of forward guidance on the macroeconomy -- a phenomenon we call the "forward guidance puzzle." We explain why this is the case and describe one approach to addressing this issue.

Key words: forward guidance, DSGE models

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1 Introduction

For decades, macroeconomists have attempted to quantify the effects of monetary policy actions on the economy. By now, a very large number of papers has documented the transmission mechanism of surprise changes in short-term interest rates onto the economy, using either VARs or DSGE models (e.g., Sims (1980), Christiano et al. (1999), Christiano et al. (2005)). While we arguably have some understanding of the effects of short-term interest rates, these have been constrained by the zero lower bound (ZLB) for a few years in most developed economies, so that for the time being they are no longer part of the policymakers' toolkit. Instead, many central banks have used other tools such as announcements about the future path of the policy rate ("forward guidance"), or "quantitative easing" measures involving a change in the size and especially the composition of the central bank balance sheet. Forward guidance has been used extensively and explicitly by the Federal Reserve since the FOMC meeting of December 16, 2008, so as to affect long-term bond yields and stimulate aggregate expenditures (see Woodford (2012) and Campbell et al. (2012a)).[1] Moreover, Woodford (2012), building on results by Krishnamurthy and Vissing-Jorgensen (2011) and Bauer and Rudebusch (2011), emphasizes the "signaling channel" of the Fed's asset purchases – that is, he argues that quantitative easing itself can at least in part be interpreted as implicit forward guidance.

While the literature has provided strong theoretical justifications for the use of such forward guidance (e.g., Eggertsson and Woodford (2003)), the evidence on the quantitative effects of such a policy tool on the macroeconomy is still limited. This may not be too surprising in light of the fact that the identification problem that needs to be surmounted in the case of contemporaneous policy shocks may be even more challenging in the case of shocks that are anticipated. In fact, an announcement by policymakers that they will maintain the policy rate at the ZLB for longer than initially anticipated by market participants may have

[1] At that meeting, the FOMC's statement mentioned that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate for some time." Three months later, the FOMC reinforced its forward guidance by stating that the exceptionally low levels of the federal funds rate would likely be warranted "for an extended period." This sentence was reiterated in each subsequent FOMC statement until August 9, 2011, when the FOMC argued that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate at least through mid-2013." That sentence was maintained in subsequent statements until January 25, 2012, when the date was pushed forward to "late 2014."

two types of effects. On the one hand, it could be interpreted as more monetary stimulus: it should lower the market's expectation of future federal funds rate (FFR), which contributes to lower longer term yields, hence stimulates economic activity and puts upward pressure on inflation. On the other hand, such an announcement could be interpreted by market participants as revealing negative news about the state of the economy, if they believe that the FOMC has access to information not shared by market participants. In this case, such an announcement would be associated with lower long-term yiels and lower projections of economic activity. The interpretation chosen by the market participants surely depends in very subtle ways on the FOMC communication.[2]

Empirically, Gürkaynak et al. (2005) and more recently Campbell et al. (2012a) find strong evidence that FOMC announcements move asset prices. Yet when Campbell et al. (2012a) try to assess the impact of exogenous anticipated changes in monetary policy on the macroeconomy, they find that this has the opposite sign than expected, highlighting these identification challenges. Moreover, even if it was possible to identify the impact of, say, four quarters-ahead forward guidance, its effect would not necessarily be the same as, say, that of eight-quarters ahead forward guidance (Campbell et al. (2012a) consider one through four quarters ahead forward guidance; current forward guidance in the U.S. goes through the end of 2014, and hence amounts to approximately eight quarters). Given that policymakers seldom if ever experimented with forward guidance this far in the future, there is little data to guide them.

New Keynesian DSGE models following the work of Christiano et al. (2005) and Smets and Wouters (2007) are in principle well suited to study the effects of forward guidance. Such models have been found to fit the data reasonably well and to provide a good forecasting performance relative to reduced form models such as VARs, private forecasters, or the Greenbook (see Smets and Wouters (2007), Del Negro et al. (2007), Edge and Gürkaynak (2010), and Del Negro and Schorfheide (forthcoming)). Most importantly, being laboratory economies, they can be used to study the impact of policy experiments never performed before. As shown by Laseen and Svensson (2011), forward guidance can be captured in DSGE

[2]Woodford (2012) argues that several recent announcements about the future path of policy rates have not indicated a clear commitment to maintaining short-term rates low, so that they run the risk of being interpreted as reflecting a deteriorating forecast for output and or inflation.

models using anticipated policy shocks. Such shocks reflect deviations of the short-term interest rate from the historical policy rule that are anticipated by the public. They can be affected by policymakers' announcements about their intentions regarding the future path of the policy rate. Milani and Treadwell (2011) study the impulse responses to anticipated policy shocks using a simple three-equations New Keynesian DSGE model. Campbell et al. (2012b) go quite a few steps further. They investigate the impact of forward guidance on the macroeconomy by estimating a medium scale DSGE model broadly similar to the one in this paper using data on market expectations for the federal funds rate, in addition to a standard set of macro variables, for the sample 1987-2007. They find that forward guidance explains about 9 percent of output and hours fluctuations at the business cycle frequency, and more than 50 percent of the movements in the federal funds rate. Their results indicate that even in the pre-Great Recession period forward guidance played a large role in monetary policy – a finding that echoes that of Gürkaynak et al. (2005) – and a significant role in terms of business cycle fluctuations.

The problem with DSGE models, however, is that they appear to deliver unreasonably large responses of key macroeconomic variables to central bank announcements about future interest rates – a phenomenon we can call the "forward guidance puzzle". Carlstrom et al. (2012) show that the Smets and Wouters model would predict an explosive inflation and output if the short-term interest rate were pegged at the ZLB between eight and nine quarters. This is an unsettling finding given that the current horizon of forward guidance by the FOMC is of at least eight quarters.

This paper has two contributions. First, we characterize the quantitative implications of forward guidance in a setting that is arguably more realistic than that adopted by Carlstrom et al. (2012). In their experiment, these authors assess the impact of fixing the interest rate to the zero lower bound relative to the steady state baseline. Given the current state of the economy, we view the assumption that interest rates would be at steady state in absence of forward guidance as unrealistic. We instead incorporate current market expectations for the short rate in our baseline forecast using the approach described in Del Negro and Schorfheide (forthcoming). Specifically, we use the FFR expected path through mid-2015 implied by OIS rates as of August 28, 2012. Doing so allows us to incorporate valuable information for the estimation of the state of the economy. We then investigate the effect of extending the forward guidance by two quarters, from the end of 2014 to mid-2015. Using the FRBNY-DSGE model we show that even for this much more modest (relative to Carlstrom et al. (2012)) experiment, these authors' findings is confirmed: the response of macroeconomic variables is unrealistically large.

The second contribution of the paper is to point to the source of the problem and suggest a solution. Credible announcements about future short-term policy rates should affect the current long-term bond yields, and these in turn affect economic activity and inflation. However, the model predicts an excessive response of the long-term bond yield to policy announcements, compared to what is observed in the data. For instance, the relatively modest (two quarters) change in forward guidance delivers in the model a 25 basis points drop in the 10-year nominal yield. In comparison, the January 25, 2012, change in forward guidance, which shifted the announced lift-off date by more than four quarters (mid-2013 to end of 2014), produced a drop in the same rate by only 7 basis points. Why this excessive response of the long rate in the model relative to the data? Interestingly, the model tends to underestimate the response of bond yields with maturities of 1 to 5 years. Instead, it predicts excessive responses in the maturities much farther in the future.

We view this response to forward guidance of the expected short term rates beyond 5 years, which leads to overestimate the impact of forward guidance, as an incredible feature of this model: it appears unlikely that policymakers are able to affect FFR expectations farther than 5 years by announcements regarding the short term rate in the next two years. To put it differently, the experiment conducted within the DSGE model is actually quite different from the one a policymaker may have in mind when changing forward guidance, i.e., the anticipated policy shocks announced at the time that the forward guidance gets extended have implausibly long-lasting effects on future short-term rates. We therefore suggest to assess the impact of forward guidance by conducting a different experiment – one whose outcome on long rates is closer to the measured impact of past announcements. We choose the sequence of anticipated policy shocks in the DSGE model so that: i) the response of the long rate is constrained to be reasonable, ii) expectations for the short rate far into the future are minimally affected. We show that under this alternative experiment the short term FFR path of the interest rate is broadly in line with the announcement, and yet the responses of inflation and output are no longer excessive.

