Deriving the Taylor Principle when the Central Bank Supplies Money*

Ceri Davies
Cardiff Business School

Max Gillman
Cardiff Business School

Michal Kejak
CERGE-EI

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Abstract

The paper presents a human-capital-based endogenous growth, cash-in-advance economy with endogenous velocity where exchange credit is produced in a decentralized banking sector, and money is supplied stochastically by the central bank. From this it derives an exact functional form for a general equilibrium 'Taylor rule'. The inflation coefficient is always greater than one when the velocity of money exceeds one; velocity growth enters the equilibrium condition as a separate variable. The paper then successfully estimates the magnitude of the coefficient on inflation from 1000 samples of Monte Carlo simulated data. This shows that it would be spurious to conclude that the central bank has a reaction function with a strong response to inflation in a 'Taylor principle' sense, since it is only meeting fiscal needs through the inflation tax. The paper also estimates several deliberately misspecified models to show how an inflation coefficient of less than one can result from model misspecification. An inflation coefficient greater than one holds theoretically along the balanced growth path equilibrium, making it a sharply robust principle based on the economy's underlying structural parameters.

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1 Introduction

What does it mean to say that the central bank should be following the Taylor (1993) rule? And what does it mean when argued that current policy is wrong if it does not follow the Taylor rule? If a central bank should adhere to a Taylor rule over the long run, then can the Taylor rule simply be added onto neoclassical models as an ad hoc feature or used as a central component of short run Phillips curve models with sticky prices in the New Keynesian mold? Further, if in the long run the central bank targets inflation at some rate such as zero or two percent, and if the Taylor rule should be followed in the long run, then does that not translate into simply saying that central banks should track the market's fluctuating real interest rate in the long run using the Taylor rule? (See Fama's, 2012, perspective on this).

It is conventional to view interest rate rules as monetary policy 'reaction functions' that represent how the central bank adjusts a short-term nominal interest rate in response to the state of the economy. The magnitude of the reaction function coefficients are interpreted to reflect a policy-maker's preferences towards variation in key macroeconomic variables such as inflation and variously defined output gaps. It has been suggested that policy-makers ought to adhere to the 'Taylor principle', whereby inflation above target is met by a more-than-proportional increase in the short-term nominal interest rate and hence an increase in the real interest rate. Such an interest rate rule forms one of the three core equations of the prominent New Keynesian modelling framework, such as in Woodford (2003), Clarida et al. (1999); and Clarida et al. (2000). One well-known finding comes from the latter paper which concludes that the Taylor principle holds for a 'Volcker-Greenspan' sample of U.S. data but that it is violated for a 'pre-Volcker' sample during which the Fed was deemed to be accommodating in its reaction to inflation. Davig and Leeper (2007) provide a reduced-from model view of how Taylor rules can hold in the long run including possible shifts between "active" and "passive" monetary regimes.

From a different angle, an historical strand of literature going back to Poole (1970), and updated by Alvarez et al. (2001) and Chowdhury and Schabert (2008), for example, considers interest rate rules and money supply rules as two ways of implementing the same monetary policy. This paper perhaps most closely follows Alvarez et al. (2001) by deriving the equilibrium nominal interest rate in 'rule form' within a general equilibrium economy in which the central bank conducts monetary policy by stochastically supplying money. Instead of an exogenous fraction of agents being able to use bonds as in Alvarez et al., here the consumer purchases goods with an endogenous fraction of bank-supplied intratemporal credit that avoids the inflation tax on exchange. This cash-in-advance monetary economy is also extended to include endogenous growth, along with endogenous velocity, as in Benk et al. (2010). The resulting equilibrium condition for the nominal interest rate 'nests' the standard Taylor rule within a more general forward-looking setting that endogenously includes traditional monetary elements, such as the (exogenous) velocity in Alvarez et al., and the money demand in McCallum and Nelson (1999).