The paper proceeds as follows. Section 2 briefly summarizes the DSGE model used, its estimation, how we formalize the introduction of a fixed interest-rate path, and describes the model's excessive response to interest-rate pegs. Section 3 proposes the solution to this problem. Section 4 concludes.

2 The macroeconomic implications of interest rate announcements

We now proceed with an evaluation of the effects of extending the forward guidance focusing on the stimulative effects of policy, and abstracting from the possible effects of information conveyed by the FOMC regarding the assessment the state of the economy. In this section, we first briefly describe the DSGE model, its estimation, and the baseline forecasts. In particular we discuss the modification of the standard feedback rule describing monetary policy to allow for anticipated policy shocks, and how we incorporate current FFR market expectations into the forecast. Next, we describe the algorithm used for conditioning the foercast on a specific interest-rate path. We show that it produces results that are hardly credible and explain why this is the case.

2.1 Model and baseline forecasts

The FRBNY DSGE model is a medium-scale, one-sector, dynamic stochastic general equilibrium model. It builds on the neoclassical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model follows the work of Christiano et al. (2005) and Smets and Wouters (2007), but also includes credit frictions, as in the financial accelerator model developed by Bernanke et al. (1999). The actual implementation of the credit frictions closely follows Christiano et al. (2009). Detailed information about the equilibrium, the data, and the priors used in the Bayesian estimation of this model are contained in Del Negro et al. (2012).

The appendix to this paper also includes the list of log linearized equilibrium conditions, as well as the priors and posteriors for the estimated parameters. In this section we focus on the features of the model that are needed to properly describe this exercise. In particular, we discuss: i) the state-space representation of the linearized DSGE model, ii) anticipated policy shocks, iii) incorporating market's FFR expectations into the baseline forecast.

The solution to the log-linear approximation of the model's equilibrium conditions around the deterministic steady state (obtained using the method in Sims (2002)) yields the following transition equation:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \tag{1}$$

where s_t is the model's vector of "state" variables, the matrices Φ_1 and Φ_ϵ are functions of the vector of all model parameters θ, and ϵ_t is the vector of structural shocks. The vector of observables y_t described below is in turn related to the states according to the system of measurement equations:

$$y_t = \Psi_1(\theta) + \Psi_2(\theta)s_t. \tag{2}$$

The variables included in y_t are: 1) annualized real GDP per capita growth, where the real gross domestic product is computed as the ratio of nominal GDP (SAAR) to the chain-type price index from the BEA;[3] 2) the log of labor hours, measured as per capita hours in non-farm payroll; 3) the log of labor share, computed as the ratio of compensation of employees to nominal GDP, from the BEA; 4) the annualized rate of change of the core PCE deflator (PCE excluding food and energy, but including purchased meals and beverages), seasonally adjusted; 5) the effective federal funds rate, percent annualized, computed from daily data; and 6) the spread between the Baa rate and the rate on 10 year Treasuries. We estimate the vector of model parameters θ using data from 1984Q1 to 2012Q3 using Bayesian methods as described in Del Negro and Schorfheide (2010), applied to the state-space representation of the linearized DSGE model provided by equations (1) and (2).

Starting in 2008Q3 (one period before the implementation of the zero lower bound) we incorporate FFR market expectations, as measured by OIS rates, into our outlook following

[3] Per capita variables are obtained by dividing through the civilian non-institutionalized population over 16. We HP-filter the population series in order to smooth out the impact of Census revisions.

the approach described in Section 5.4 of Del Negro and Schorfheide (forthcoming). Specifically, we take FFR expectations up to K quarters ahead into account by augmenting the measurement equation (2) with the expectations for the policy rate:

$$\begin{aligned} FFR^e_{t,t+k} &= 400\left(I\!E_t\widehat{R}_{t+k} + \ln R_*\right) \\ &= 400\left(\Psi_{R,2}(\theta)\Phi_1(\theta)^k s_t + \Psi_{R,1}(\theta)\right), \quad k = 1,..,K \end{aligned} \tag{3}$$

where $FFR^e_{t,t+k}$ are the market's expectations for the FFR k quarters ahead, $\Psi_{R,2}(\theta)$ and $\Psi_{R,1}(\theta)$ are the rows of $\Psi_2(\theta)$ and $\Psi_1(\theta)$, respectively, corresponding to the interest rate, and R_* is the gross steady state nominal interest rate. This observation equation contains valuable information for the estimation of the state of the economy. The market expectations of continued low interest rates reflect both a relatively weak economy as well as an accommodative monetary policy.

In order to incorporate the forward guidance, which partly drives these market expectations, we also modify the policy rule followed by the Central Bank. In absence of forward guidance we assume that the central bank follows a standard feedback rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \psi_y \sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon^R_t, \tag{4}$$

where $\sum_{j=0}^{3} \hat{\pi}_{t-j}$ is 4-quarter inflation expressed in deviation from the Central Bank's objective π_* (which corresponds to steady state inflation), $\sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$ is 4-quarter growth rate in real GDP expressed in deviation from steady state growth, and $\epsilon_{R,t}$ is the standard contemporaneous policy shock, where $\epsilon^R_t \sim N(0, \sigma^2_r)$, *i.i.d.*.[4] We modify this rule to allow for forward guidance following Laseen and Svensson (2011):

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \psi_y \sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon^R_t + \sum_{k=1}^{K} \epsilon^R_{k,t-k}, \tag{5}$$

where $\epsilon^R_{k,t-k}$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume that $\epsilon^R_{k,t-k} \sim N(0, \sigma^2_{k,r})$, *i.i.d.*. We express

[4] The economy displays a stochastic trend, so if $\hat{y}_{t-j}$ is output in deviation from this trend and $\hat{z}_t$ corresponds to the growth rate of technology in deviations from steady state, then the growth rate of output in period t is $\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$.

the anticipated shocks in recursive form by augmenting the state vector s_t with K additional states $\nu_t^R, \ldots, \nu_{t-K}^R$ whose law of motion follows[5]

$$\begin{aligned} \nu_{1,t}^R &= \nu_{2,t-1}^R + \epsilon_{1,t}^R \\ \nu_{2,t}^R &= \nu_{3,t-1}^R + \epsilon_{2,t}^R \\ &\vdots \\ \nu_{K,t}^R &= \epsilon_{K,t}^R. \end{aligned} \tag{6}$$

We also augment the vector of shocks ϵ_t in equation (1) with the anticipated shocks $[\epsilon_{1,t}^R, .., \epsilon_{K,t}^R]'$ and resolve the model to compute the matrices $\Phi_1(\theta)$ and $\Phi_\epsilon(\theta)$ appropriately. Note that we make the – arguably counterfactual – assumption that the anticipated shocks are independent from one another. Campbell et al. (2012b) forcefully argue, based on their own findings as well as Gürkaynak et al. (2005)'s, that anticipated shocks follow a factor structure. It would be important to relax the independence assumption if we were to estimate the model with forward guidance shocks. However, this assumption bears no implications in the policy exercise described in sections 2.2 and 3: in the exercise the magnitude of the anticipated shocks is chosen to obey a certain set of restrictions – their variance-covariance matrix is irrelevant.[6]

For simplicity we estimate the model parameters assuming no forward guidance – that is, using equation (4) instead of (5) and without adding (3) to the system of measurement equations. Implicitly we are assuming that forward guidance has little impact on the estimated model parameters. We are however recognizing that it has a potentially large impact on our inference about the state of the economy s_t in the 2008Q3-2012Q3 period (conditional on the estimated parameters), and hence on the model's forecasts. We are therefore re-estimating s_t during this period in light of the information provided by (3).[7] Our baseline forecast, which is described in Table 1 and Figure 1, is therefore obtained using data released through 2012Q2 augmented for 2012Q3 with observations on the federal funds rate and the

[5]It is easy to verify that $\nu_{1,t-1}^R = \sum_{k=1}^K \epsilon_{k,t-k}^R$, that is, $\nu_{1,t-1}^R$ is a "bin" that collects all anticipated shocks that affect the policy rule in period t.

[6]Moreover, in this log-linearized model the variance-covariance matrix of the shocks does nor affect the equilibrium conditions.

[7]The only extra parameters introduced by the forward guidance are the standard deviations $\sigma_{k,r}$ of the anticipated shocks. Since we do not have estimates for these parameters, we assume that these shocks have the same standard deviation as the contemporaneous shock: $\sigma_{k,r} = \sigma_r$. Importantly, note that the parameters $\sigma_{k,r}$ do not enter *any* of the policy experiments described below.

Baa corporate bond spread, and with market's FFR expectations through mid-2015 (hence $K = 11$ in equation (3)) as measured by OIS rates on August 28, 2012.[8]

Figure 1 shows the model's predictions for real GDP growth, core PCE inflation and the federal funds rate, conditional on alternative assumptions regarding the federal funds rate. These forecasts are obtained using the mode of the posterior distribution for θ and s_t, although these modal forecasts in the baseline case essentially coincide with the mean of the forecast distribution obtained by drawing from the full posterior of θ and s_t. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. In this forecast, GDP growth is 1.9 percent in 2012 (Q4/Q4), rises to 2.2 percent in 2013 but remains mostly below 2 percent throughout the rest of the forecast horizon (see the first row in each of the three panels of Table 1). Core PCE inflation is predicted to be at 1.6 percent in 2012 and is also expected to remain below 2 percent throughout the forecast horizon.

2.2 Using anticipated shocks to condition on an interest-rate path

We now proceed with our counterfactual policy experiment in which the federal funds rate is set to 25 basis points (the current rate paid on excess reserves held at the central bank, or IOR) until 2015Q2, and that it follows the historical policy rule after that.[9] We first summarize the procedure used to condition the model's predictions on a given interest-rate path, which is taken from section 6.3 of Del Negro and Schorfheide (forthcoming), and then describe the outcome of the experiment.

Suppose that at the end of period T, after time T shocks are realized, the central bank announces its intention to commit to a given interest-rate path: $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$. For the

[8]As 2012Q3 observations for the the FFR and the Baa corporate bond spread we are using the average of daily rates during the quarter up to this date.

[9]At the time we wrote the paper, this was one policy option discussed by market commentary for the upcoming FOMC meeting, see the September 10, 2012 WSJ "MarketBeat" Blog at `blogs.wsj.com/marketbeat//qe3-what-everybody-that-matters-on-wall-street-expects/`. We chose 25 basis points for simplicity as it coincides with the IOR, but of course choosing any lower rate would make the results even stronger as the policy would be even more accommodative.

agents, the announcement is a one-time surprise in period $T+1$. This corresponds to the realization of a single unanticipated monetary policy shock ϵ^R_{T+1} and a sequence of anticipated shocks $\{\epsilon^R_{1,T+1}, \epsilon^R_{2,T+1}, \ldots, \epsilon^R_{K,T+1}\}$ where $K = \bar{H} - 1$. Notice that all policy shocks that are used to implement the interest rate path are dated $T+1$. We denote by ϵ_t the vector that collects the innovations of the unanticipated shocks (both policy and non policy shocks), and by $\epsilon^R_{1:K,t}$ the vector of anticipated policy shocks. The following algorithm determines the time $T+1$ monetary policy shocks as a function of the desired interest rate sequence $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$ to generate predictions conditional on an announced interest rate path. The announced interest rate path will be attained in expectation.

Algorithm 1. Drawing Counterfactual Forecasts via Anticipated Shocks.[10]

1. Use the Kalman filter to compute the mean $s_{T|T}$ of the distribution $p(s_T|\theta, Y_{1:T})$.

2. Consider the following system of equations, omitting the θ argument of the system matrices:

$$
\begin{aligned}
\bar{R}_{T+1} &= \Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T + \Psi_{R,2}\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]' \\
\bar{R}_{T+2} &= \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^2 s_T + \Psi_{R,2}\Phi_1\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]' \\
&\vdots \\
\bar{R}_{T+\bar{H}} &= \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T + \Psi_{R,2}(\Phi_1)^{\bar{H}-1}\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]'
\end{aligned}
\tag{7}
$$

This linear system of $\bar{H}$ equations with $\bar{H}$ unknowns can be solved for for the vector of policy shocks $\bar{\epsilon}^R = [\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\prime}_{1:K,T+1}]'$. Specifically, rewrite the system (7) as

$$
b = M_{\bar{H}}\bar{\epsilon}^R, \tag{8}
$$

where

$$
\begin{aligned}
b &= [\bar{R}_{T+1}, ..., \bar{R}_{T+\bar{H}}]' - [\Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T, ..., \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T]', \\
M_{\bar{H}} &= [\Psi_{R,2}, \Psi_{R,2}\Phi_1, ..., \Psi_{R,2}(\Phi_1)^{\bar{H}-1}]\Phi_{\epsilon,R},
\end{aligned}
\tag{9}
$$

and $\Phi_{\epsilon,R}$ collects the columns of the matrix Φ_ϵ corresponding to the vector of policy shocks $\bar{\epsilon}^R$. The solution of (7) is then

$$
\bar{\epsilon}^R = M^{-1}_{\bar{H}} b. \tag{10}
$$

[10] The algorithm in Del Negro and Schorfheide (forthcoming) describes how to draw from the entire counterfactual predictive distribution, conditional on draws of θ from the posterior density. Here we focus on the mode of the posterior density for θ.

3. Starting from $s_{T|T}$, iterate the state transition equation (1) forward to obtain a sequence $s_{T+1:T+H|T}$:

$$s_{t|T} = \Phi_1(\theta^{(j)})s_{t-1|T} + \Phi_\epsilon(\theta^{(j)})[\epsilon_t^R, 0, \ldots, 0, \epsilon_{1:K,t}^{R\,\prime}]', \quad t = T+1, \ldots, T+H,$$

where (i) $\epsilon_{T+1}^R = \bar{\epsilon}_{T+1}^R$ and $\epsilon_t^R = 0$ for $t = T+2, \ldots, T+\bar{H}$; (ii) $\epsilon_{1:K,T+1}^R = \bar{\epsilon}_{1:K,T+1}^R$ and $\epsilon_{1:K,t}^R = 0$ for $t = T+2, \ldots, T+\bar{H}$ (that is, in both cases use solved-for values in period $T+1$ and zeros thereafter).

4. Use the measurement equation (2) to compute $y_{T+1:T+H}$ based on $s_{T+1:T+H|T}$. □

The solid blue lines show in turn the model's predictions in our counterfactual policy experiment. Such a policy change would imply a reduction in the expected federal funds rate of 15 basis points at the end of 2014 compared to the baseline forecast. According to the model, this alternative policy assumption generates a massive stimulus in 2012 and 2013. Indeed, in this alternative scenario, real GDP growth is forecast to jump to 3.5 percent in 2012 (Q4/Q4), and to 4.9 percent in 2013. GDP growth is however lower than under the baseline scenario in 2014 and 2015, as the effects of the policy stimulus fade over time and the GDP *level* returns to the level it would have had without the stimulus (see the second row in each of the three panels of Table 1). The stimulative effect of policy also raises inflation in 2012 and 2013 to respectively 1.8 percent (Q4/Q4) and 1.9 percent, but inflation is also forecast to remain below 2 percent in 2014 and 2015. The model seems to be generating an implausibly large response of real GDP growth and inflation to an apparently small change in the federal funds rate. What is responsible for this?

2.3 What is the excessive response due to?

To understand this, consider a simplified version of the FRBNY DSGE model in which there is no habit persistence. In this case, the consumption Euler equation reduces to the conventional expression

$$\widehat{c}_t = I\!E_t[\widehat{c}_{t+1}] - (\widehat{R}_t \; - \; I\!E_t[\widehat{\pi}_{t+1}] \; - \; I\!E_t[\hat{z}_{t+1}]), \tag{11}$$

where $\widehat{c}_t$ denote consumption deviations from steady state. Iterating this equation forward to eliminate expected future consumption, and abstracting from fluctuations in technology,

$\hat{z}_t$, we obtain

$$\widehat{c}_t = -\sum_{j=0}^{\infty} \mathbb{E}_t[\widehat{R}_{t+j} \; - \; \widehat{\pi}_{t+1+j}], \tag{12}$$

so that contemporaneous consumption is directly negatively related to the long-term real interest rate (at infinite maturity), which is given by the negative of the right-hand side of (12). Similarly, real investment is also related to the long-term real interest rate. A natural question is then whether the strong response of economic activity in the model to changes in the near-term path of the short-term interest rate is due to too strong a response of consumption and inflation to given changes in the long-term interest rate, or alternatively to too strong a response of the long-term interest rate.