The endogenous growth aspect implies that the 'target' terms of the equilibrium 'Taylor condition', such as the inflation target or the 'potential' output level, are the balanced growth path (BGP) equilibrium values of the relevant variables. In addition, the coefficients of the Taylor condition are a function of the model's utility and technology parameters along with the BGP money supply growth rate. This in essence fulfills Lucas's (1976) goal of postulating policy rules with coefficients that depend explicitly upon the economy's underlying utility and technology coefficients plus a key policy choice, in this case the BGP rate of money supply growth. This differs from Leeper and Zha's (2003) novel approach to the Lucas critique in monetary policy in that they study whether the variance of the money supply growth may be changing, interpreting the Lucas critique as holding when the variance is unchanged. Our model assumes a single money supply variance for the entire period, and within this context shows how the average growth rate of money supply on the BGP is a structural policy parameter that the consumer understands as part of the equilibrium conditions used to determine their behavior. The Taylor principle results from the structural parameters within a *policy function,* or optimal control law, or policy rule, as Lucas and Sargent (1981) state it variously, given a certain distribution of policy control processes, in this case the money supply growth rate.

The Lucas critique framing of the Taylor condition given for a constant variance distribution could be said to complement the Leeper and Zha (2003) focus on possible changes in the money supply rule variance distribution per se. It provides a solid theoretical result that could be said to exist within each regime, or within just a single regime if that holds over the entire period, as is modeled here: a structural derivation of the 'Taylor principle' whereby the coefficient on the inflation term always exceeds one for any given non-Friedman (1969) optimum BGP money supply growth rate, equals one only at the Friedman optimum, and never falls below one. Equivalently, the inflation coefficient always exceeds one when the endogenous velocity exceeds one since the cash-in-advance velocity rises above one for any non-Friedman optimal rate of money supply growth. In general, the inflation coefficient rises with the BGP velocity level. Another central result is that the expected velocity growth rate itself enters the Taylor condition as an additional term, in contrast to standard Taylor rules. Omitting this term can cause misspecification bias in estimated Taylor rules within the economy.

Having derived the Taylor condition, the paper then estimates it by applying three conventional estimation procedures to one thousand samples of artificial data simulated from the baseline model, where the simulated data is passed through three standard statistical filters prior to estimation. The results verify the theoretical form of the Taylor condition along several key dimensions. In particular, the coefficient on inflation is greater than one and close to its theoretical magnitude for all three estimation techniques and for all three data filters. Robustness tests explore the impact of estimating two alternative Taylor conditions. This involves the use of two *ad hoc*, deliberately misspecified equations which differ from the 'true' theoretical expression: the first changes just one of the variables in the Taylor condition while the second posits a standard Taylor rule which involves multiple misspecification errors. Using the same artificial data, the two misspecified models produce an estimated coefficient on inflation which falls below one, a violation of the 'Taylor principle'. In the context of actual data, this result would typically be interpreted as the central bank being 'passive', or 'weak', or 'accommodative' towards inflation. Here, the paper shows that such an interpretation could be spurious as this result occurs simply due to misspecification in the estimating equation.[1]

The estimated 'Taylor rule' emerges even though the central bank is merely satisfying the government's fiscal needs through the inflation tax. This implies the central point of the paper: it would be spurious within this economy to associate the Taylor condition with a 'reaction function' for the nominal interest rate since in the model the central bank just stochastically prints money. Also, failure of the so-called Taylor principle in numerous published empirical studies may be a result of model misspecification rather than behavioral changes by the central bank per se. Indeed, our current preliminary extension of this work, not presented here, shows that estimation with actual U.S. data of Taylor rules which include the unconventional terms implied by the theory of this paper - particularly velocity growth - can reverse the result that the coefficient on inflation falls below unity during periods of macroeconomic instability.[2]

Related work is vast but includes Taylor (1999), who alludes to the possibility that an interest rate rule can be derived from the quantity theory of money. Sørensen and Whitta-Jacobsen (2005, pp.502-505) present such a derivation under the assumption of constant money growth whereby the coefficients of the 'rule' relate to elasticities of money demand rather than the preferences of policy-makers. Fève and Auray (2002) and Schabert (2003) consider the link between money supply rules and interest rate rules in standard cash-in-advance models with velocity fixed at unity. Alternatively, the paper could be viewed in the context of Canzoneri et al's. (2007) account of the shortcomings of estimated Euler equations because it shows how the Euler equation in combination with the stochastic asset pricing kernel can be used to derive a Taylor condition which can be estimated successfully.

Section 2 describes the economy, as in Benk et al. (2008, 2010). Section 3 derives the model's 'Taylor condition' and Section 4 provides the baseline calibration. Section 5 describes the econometric methodology applied to model-simulated data and presents the estimation results. Section 6 derives theoretical special cases of the more general (Section 2) model to show how alternative Taylor conditions can be derived. Section 7 presents a discussion and Section 8 concludes.