Looking at the model's interest-rate projections farther into the future provides valuable insights. Figure 2 shows the paths of short-term interest rates under the baseline projection (red dashed lines), and the counterfactual policy (blue solid line) until 2027Q4. This figure reveals that while the expected short-term rate is only 15 basis points lower in the counterfactual than in the baseline at the end of 2014, the difference between the two interest-rate paths is expected to be much larger farther in the future, in particular between 5 and 10 years following the current policy announcement. These large drop far in the future of the expected future short-term rate compared to the baseline path is in turn resulting in a large drop of the long-term interest rate.

To see this more clearly, we compute the long-run interest rate response, proceeding as follows. At the end of period T, after the realization of all period-T shocks, the pre-intervention interest rate with maturity L at date $T+1$ is computed as the average of future short-term rate over the relevant horizon, and is given by the following expression:

$$\begin{aligned} R^L_{T+1} &= \tfrac{1}{L}\textstyle\sum_{j=1}^{L} \mathbb{E}_T[R_{T+j}] \\ &= \Psi_{R,1} + \tfrac{1}{L}\Psi_{R,2}(I-\Phi_1)^{-1}(I-\Phi_1^L)\Phi_1 s_T. \end{aligned} \tag{13}$$

The post-intervention 10-year rate R^{L*}_T, i.e., the rate obtained after the announcement of period-$T+1$ policy shocks $(\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\,\prime}_{1:K,T+1})$ is given by:

$$\begin{aligned} R^{L,*}_{T+1} &= \tfrac{1}{L}\textstyle\sum_{j=1}^{L} \mathbb{E}_T[R_{T+j}|\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}] \\ &= R^L_{T+1} + \Psi_{R,2}\tfrac{1}{L}(I-\Phi_1)^{-1}(I-\Phi_1^L)\Phi_{\epsilon,R}[\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\,\prime}_{1:K,T+1}]'. \end{aligned} \tag{14}$$

Call $\Delta R^L_{T+1} = R^{L,*}_{T+1} - R^L_{T+1}$ the impact of the intervention on rate with maturity L. It satisfies:

$$\Delta R^L_{T+1} = N_L[\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\ \prime}_{1:K,T+1}]' \tag{15}$$

where

$$N_L = \Psi_{R,2}\frac{1}{L}(I - \Phi_1)^{-1}(I - \Phi_1^L)\Phi_{\epsilon,R}. \tag{16}$$

In the counterfactual experiment, the 5-year yield falls by 16 basis points upon the announcement, compared to the baseline scenario, and the 10-year yield falls by as much as 25 basis points. The fact that the 10-year yields falls by more than the 5-year yield is simply a reflection, again, that the short-term interest rate is expected to deviate more from the baseline at long horizons than in the near term, as shown in Figure 2.

The model-implied responses for the long-term rate do not seem to match the 5 and 10-year yield responses observed in the data, however. Following the January 25, 2012 FOMC meeting, for instance, the statement reinforced the forward guidance about the federal funds rate by announcing an extension of the first liftoff date. This resulted in a reduction in 5 and 10-year yields of 8 and 7 basis points, respectively.[11]

Figure 3 shows the impulse response functions to contemporaneous and anticipated policy shocks in order to provide some more intuition for what is happening. Specifically, the figure shows the response of the short term interest rate, the 10-year nominal rate, the level of output, and inflation to expansionary 50 basis points shocks. The difference between the three columns in Figure 3 is that this 50 basis points shock is either contemporaneous (left

[11] As mentioned in the introduction, Carlstrom et al. (2012) assess the impact of pegging the policy rate in three variants of the New Keynesian model. They start from the steady-state equilibrium and analyze the effect of lowering the policy rate to the ZLB for K quarters. In their calibration, this amounts to lowering the policy rate by 4 percentage points for K quarters. In the simple version of the New Keynesian model, it can be shown that the response of inflation or output is directly proportional the distance between the steady state interest rate and the level at the ZLB, and the response of inflation and output grows exponentially with the number of periods that the policy rate is expected to be maintained at the ZLB. Since the short-term rate is assumed to return back to steady state $K + 1$ periods after the announcement, the 10-year long-run nominal rate is assumed to fall by $400K/40 = 10K$ basis points in their experiment. Concretely, their experiment assumes that a forward guidance of 8 quarters would imply a drop of 80 basis points in the 10-year bond yield.

column) or anticipated 4 and 8 periods ahead (middle and right columns, respectively). We want to highlight four features of Figure 3: i) Since the anticipated expansionary shock leads to higher inflation and output *before the shock takes place*, and since the policy authorities are bound to follow the rule before that date, the interest rate follows a zig-zag pattern, where it first rises and then falls. If this pattern of interest rates appears awkward, bear in mind that we are unlikely to see an eight periods-ahead shock *in isolation* (e.g., Campbell et al. (2012b)). ii) The response of the 10-year rate, quite understandably, reaches its lowest point at the time the shock takes place, and the trough decreases monotonically with the anticipation horizon. iii) The peaks in the response of output roughly coincide (with a slight delay) with the peaks in the response of the 10-year rate, in agreement with equation (12). In addition, the effect on output increases monotonically with the horizon. The delay is due to features like habit persistence. iv) The impact on inflation also increases monotonically with the horizon – not a surprising finding given the output responses.

The responses in Figure 3 provide some economic intuition behind the finding of Carlstrom et al. (2012) that the response of macroeconomic variables to an interest rate peg is a convex function of the horizon of the peg. Imagine the policymakers want to lower interest rates by 50 basis points for 7 periods. This can be implemented with a sequence of contemporaneous and anticipated shocks up to 7 periods ahead. Now imagine they decide to extend the peg one extra period. Because of the zig-zag feature of the 8th period impulse response, that decision will tend to lift the short-term rate in quarters 0 to 7 and so requires a cascade of shocks over that period to push the interest rate back down. In light of these impulse responses (and the related impact on the long rate) it is not surprising that even a modest amount of forward guidance produces large effects, as long as it extends far enough into the future.

3 Constraining the 10-year rate response to the announcement

As discussed above, the model's excessive response to the extension of forward guidance is attributable to the large response of the long-term bond yield. This suggests that the effects of forward guidance depend not only on the peg, but also very much on the way policy is conducted *after* the commitment to pegging the short rate has expired, that is, on the policy rule that is adopted after the rate's liftoff date. We assume here that policy will again be conducted according to the estimated historical rule and that the policymaker's communication is focused on altering the market participants' expectations about future rates in the near term, but it is not attempting to affect these expectations very far into the future.[12] Formally, we choose a set of anticipated policy shocks (i.e., a central bank communication policy) that has a given impact on the rate at the L-th maturity. Specifically, consider an estimate of the impact ΔR^{40}_{T+1} of the announcement on the 10-year yield. We want to choose the vector $\bar{\epsilon}^R$ that minimizes the (weighted) deviations from the baseline federal funds rate path over the next 40 quarters, $M_{40}\bar{\epsilon}^R$, subject to delivering a drop of the 10-year bond yield, $N_{40}\bar{\epsilon}^R$, of the given magnitude. Our problem is

$$\min_{\bar{\epsilon}^R} \bar{\epsilon}^{R'} M'_{40} W M_{40} \bar{\epsilon}^R - \lambda (N_{40}\bar{\epsilon}^R - \Delta R^{40}_{T+1})$$

where W is a diagonal weighting matrix and λ is the Lagrange multiplier, and the matrices M_{40} and N_{40} are defined respectively in (9) and (16). The solution is:[13]

$$\bar{\epsilon}^R = \left(M'_{40} W M_{40}\right)^{-1} N'_{40} \left(N_{40} \left(M'_{40} W M_{40}\right)^{-1} N'_{40}\right)^{-1} \Delta R^{40}_{T+1}. \tag{17}$$

We therefore replace equation (10) in step 3 of Algorithm 1 with (17).

The two inputs of our proposed approach are the impact ΔR^{40}_{T+1} of the announcement on the 10-year yield and the diagonal weighting matrix W which penalizes deviations of the model-based federal funds rate path from the pre-announcement market expectations. We have found that the former matters most in term of the impact of the macroeconomy. How should one pick ΔR^{40}_{T+1}? Previous episodes of forward guidance provide some evidence regarding plausible values of ΔR^{40}_{T+1}. For instance, the forward guidance at the January 25, 2012 FOMC meeting – a six quarter extension of the horizon for which the FOMC "anticipates that exceptionally low levels for the federal funds rate" – yielded a 7 basis points

[12] An alternative analysis of the long-term rate's excessive response may involve an evaluation of the effects of different policy rules after the liftoff date. We leave such analysis for future work.

[13] The first-order condition with respect to $\bar{\epsilon}^R$ is $2\bar{\epsilon}^{R'} \left(M'_{40} W M_{40}\right) = \lambda N_{40}$, which implies: $\bar{\epsilon}^R = (\lambda/2) \left(M'_{40} W M_{40}\right)^{-1} N'_{40}$. Pre-multiplying on both sides by N_{40}, using (15) to replace $N_{40}\bar{\epsilon}^R$ with the desired change in the 10-year bond yield ΔR^{40}_{T+1}, and solving for λ, we obtain: $(\lambda/2) = \left(N_{40} \left(M'_{40} W M_{40}\right)^{-1} N'_{40}\right)^{-1} \Delta R^{40}_{T+1}$. We then use this to eliminate λ in the first-order condition to obtain the solution for $\bar{\epsilon}^R$.

decline in the 10-year rate. Even if nailing down the exact response of the 10-year rate to forward guidance can be challenging, the the policymakers can use this approach to construct upper and lower bounds on the macroeconomic impact of the announcement based on what they regard as reasonable bounds for the effect on the 10-year rate.

For given effect on ΔR^{40}_{T+1} the choice of W does not matter much in term of the responses of output and inflation. It matters however in terms of the resulting expected path of the short rate, and hence we want to choose W in such a way that the latter is close to what policymakers would expect from the announcement. Specifically, we choose weights so as to penalize highly these deviations in the short run, i.e., in the first 3-4 quarters, as the federal funds rate is already in line with desired policy path over that period, and the announcement is unlikely to change this. We also also penalize highly the deviations expected future short term rate in the long-run, that is 8 years after the announcement and thereafter, so that the federal funds rate is expected to revert to its baseline path in the long-run. Indeed, we consider it as unlikely for the announcement of an extension of the forward guidance by two quarters to have large effects at that horizon. However, we provide smaller penalties on deviations of the interest rate path over the intermediate horizon. Figure 4 plots the weights used for the different horizons.

Figure 1 reports with the solid red lines the model's predictions conditional on the proposed alternative experiment (recall that the dashed red lines correspond to the FRBNY DSGE model's baseline forecast). In this scenario, we assume that the extension of the forward guidance by two more quarters (from late-2014 to mid-2015) would yield a decline in the 10-year rate by at most 10 basis points, which we view as an upper bound based on the evidence from the January 25, 2012 FOMC meeting. Such a policy change reduces the expected federal funds rate from 40 basis points to 13 basis points at the end of 2014. Note that the FFR path in the short term is actually lower than what we would obtain by fixing the FFR at 25 basis points, and is therefore quite consistent with the likely impact of credible forward guidance. According to the model, this alternative policy has a stimulative effect in 2012 and 2013, with real GDP growth increase by 1/2 a percentage point in 2012 (Q4/Q4) relative to the baseline, and by 0.8 percentage point in 2013 (see Table 1). GDP growth is however somewhat lower than in the baseline scenario in 2014 and 2015, as the effects of the policy stimulus fade over time. This response is still quite large, but as discussed above we view this as an upper bound. The stimulative effect of policy has little effect on core PCE inflation with a 0.2 percentage point increase in 2013 inflation but no change in other years.[14]

Finally, the reader who is bothered by the rather subjective choices of ΔR^{40}_{T+1} and W in our procedure should note that a policymaker could in principle do away with these choices altogether if she (or her staff) is willing to hand-pick the anticipated shocks so to deliver what she regards as "likely path" in FFR forecasts following the announcement. The point of this section is to show that as long as this "likely path" does not involve shifts in FFR expectations far into the future, the DSGE model will yield reasonable answers as to the macroeconomic effect of the experiment. If she is not too picky on the exact path however, but is willing to make an educated guess on the impact on the 10-year rate, the procedure in (17) will deliver a straightforward way of choosing the anticipated shocks.

4 Conclusion and postscript

Our proposed solution to the "forward guidance puzzle" is based on the realization that the apparently straightforward experiment *"let us fix the short term interest rate to x percent for K periods"* has implications for the short term rate that go well beyond the K-th period in medium scale DSGE models. As a consequence, these counterfactuals appear to have an over-sized effect on the macroeconomy. We view the implications of these experiments of short term interest rate in the far future as incredible. They are at odds with both common sense and the empirical evidence of the effects of announcements. We therefore propose to capture the effect of the forward guidance via a slightly different implementation of the experiment, one where we constrain the overall impact of the guidance on long term rates. We show that this alternative approach produces quite reasonable effects on the macroeconomy, as well as a path for short term rates that arguably agrees with the likely impact of credible forward guidance.

[14]In this model the estimated output-inflation trade-off, or sacrifice ratio, is quite favorable to the central bank.

Postscript: On September 13, 2012, the FOMC actually did extend forward guidance through mid-2015, as in our counterfactual experiment. As always, controlled experiments are hard to come by in macroeconomics, and the statement of September 13, 2012 can hardly be characterized as a test of our theory. This is for a number of reasons. First, between August 28 (the date for which we collected market expectations for the baseline forecast) and September 13 there was the release of the employment report. As the news were bad, FFR market expectations had adjusted to incorporate further accommodation on the part of the central bank. Moreover, the FOMC statement also contained language concerning additional long-term asset purchases, information that these purchases would be in MBS securities as opposed to Treasuries, and an indication that policy accommodation would continue until labor market conditions have improved.[15] In general it will always be hard, if not impossible, to test the predictions of DSGE models by looking at the outcome of policy counterfactuals such as the ones in our paper: even if the counterfactual is implemented, this will not occur in a controlled environment. Fortunately, we have other ways of testing DSGE models (see Christiano et al. (2005), Del Negro et al. (2007)). Nonetheless, we argue that counterfactuals like the one performed here are useful for policy makers in order to quantify the potential effects of their policies, particularly when alternative approaches are lacking as is the case here.

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[15]From the FOMC statement: "If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability."