[1] Estimation of simulated data is conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model. We are indebted to Warren Weber for the suggestion to follow such an approach here.

[2] Clarida et al.'s (2000) 'pre-Volcker' sample, for example, corresponds to a period of high and variable inflation.

2 Stochastic Endogenous Growth with Banking

The representative agent economy is as in Benk et al (2008, 2010) but with a decentralized banking sector that produces credit as in Gillman and Kejak (2011). By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically in Gillman et al. (2004) and Fountas et al. (2006), for example. Further, money supply shocks can cause inflation at low frequencies, as in Haug and Dewald (2012) and as supported by Sargent and Surico (2008, 2011), which can lead to output growth effects if the shocks are persistent and repeated. This allows shocks over the business cycle to cause changes in growth rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to credit sector productivity exists by virtue of the model's endogenous money velocity. Exchange credit is produced via a functional form used extensively in the financial intermediation microeconomics literature starting with Clark (1984) and promulgated by Berger and Humphrey (1997) and Inklaar and Wang (2013), for example.

The shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process. For goods sector productivity, z_t, the money supply growth rate, u_t, and bank sector productivity, v_t:

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \tag{1}$$

where the shocks are $Z_t = [z_t \; u_t \; v_t]'$, the autocorrelation matrix is $\Phi_Z = diag\{\varphi_z, \varphi_u, \varphi_v\}$ and $\varphi_z, \varphi_u, \varphi_v \in (0,1)$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{Zt} = [\epsilon_{zt} \; \epsilon_{ut} \; \epsilon_{vt}]' \sim N(\mathbf{0}, \mathbf{\Sigma})$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix $\mathbf{\Sigma}$. These shocks affect the economy as described below, and as calibrated in Benk et al. (2010).

2.1 Consumer Problem

A representative consumer has expected lifetime utility from consumption of goods, c_t, and leisure, x_t; with $\beta \in (0,1)$, $\psi > 0$ and $\theta > 0$, this is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^{\psi})^{1-\theta}}{1-\theta}. \tag{2}$$

Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in goods production (G) and human capital investment (H), whereby:

$$s_{Gt} + s_{Ht} = 1. \tag{3}$$

The agent allocates a time endowment of one between leisure, x_t, labor in goods production, l_{Gt}, time spent investing in the stock of human capital, l_{Ht}, and time spent working in the bank sector (F subscripts for Finance), denoted by l_{Ft}:

$$l_{Gt} + l_{Ht} + l_{Ft} + x_t = 1. \quad (4)$$

Output of goods can be converted into physical capital, k_t, without cost and is thus divided between consumption goods and investment, denoted by i_t, net of capital depreciation. The capital stock used for production in the next period is therefore given by:

$$k_{t+1} = (1-\delta_k)k_t + i_t = (1-\delta_k)k_t + y_t - c_t. \quad (5)$$

The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $l_{Ht}h_t$, with $A_H > 0$ and $\eta \in [0,1]$, such that the human capital flow constraint is

$$h_{t+1} = (1-\delta_h)h_t + A_H(s_{Ht}k_t)^{1-\eta}(l_{Ht}h_t)^{\eta}. \quad (6)$$

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t (l_{Gt} + l_{Ft}) h_t$ and $P_t r_t s_{Gt} k_t$, a nominal transfer from the government, T_t, and dividends from the bank. The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by d_t, and the dividend per unit of deposits as R_{Ft}, the consumer receives a nominal dividend income of $P_t R_{Ft} d_t$. The consumer also pays to the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Ft} denoting the nominal price of each unit of credit, and q_t the real quantity of credit that the consumer can use in exchange, the consumer pays $P_{Ft}q_t$ in credit fees.

With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1-\delta_k)k_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1+R_t)$, where R_t is the net nominal interest rate, the consumer's budget constraint is:

$$\begin{aligned} & P_t w_t (l_{Gt} + l_{Ft}) h_t + P_t r_t s_{Gt} k_t + P_t R_{Ft} d_t + T_t \\ \geq \; & P_{Ft} q_t + P_t c_t + P_t k_{t+1} - P_t(1-\delta_k)k_t + M_{t+1} - M_t \\ & + B_{t+1} - B_t(1+R_t). \end{aligned} \quad (7)$$

The consumer can purchase goods by using either money M_t or credit services. With the lump sum transfer of cash T_t coming from the government at the beginning of the period, and with money and credit equally usable to buy goods, the consumer's exchange technology is:

$$M_t + T_t + P_t q_t \geq P_t c_t. \quad (8)$$

Since all cash comes out of deposits at the bank and credit purchases are paid off at the end of the period out of the same deposits, total deposits are equal to consumption. This gives the constraint that:

$$d_t = c_t. \tag{9}$$

Given k_0, h_0, and the evolution of M_t $(t \geq 0)$ as given by the exogenous monetary policy in equation (17) below, the consumer maximizes utility subject to the budget, exchange and deposit constraints (7)-(9).

2.2 Banking Firm Problem

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_F > 0$ and $\gamma \in (0, 1)$:

$$q_t = A_F e^{v_t} \left(l_{Ft} h_t\right)^{\gamma} d_t^{1-\gamma}, \tag{10}$$

where $A_F e^{v_t}$ is the stochastic factor productivity.

Subject to the production function in equation (10), the bank maximizes profit Π_{Ft} with respect to the labor l_{Ft} and deposits d_t:

$$\Pi_{Ft} = P_{Ft} q_t - P_t w_t l_{Ft} h_t - P_t R_{Ft} d_t. \tag{11}$$

Equilibrium implies that:

$$\left(\frac{P_{Ft}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma - 1} = w_t; \tag{12}$$

$$\left(\frac{P_{Ft}}{P_t}\right) (1-\gamma) A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma} = R_{Ft}. \tag{13}$$

These indicate that the marginal cost of credit, $\left(\frac{P_{Ft}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w_t}{\gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma-1}}$, and that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by $\left(\frac{P_{Ft}}{P_t}\right)(1-\gamma)\left(\frac{q_t}{d_t}\right)$.

2.3 Goods Producer Problem

The firm maximizes profit given by $y_t - w_t l_{Gt} h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t} (s_{Gt} k_t)^{1-\alpha} (l_{Gt} h_t)^{\alpha}. \tag{14}$$

The first order conditions for the firm's problem yield the standard expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_{Gt} h_t} \right)^{1-\alpha}, \tag{15}$$

$$r_t = (1-\alpha) A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_{Gt} h_t} \right)^{-\alpha}. \tag{16}$$

2.4 Government Money Supply

It is assumed that government policy includes sequences of nominal transfers as given by:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1) M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}, \tag{17}$$

where Θ_t is the growth rate of money and Θ^* is the stationary gross growth rate of money.

2.5 Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,x,l_G,l_H,l_F,s_G,s_H,q,d,k',h',M'} \{u(c,x) + \beta E V(s')\} \tag{18}$$

subject to the conditions (3) to (9), where the state of the economy is denoted by $s = (k, h, M, B; z, u, v)$ and a prime (') indicates next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l_G(s)$, $l_H(s)$, $l_F(s)$, $s_G(s)$, $s_H(s)$, $q(s)$, $d(s)$, $k'(s)$, $h'(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $r(s)$, $R_F(s)$, $P_F(s)$ and a value function $V(s)$, such that:

(i) the consumer maximizes utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (18);

(ii) the goods producer maximizes profit similarly, with the resulting functions for w and r being given by equations (15) and (16);

(iii) the bank firm maximizes profit similarly in equation (11) subject to the technology of equation (10)

(iv) the goods, money and credit markets clear, in equations (7) and (14), and in (8), (17), and (10).

3 General Equilibrium Taylor Condition

The 'Taylor condition' is now derived as an equilibrium condition of the Benk et al. (2010) model described in the previous section. Beginning from the first-order conditions of the model, we obtain:

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta} x_{t+1}^{\psi(1-\theta)}}{c_t^{-\theta} x_t^{\psi(1-\theta)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{19}$$

where R and π are gross rates of nominal interest and inflation, respectively. The term $\tilde{R}_t$ represents (one plus) a 'weighted average cost of exchange' as follows:

$$\tilde{R}_t = 1 + \frac{m_t}{c_t}(R_t - 1) + \gamma\left(1 - \frac{m_t}{c_t}\right)(R_t - 1).$$

where a weight of $\frac{m}{c}$ is attached to the opportunity cost of money $(R_t - 1)$ and a weight of $(1 - \frac{m}{c})$ is attached to the average cost of credit, $\gamma(R_t - 1)$, and $\frac{m_t}{c_t}$ is the real consumption normalised demand for money (i.e. the inverse of the consumption velocity of money). In effect, equation (19) augments a standard consumption Euler equation with the (growth rate of) the weighted average cost of exchange. If all goods purchases are conducted using money $(m_t/c_t = 1)$ then equation (19) reverts back to the familiar consumption Euler equation which would constitute an equilibrium condition of a standard, unit velocity cash-in-advance model without a money alternative.