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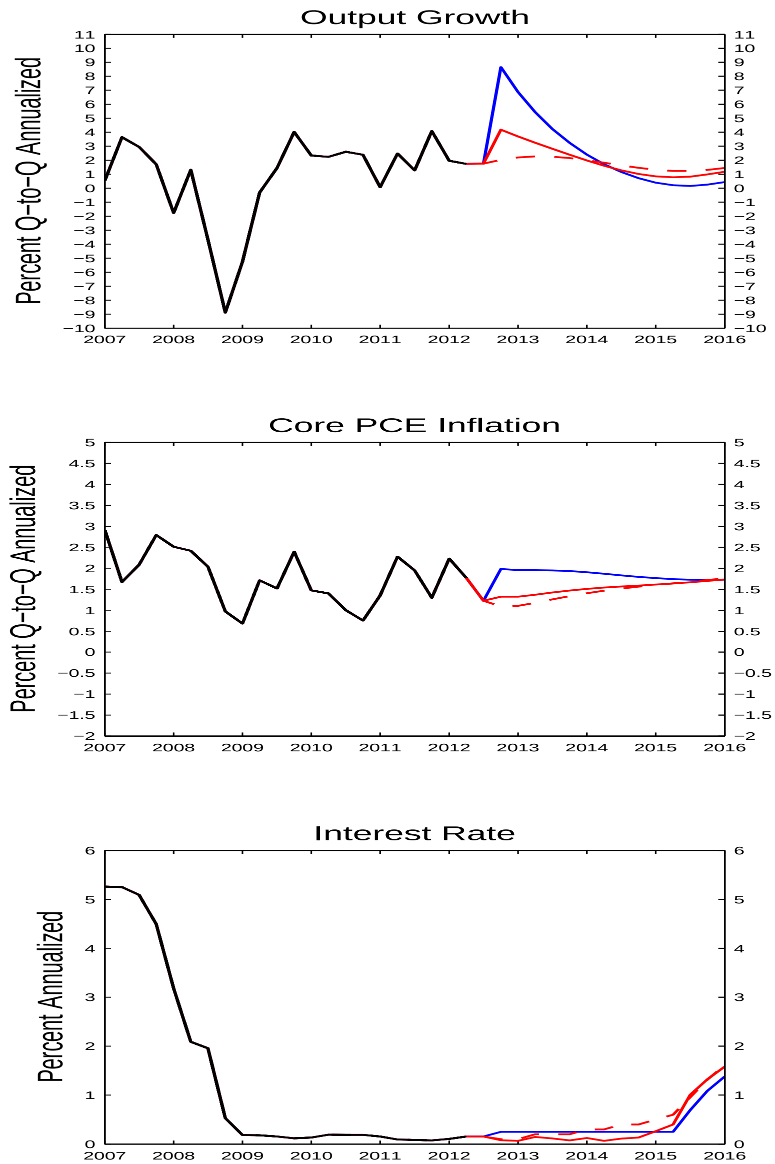
Table 1: The macroeconomic consequences of forward guidance

	2012 (Q4/Q4)	2013 (Q4/Q4)	2014 (Q4/Q4)	2015 (Q4/Q4)
GDP growth				
Baseline	1.9	2.2	1.7	1.3
FFR at 25bp	3.5	4.9	1.5	0.3
Forward guidance with constrained 10y yield	2.4	3.0	1.5	0.9
Core PCE inflation				
Baseline	1.6	1.2	1.5	1.6
FFR at 25bp	1.8	1.9	1.8	1.7
Forward guidance with constrained 10y yield	1.6	1.4	1.5	1.6
Federal funds rate				
Baseline	0.10	0.20	0.40	1.32
FFR at 25bp	0.25	0.25	0.25	1.09
Forward guidance with constrained 10y yield	0.08	0.08	0.13	1.32

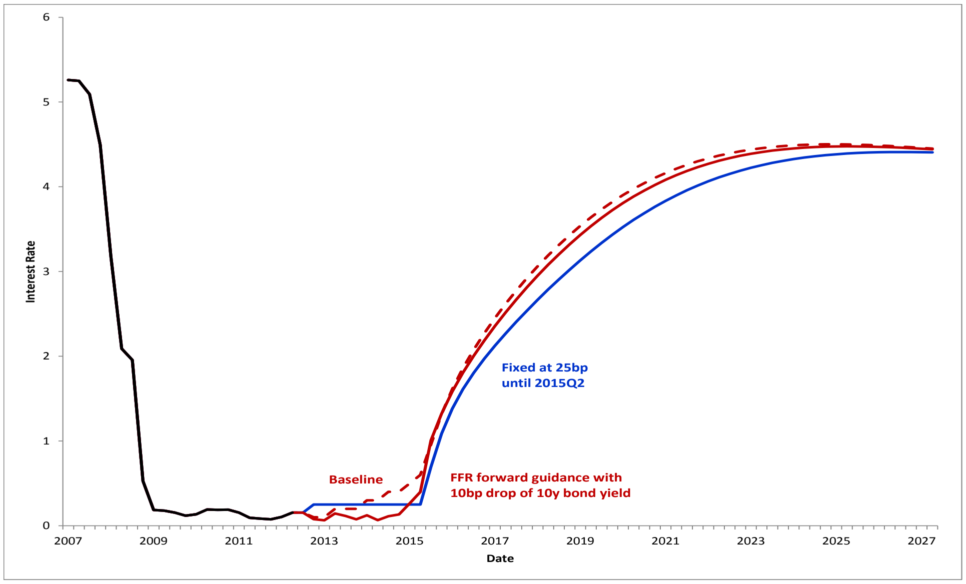
Notes: The table reports the model's predictions conditional on alternative assumptions regarding the federal funds rate: the baseline forecast, a counterfactual policy experiment in which the federal funds rate is maintained at 25 basis points until 2015Q2, and a counterfactual policy experiment in which more forward guidance is provided about the federal funds rate such that the 10-year bond yield falls by 10 basis points.

Figure 1: The macroeconomic consequences of forward guidance



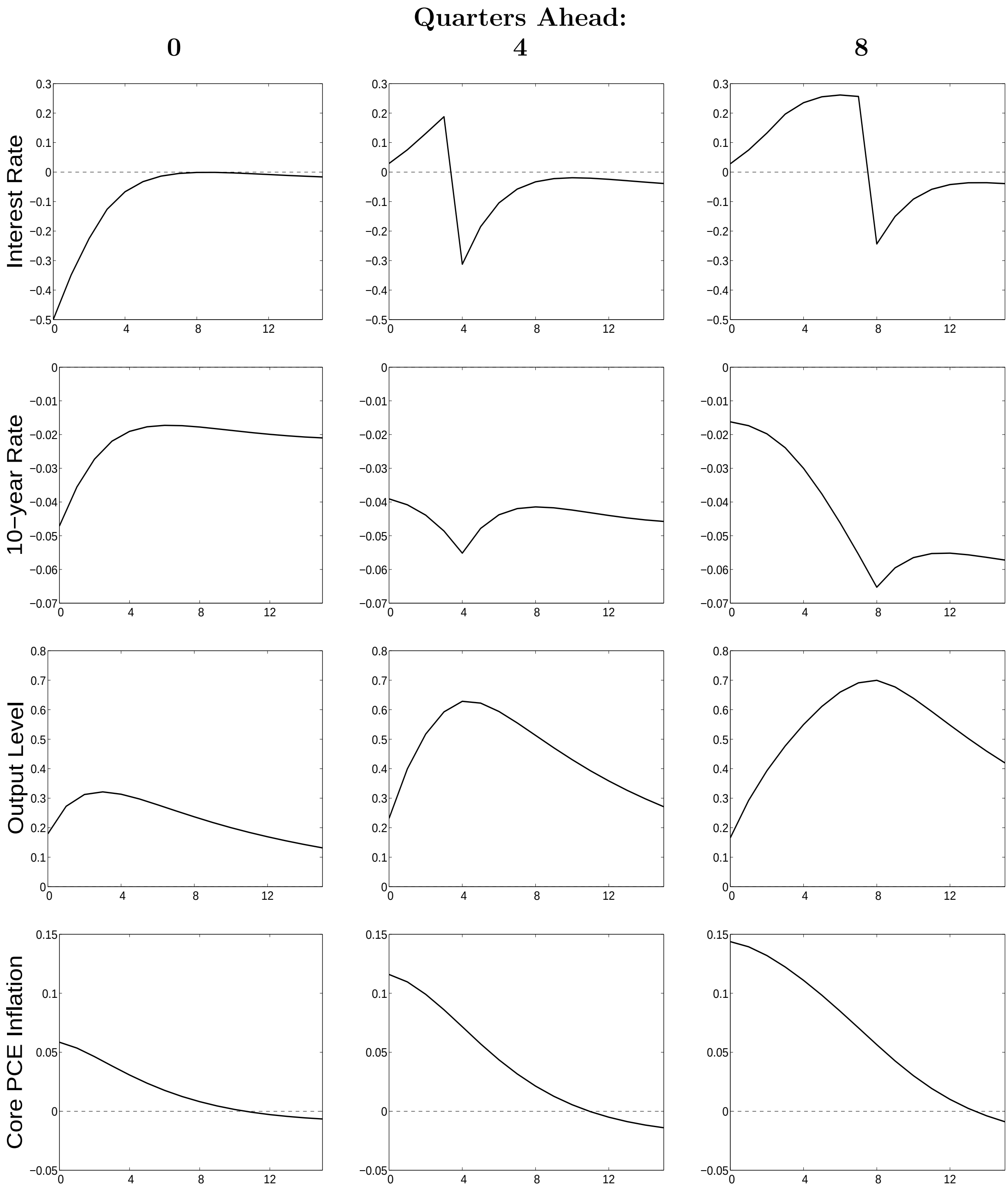
Notes: The figure shows the model's predictions conditional on alternative assumptions regarding the federal funds rate. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. The solid blue lines show in turn the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2. The solid red lines show the model's predictions in a counterfactual policy experiment in which more forward guidance is provided about the federal funds rate such that the 10-year bond yield falls by 10 basis points.