For any variable z_t, define $\widehat{z}_t \equiv \ln z_t - \ln z$, where the absence of a time subscript denotes a BGP stationary value, and define $\widehat{g}_{z,t+1} \equiv \ln z_{t+1} - \ln z_t$, which approximates the growth rate at time $t+1$ for sufficiently small z_t. Consider a log-linear approximation of (19) evaluated around the BGP:

$$0 = E_t\left\{\theta\widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} + \widehat{g}_{\tilde{R},t+1} - \widehat{R}_{t+1} + \widehat{\pi}_{t+1}\right\}.$$

Rearranging this in terms of $\widehat{R}_t$ gives the Taylor condition expressed in log-deviations from the BGP equilibrium:

$$\begin{aligned}\widehat{R}_t \;=\; & E_t\{\Omega\widehat{\pi}_{t+1} + \Omega\theta\widehat{g}_{c,t+1} - \Omega\psi(1-\theta)\widehat{g}_{x,t+1} \\ & + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\left[(R-1)\frac{\frac{m}{c}}{1-\frac{m}{c}}\widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1}\right]\right\},\end{aligned} \quad (20)$$

where $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1$. The Taylor condition (20) can now be expressed in net rates (denoted by over-barred terms) and absolute deviations from the BGP equilibrium, as demonstrated by the following proposition.

Proposition 1 *An equilibrium condition of the economy takes the form of a Taylor Rule which sets deviations of the short-term nominal interest rate from some baseline path in proportion to deviations of variables from their targets:*

$$\begin{aligned}\overline{R}_t - \overline{R} \;=\; & \Omega E_t(\overline{\pi}_{t+1} - \overline{\pi}) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi(1-\theta)E_t\overline{g}_{x,t+1} \\ & + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\left[(R-1)\frac{\frac{m}{c}}{1-\frac{m}{c}}E_t\overline{g}_{\frac{m}{c},t+1} - E_t\left(\overline{R}_{t+1} - \overline{R}\right)\right].\end{aligned} \quad (21)$$

where $\Omega \geq 1$, and for a given w, then $\frac{\partial\Omega}{\partial R} > 0$ and $\frac{\partial\Omega}{\partial A_F} > 0$, and the target values are equal to the balanced growth path equilibrium values.[3]

[3] This is the the Brookings project form of the Taylor rule as described in Orphanides (2008).

Proof. Since the *BGP* solution for normalized money demand is:

$$0 \leq \frac{m}{c} = 1 - A_F \left(\frac{(R-1)\,\gamma A_F}{w} \right)^{\frac{\gamma}{1-\gamma}} \leq 1,$$

then $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1$ and, given w, $\frac{\partial \Omega}{\partial R} \geq 0$ and $\frac{\partial \Omega}{\partial A_F} \geq 0$. ■

For a linear production function of goods w is the constant marginal product of labor but more generally w is endogenous and will change; however this change in w is quantitatively small compared to changes in R and A_F, so that the derivatives above almost always hold true. Note that for a unitary consumption velocity of money, the velocity growth and forward interest terms drop out of equation (21)

The term $\bar{\pi}$ in equation (21) can be compared to the inflation target that features in many interest rate rules (e.g. Taylor, 1993; Clarida et al., 2000). This is usually set as an exogenous constant in a conventional rule but represents the *BGP* rate of inflation in the Taylor condition.[4] The term in consumption growth is similar, but not identical to, the first difference of the output gap that features in the so-called 'speed limit' rule (Walsh, 2003). Alternatively, the term in the growth rate of leisure time can be compared to the unemployment rate which sometimes features in conventional interest rate rules in place of the output gap.[5]

Equation (21) also contains two terms which are not usually found in standard monetary policy reaction functions. Firstly, there is a term in the growth rate of the real (consumption normalized) demand for money. Conventional interest rate rules are usually considered in the context of models which omit monetary relationships and thus money demand does not feature directly in the model.[6] Secondly, the Taylor condition contains a term in the expected future nominal interest rate. This contrasts with the lagged nominal interest term which is often used to capture 'interest rate smoothing' in a conventional rule (e.g. Clarida et al., 2000).

In general, the coefficient on inflation in (21) exceeds unity ($\Omega > 1$). This replicates the 'Taylor principle' whereby the nominal interest rate responds more than one-for-one to (expected future) inflation deviations from 'target'. However, the inflation coefficient in the Taylor condition does not reflect policymakers' preferences. Rather, it is a function of the *BGP* nominal interest rate (R), the consumption normalized demand for real money balances (m/c) and the efficiency with which the banking sector transforms units of deposits into units of the credit service, as reflected by the magnitude of ($1-\gamma$). Furthermore, higher productivity in the banking sector (A_F) causes a higher velocity

[4] Although see Ireland (2007) for an example of a conventional interest rate rule with a time-varying inflation target.

[5] For example, Mankiw (2001) includes the unemployment rate in an interest rate rule and Rudebusch (2009) includes the 'unemployment gap'.

[6] Specifically, shifts in the demand for money are perfectly accommodated by adjustments to the money supply in order to maintain the rule-implied nominal interest rate. This, it is claimed, renders the evolution of the money supply an operational detail which need not be modelled directly (e.g. Woodford, 2008).

and implies a larger inflation coefficient in the Taylor condition. The magnitude of Ω clearly does not reflect a response to inflation in the conventional 'reaction function' sense.[7]

Equation (21) can alternatively be rewritten in terms of the consumption velocity of money, $V_t \equiv \frac{c_t}{m_t}$, and the productive time, or 'employment', growth rate ($l \equiv l_G + l_H + l_F = 1 - x$). Using the fact that $\widehat{x}_t = -\frac{1-x}{x}\widehat{l}_t$:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_V E_t \overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \qquad (22)\end{aligned}$$

Where over-barred terms again denote net rates and:

$$\Omega_V \equiv \frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\gamma + (1-\gamma)\frac{m}{c}}\right).$$

Proposition 2 *For the Taylor condition of equation (22), it is always true that* $0 \leq \Omega_V \leq 1 \leq \Omega$.

Proof.

$$\begin{aligned}\Omega &\equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R[1-(1-\gamma)\left(1-\frac{m}{c}\right)]} \geq 1; \; \frac{m}{c} = 1 - A_F^{\frac{1}{1-\gamma}}\left[\frac{(R-1)\gamma}{w}\right]^{\frac{\gamma}{1-\gamma}} \leq 1; \\ 1 &\geq (1-\gamma)\left(1-\frac{m}{c}\right) \geq 0; \Rightarrow 0 \leq \Omega_V \equiv \frac{(R-1)(1-\gamma)}{R}\left(\frac{\frac{m}{c}}{1-(1-\gamma)\left(1-\frac{m}{c}\right)}\right) \leq 1; \\ &\Rightarrow \; 0 \leq \Omega_V \leq 1 \leq \Omega.\end{aligned}$$

■

At the Friedman (1969) optimum for the gross nominal interest rate ($R = 1$), $\frac{m}{c} = 1$, $\omega = 0$, and the velocity coefficient (Ω_V) takes a value of zero. The velocity growth term only enters the Taylor condition when the nominal interest rate differs from the Friedman (1969) optimum and fluctuates. In turn, this has implications for $\Omega = 1 + \left(\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right)$, since when $R = 1$, $(1-\gamma)\left(1-\frac{m}{c}\right) = 0$, and $\Omega = 1$. For $\frac{m}{c}$ below one (velocity above one), which is true for most practical experience, the model's equivalent of the 'Taylor principle' ($\Omega > 1$) holds.

Corollary 3 *Given* w*, then* $\frac{\partial\Omega}{\partial R} \geq 0$, $\frac{\partial\Omega_V}{\partial R} \geq 0$, $\frac{\partial\Omega}{\partial A_F} \geq 0$, $\frac{\partial\Omega_V}{\partial A_F} \leq 0$.

[7] Unlike Sørensen and Whitta-Jacobsen's (2005, pp.502-505) quantity theory based equilibrium condition, the inflation coefficient in (21) exceeds unity for any (admissible) interest elasticity of money demand. In their expression, the inflation coefficient falls below unity if the interest (semi) elasticity of money demand exceeds one in absolute value. In the Benk et al. (2010) model, the coefficient on inflation would exceed unity even in this case but the central bank would not wish to increase the money supply growth rate to this extent because seigniorage revenues would begin to recede as the elasticity increases beyond this point.