Figure 2: Interest-rate projections farther into the future



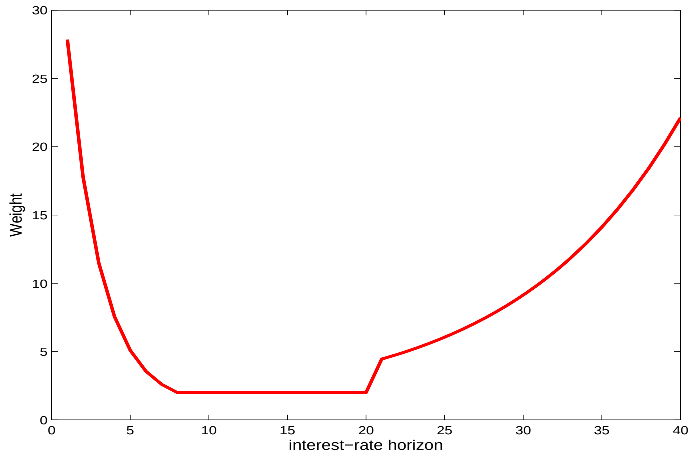
Notes: The figure shows the model's predictions for the federal funds rate farther into the future. The black solid line shows the historical data. The dashed red line shows the FRBNY DSGE model's baseline forecast. The solid blue line shows the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2. The solid red line shows the model's predictions in a counterfactual policy experiment in which the 10-year bond yield falls by 10 basis points.

Figure 3: Impulse response functions to contemporaneous and anticipated policy shocks



Notes: The figure shows the percent change over a 12 quarter horizon of the short term interest rate, the 10-year nominal rate, the level of output, and Core PCE inflation in response to a contemporaneous, 4 quarter and 8 quarter ahead negative 50 basis points policy shock.

Figure 4: Penalty for interest rate deviations from baseline path at various horizons



Notes: The figure shows the weights that penalize deviations of the model-based federal funds rate path from the pre-announcement market expectations at horizons of 1 to 40 quarters.

A Appendix

A.1 Model Description

In this appendix, we summarize the log-linear equations that characterize the FRBNY DSGE model. Because the model has a source of non-stationarity in the process for technology Z_t, to solve the model we first rewrite its equilibrium conditions in terms of stationary variables, and then solve for the non-stochastic steady state of the transformed model. Finally we take a log-linear approximation of the transformed model around its steady state.

1. *Evolution of Marginal Costs (or labor share):*

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}_t^k$$

   where $\hat{mc}_t$ is nominal marginal cost, $\hat{w}_t$ nominal wage, α the elasticity of output to capital and $\hat{r}_t^k$ the rental rate of capital.

2. *Philips curve:*

$$\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p}\hat{mc}_t + \frac{1}{\zeta_p}\tilde{\lambda}_{f,t}$$

   where $\hat{\pi}_t$ is inflation, β is the discount factor, and ζ_p is the Calvo parameter, representing the fraction of firms that cannot adjust their prices each period. We use the following re-parameterization of the cost-push shock: $\tilde{\lambda_{f,t}} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)]\lambda_{f,t}$ where λ_f is the steady state value of the markup shock.

3. *Evolution of the Capital Stock:*

$$\hat{\bar{k}}_t = -(1-\tfrac{i^*}{k^*})\hat{z}_t + (1-\tfrac{i^*}{k^*})\hat{\bar{k}}_{t-1} + \tfrac{i^*}{k^*}\hat{\mu}_t + \tfrac{i^*}{k^*}\hat{i}_t$$

   where $\hat{\bar{k}}_t$ is installed capital , $\hat{z}_t$ is the growth rate of productivity, i^* and k^* are steady state investment and level of capital, respectively, and $\hat{\mu}_t$ is the exogenous process that affects the efficiency by which a foregone unit of consumption contributes to capital utilization.

4. *Evolution of Effective Capital:*

$$\hat{k}_t = \hat{u}_t - \hat{z}_t + \hat{\bar{k}}_{t-1}$$

where $\hat{k}_t$ is the capital and $\hat{u}_t$ is the level of capital utilization.

5. *Marginal Utility of Consumption:*

$$(e^\gamma - h\beta)(e^\gamma - h)\hat{\xi}_t = e^\gamma(e^\gamma - h)\hat{b}_t \; - \; (e^{2\gamma} + \beta h^2)\hat{c}_t + he^\gamma\hat{c}_{t-1} \; - \; he^\gamma\hat{z}_t - \beta h(e^\gamma - h)\mathbb{E}_t[\hat{b}_{t+1}] + \beta he^\gamma\mathbb{E}_t[\hat{c}_{t+1}] \; + \beta he^\gamma\mathbb{E}_t[\hat{z}_{t+1}]$$

where $\hat{b}_t$ represents the government bond holdings, $\hat{c}_t$ is consumption, e^γ is the growth rate of the economy, h captures habit persistence in consumption, and $\hat{\xi}_t$ is the marginal utility of consumption.

6. *Optimal Demand for Money:*

$$v_m\hat{m}_t = \hat{\chi}_t + \hat{b}_t - \tfrac{1}{R^*-1}\hat{R}_t - \hat{\xi}_t$$

where $\hat{m}_t$ is money, $\hat{\chi}_t$ is a stochastic preference shifter that affects the marginal utility from real money balances and $\hat{b}_t$ another stochastic preference shifter that scales the overall period utility.

7. *Euler Equation:*

$$\hat{\xi}_t = \hat{R}_t + \mathbb{E}_t[\hat{\xi}_{t+1}] - \mathbb{E}_t[\hat{z}_{t+1}] - \mathbb{E}_t[\hat{\pi}_{t+1}]$$

where $\hat{R}_t$ is the gross nominal interest rate on government bonds

8. *Optimal Investment:*

$$\hat{i}_t = \tfrac{1}{1+\beta}\mathbb{E}_t[\hat{\imath}_{t-1} - \hat{z}_t] + \tfrac{\beta}{1+\beta}\mathbb{E}_t[\hat{\imath}_{t+1} + \hat{z}_{t+1}] + \tfrac{1}{(1+\beta)S''e^{2\gamma}}\hat{q}^k_t + \tfrac{1}{(1+\beta)S''e^{2\gamma}}\hat{\mu}_t$$

where $\hat{i}_t$ is investment, S is the cost of adjusting capital, with S'' positive, and $\hat{q}^k_t$ is the price of capital.

9. *Capital Utilization:*

$$r^k_*\hat{r}^k_t = a''\hat{u}_t$$

where r^k_* is the steady state rental rate of capital. The function a describes the cost of capital utilization as a function of the level of capital utilization.

10. *Wage Equation:*

$$(1+\nu_l \tfrac{1+\lambda_w}{\lambda_w})\hat{\tilde{w}}_t \;+\; (1+\zeta_w\beta\nu_l(\tfrac{1+\lambda_w}{\lambda_w}))\hat{w}_t = \zeta_w\beta(1+\nu_l\tfrac{1+\lambda_w}{\lambda_w})\mathbb{E}_t[\hat{\tilde{w}}_{t+1}+\hat{w}_{t+1}] \;+\; (1-\zeta_w\beta)(e^{2\gamma}+h^2\beta)\tfrac{e^{-\gamma}}{e^{\gamma}-h}\hat{b}_t \;+\; \hat{\varphi}_t \;+\; (1-\zeta_w\beta)(\nu_l\hat{L}_t-\hat{\xi}_t) \;+\; \zeta_w\beta(1+\nu_l\tfrac{1+\lambda_w}{\lambda_w})\mathbb{E}_t[\hat{\pi}_{t+1}+\hat{z}_{t+1}]$$

where λ_w affects the elasticity of substitution between differentiated labor services, $\hat{\tilde{w}}_t$ is the optimal wage chosen by households that can set their optimal wage and $\hat{\varphi}_t$ is a stochastic preference shifter affeting the marginal utility of leisure. It is assumed to follow an AR(1) process.