Proof. This comes directly from the definitions of parameters above. ■

A higher target nominal interest rate can be accomplished only by a higher BGP money supply growth rate. This would in turn make the inflation and consumption growth coefficients larger and the forward interest rate and velocity coefficients would become more negative. A higher credit productivity factor A_F, and so a higher velocity, leads to a higher inflation coefficient and a more negative response to the forward-looking interest term but a less negative coefficient on the velocity growth term.

The Taylor condition above would look identical with exogenous growth. However, under exogenous growth the targeted inflation rate and growth rate of the economy are unrelated and exogenously specified. Under endogenous growth the targets are instead the endogenously determined BGP values for inflation, the growth rate, and the nominal interest rate and each of these are determined, in part, by the long run stationary money supply growth rate Θ^*, which is exogenously given. In turn, Θ^* translates directly into a long run inflation target accepted by the central bank, such as the two percent target often incorporated into conventional interest rate rules (for example, Taylor, 1993). So the model assumes only a long run money supply growth target, or alternatively, a long run inflation rate target.

3.1 Misspecified Taylor Condition with Output Growth

It is not surprising to find that the growth rate of consumption appears in equation (22) rather than the output growth rate given that the derivation of the Taylor condition begins from the consumption Euler equation (19). However, the Taylor condition can be rewritten to include an output growth term and hence correspond more closely to standard Taylor rule specifications, in particular the 'speed limit' rule considered by Walsh (2003). To derive this alternative rule, consider that the identity $y_t = c_t + i_t$ implies that $\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t$, where $\widehat{i}_t = \frac{k}{i}\left[\widehat{k}_t - (1-\delta)\,\widehat{k}_{t-1}\right]$. The growth rate of investment can be understood as the acceleration of the growth of capital gross of depreciation. The Taylor condition can be rewritten as:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[\frac{y}{c}E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) - \frac{i}{c}E_t\left(\overline{g}_{i,t+1} - \overline{g}\right)\right] \qquad (23) \\ &\quad + \Omega\psi\,(1-\theta)\,\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - (\Omega - 1)\,E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned}$$

A term in investment growth does not appear in standard Taylor rules but plays a role as part of what is interpreted as the output gap growth rate in this modified Taylor condition. Equation (23) forms the basis for the two misspecified estimating equations considered in Section 5. The first misspecified estimating equation simply replaces the consumption growth term in equation (22) with an output growth term as follows:

$$\overline{R}_t - \overline{R} = \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \tag{24}$$
$$+\Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right).$$

Comparing equation (23) and equation (24) shows that the latter erroneously overlooks the weighting on the output growth rate ($\frac{y}{c}$) and omits the term in the investment growth rate. Replacing consumption growth with output growth without the additional term in investment therefore misrepresents the structure of the underlying Benk et al. (2010) model and as such equation (24) is misspecified. Note that with no physical capital in the economy, equation (24) would be a valid equilibrium condition of the economy.

3.2 Misspecified Standard Taylor Rule

The second misspecified model erroneously imposes the same restrictions used to arrive at equation (24) but also drops the terms in productive time and velocity, giving:

$$\overline{R}_t - \overline{R} = \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \tag{25}$$

This can be interpreted as a conventional interest rate rule with a forward-looking 'interest rate smoothing' term; the additional restriction that $\Omega = 1$ would replicate a standard interest rate rule without interest rate smoothing. Once again, equation (25) does not accurately represent an equilibrium condition of the Benk et al. (2010) economy and is therefore misspecified. Equation (25) with $\Omega = 1$ would be the correct equilibrium condition if the economy featured neither physical capital nor exchange credit.

4 Calibration

We follow Benk et al. (2010) in using postwar U.S. data to calibrate the model (Table 1) and calculate a series of 'target values' consistent with this calibration (Table 2); see Benk et al. for the shock process calibration.