11. *Aggregate Wage Equation:*

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \tfrac{1-\zeta_w}{\zeta_w}\hat{\tilde{w}}_t$$

where ζ_w is the fraction of workers who cannot adjust their wages in a given period.

12. *Capital-Labor Ratio:*

$$\hat{k}_t = \hat{w}_t - \hat{r}^k_t + \hat{L}_t$$

13. *Resource Constraint:*

$$\hat{y}_t = \hat{g}_t + \tfrac{c^*}{c^*+i^*}\hat{c}_t + \tfrac{i^*}{c^*+i^*}\hat{\imath}_t + \tfrac{r^{k*}k^*}{c^*+i^*}\hat{u}_t$$

where $\hat{y}_t$ is output and $\hat{g}_t$ is government spending, which we assume follows an AR(1) process.

14. *Production Function:*

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)\hat{L}_t$$

15. *Policy Rule:*

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi\sum_{j=0}^{3}\hat{\pi}_{t-j} + \psi_y\sum_{j=0}^{3}(\hat{y}_{t-j}-\hat{y}_{t-j-1}+\hat{z}_{t-j})\right) + \epsilon^R_t + \sum_{k=1}^{K}\epsilon^R_{k,t-k}$$

where ψ_π and ψ_y are the central bank's response deviations of inflation from target and deviations of output growth from stead state, respectively. ρ_R captures the persistence in the policy rule and ϵ_t^R is a monetary policy shock s.t. $\epsilon_t^R \sim N(0, \sigma_r^2),\ i.i.d.$

16. *Expected Excess Return on Capital:*

$$\mathbb{E}_t\left[\hat{\tilde{R}}^k_{t+1} - \hat{R}_t\right] = \zeta_{sp,b}\left(\hat{q}_t^k + \hat{\bar{k}}_t - \hat{n}_t\right) + \tilde{\sigma}_{\omega,t}$$

where $\zeta_{sp,b}$ is the elasticity of spread with respect to leverage, $\hat{n}_t$ is the entrepreneur's net worth expressed in nominal terms, and $\tilde{\sigma}_{\omega,t}$ is the volatility of the idiosyncratic random productivity, which changes over time according to an AR(1) process. This equation therefore defines the expected excess return on capital as a function of the leverage of the firms (the ratio of the value of capital to net worth) and exogenous shocks.

17. *Evolution of the Entrepreneurs' Net Worth:*

$$\hat{n}_t = \zeta_{n,\tilde{R}^k}\left(\hat{\tilde{R}}^k_t - \pi_t\right) - \zeta_{n,R}\left(\hat{R}_{t-1} - \pi_t\right) + \zeta_{n,qK}\left(\hat{q}_{t-1}^k + \hat{\bar{k}}_{t-1}\right) + \\ \zeta_{n,n}\hat{n}_{t-1} - \gamma^e \tfrac{v_*}{n_*}\hat{z}_t - \tfrac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\tilde{\sigma}^2_{\omega,t-1}$$

where $\zeta_{n,\tilde{R}^k}$, $\zeta_{n,R}$, ζ_{n,q^K}, $\zeta_{n,n}$, and ζ_{n,σ_ω} are the elasticities of net worth to installed capital, nominal interest rate, cost of capital, itself and the volatility of productivity, respectively. ζ_{sp,σ_ω} is the elasticity of the spread with respect to the volatility of productivity and γ^e is the fraction of entrepreneurs who survive each period.

18. *Realized Return on Capital:*

$$\hat{\tilde{R}}^k_t - \pi_t = \frac{r_*^k}{r_*^k + (1-\delta)}\hat{r}_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)}\hat{q}_t^k - \hat{q}_{t-1}^k$$

where δ is the rate of capital depreciation. This equation links the realized return on capital to the capital rental rate and the evolution of the price of capital

We use the method in Sims (2002) to solve the system of log-linear approximate equilibrium conditions. We collect all the DSGE model parameters in a vector θ and stack the structural shocks in a vector ϵ_t. The vector of ϵ_t is composed of seven exogenous shocks: a productivity shock z_t, a labor shock φ_t, a marginal efficiency of investment (MEI) shock μ_t, a government policy shock g_t, a price mark-up shock $\lambda_{f,t}$, a spread shock $\sigma_{\omega,t}$, and a monetary policy shock $\epsilon_{R,t}$.

A.2 Measurement Equations

$$
\begin{array}{llcl}
\text{Real output growth (\%, annualized)} & 400(\ln Y_t - \ln Y_{t-1}) & = & 400(\widehat{y}_t - \widehat{y}_{t-1} + z_t) \\
\text{Hours (\%)} & 100 \ln L_t & = & 100(L_t + \ln L^{adj}) \\
\text{Labor Share (\%)} & 100 \ln LS_t & = & 100(\widehat{L}_t + \widehat{w}_t - \widehat{y}_t + \ln LS_*) \\
\text{Inflation (\%,annualized)} & \pi_t^{Core} & = & 400(\widehat{\pi}_t + \ln \pi_*) \\
\text{Interest Rate (\%,annualized)} & FFR_t & = & 400(\widehat{R}_t + \ln R_*), \\
\text{Spread (\%,annualized)} & SP_t & = & 400(\mathbb{E}_t \left[\widehat{\tilde{R}}^k_{t+1} - \hat{R}_t \right] + SP_*),
\end{array}
$$

where the parameter L^{adj} captures the units of measured hours.

A.3 Prior and Posterior

Parameters	Prior Mean	Prior Stdd	Post Mean	90% Lower Band	90% Upper Band
Policy Parameters					
ψ_1	2.000	0.250	2.016	1.782	2.257
ψ_2	0.200	0.100	0.273	0.175	0.369
ρ_r	0.500	0.200	0.762	0.721	0.804
σ_r	0.200	4.000	0.152	0.132	0.170
Nominal Rigidities Parameters					
ζ_p	0.750	0.100	0.879	0.852	0.908
ζ_w	0.750	0.100	0.904	0.872	0.938
Other "Endogenous Propagation and Steady State" Parameters					
α	0.330	0.020	0.350	0.345	0.355
a''	0.200	0.100	0.294	0.129	0.444
h	0.700	0.050	0.704	0.636	0.770
S''	4.000	1.500	3.121	2.247	3.994
ν_l	2.000	0.750	1.273	0.465	2.017
r^*	1.500	1.000	0.288	0.038	0.525
π^*	2.000	0.250	2.382	2.115	2.655
γ	2.750	0.500	1.687	1.307	2.063
g^*	0.300	0.100	0.195	0.090	0.300
ζ_{sp}	0.050	0.020	0.070	0.041	0.100
spr_*	2.000	0.500	1.163	0.750	1.556
ρs and σs					
ρ_z	0.400	0.250	0.487	0.369	0.605
ρ_ϕ	0.750	0.150	0.284	0.165	0.397
ρ_{λ_f}	0.750	0.150	0.470	0.364	0.572
ρ_μ	0.750	0.150	0.991	0.982	1.000
ρ_g	0.750	0.150	0.927	0.847	0.999
ρ_{sigw}	0.750	0.150	0.965	0.938	0.995
σ_z	0.300	4.000	0.788	0.695	0.877
σ_ϕ	3.000	4.000	29.403	12.878	45.636
σ_{λ_f}	0.200	4.000	0.087	0.072	0.101
σ_μ	0.750	4.000	0.358	0.271	0.450
σ_g	0.500	4.000	0.224	0.180	0.266
σ_{sigw}	0.050	4.000	0.085	0.076	0.095

Note: The following parameters are fixed: $\delta = 0.025$, $\nu_m = 2$, $\lambda_w = 0.3$, $\chi = 0.1$, $\lambda_f = 0.15$, $F(\omega) = 0.15$, $\gamma_* = 0.99$. L^{adj} has a prior mean of 253.500, with standard deviation at 5.