Subject to this calibration, we derive a set of theoretical 'predictions' for the coefficients of the Taylor condition (22). These values will subsequently be compared to the coefficients estimated from artificial data simulated from the model. Consider first the inflation coefficient (Ω). According to the calibration and target values presented in tables 1 and 2, its theoretical value is

$$\Omega = 1 + \frac{\left(1-\gamma\right)\left(1-\frac{m}{c}\right)}{R\left[1-\left(1-\gamma\right)\left(1-\frac{m}{c}\right)\right]} = 1 + \frac{\left(1-0.11\right)\left(1-0.38\right)}{1.0944\left(1-\left(1-0.11\right)\left(1-0.38\right)\right)} = 2.125$$

And for $R = 1$, only cash is used so that $\frac{m}{c} = 1$ and Ω reverts to its lower bound of 1. This also happens with zero credit productivity ($A_F = 0$), in which case only cash is used in exchange.

Preferences		
θ	1	Relative risk aversion parameter
ψ	1.84	Leisure weight
β	0.96	Discount factor
Goods Production		
α	0.64	Labor share in goods production
δ_k	0.031	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human Capital Production		
ε	0.83	Labor share in human capital production
δ_h	0.025	Depreciation rate of human capital sector
A_H	0.21	Human capital productivity parameter
Banking Sector		
γ	0.11	Labor share in credit production
A_F	1.1	Banking productivity parameter
Government		
σ	0.05	Money growth rate

Table 1: Parameters

g	0.024	Avg. annual output growth rate
π	0.026	Avg. annual inflation rate
R	0.0944	Nominal interest rate
l_G	0.248	Labor used in goods sector
l_H	0.20	Labor used in human capital sector
l_F	0.0018	Labor used in banking sector
i/y	0.238	Investment-output ratio in goods sector
m/c	0.38	Share of money transactions
x	0.55	Leisure time
$l \equiv 1 - x$	0.45	Productive time

Table 2: Target Values

The remaining coefficients, except for velocity, are simple functions of the inflation coefficient. The consumption growth coefficient is $\Omega\theta$, which with $\theta = 1$ for log-utility should simply take the same magnitude as the coefficient on inflation ($\theta\Omega = 2.125$). The coefficient on the productive time growth rate should take a value of zero with log utility. However with leisure preference calibrated at 1.84, and productive time ($1 - x \equiv l$) equal to equal to 0.45 along the BGP, the estimated value of the productive time coefficient can be interpreted as implying a certain θ factored by $\Omega\psi\frac{l}{1-l} = (2.125)(1.84)\frac{0.45}{0.55} = 3.199$. Given the magnitude of the inflation coefficient, the coefficient on the forward interest term is simply $-(\Omega - 1) = -1.125$. The velocity coefficient ($-\Omega_V$) is -0.065 using:

$$-\frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) = -\frac{(1.0944-1)}{1.0944}\left(\frac{(1-0.11)\,0.38}{(1-(1-0.11)\,(1-0.38))}\right).$$

At the Friedman (1969) optimum ($R = 1$), $\Omega_V = 0$. In this case the omission of the term in velocity growth in the estimation exercises that follow would be innocuous but this is not true in general.

5 Artificial Data Estimation

The Benk et al. (2010) model presented in Section 2 is simulated using the calibration provided in Table 1 in order to generate 1000 alternative 'joint histories' for each of the variables in equation (22), where each history is 100 periods in length. To do so, 100 random sequences for the shock vector innovations are generated and control functions of the log-linearized model are used to compute sequences for each variable. Each observation within a given history may be thought of as an annual period given the frequency considered by the Benk et al. (2010) model. The data set used to estimate the coefficients of the Taylor condition can therefore be viewed as comprising of 1000, '100-year', samples of artificial data.

5.1 Estimation Methodology

This section presents the results of estimating a 'correctly specified' estimating equation based upon the true theoretical relationship (22) against artificial data generated from the Benk et al. (2010) model.[8] In a similar manner, two alternative estimating equations are evaluated using the same data set. Since these alternative estimating equations differ from the expression based upon the true theoretical relationship, they necessarily constitute misspecified empirical models.[9]

[8] The exercise conducted here is similar to those conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model.

[9] We acknowledge that in a full information maximum likelihood estimation that uses all of the equilibrium conditions of the economy we may be able to recover the theoretical coefficients

Prior to estimation, the simulated data is filtered by either 1) a Hodrick-Prescott (HP) filter with a smoothing parameter selected in accordance with Ravn and Uhlig (2002); 2) a $3-8$ period ("year") Christiano and Fitzgerald (2003) band pass filter for 'business cycle frequencies'; or 3) a $2-15$ year Christiano and Fitzgerald (2003) band pass filter which retains more of the lower frequency trends in the data than the $3-8$ year filter, in the spirit of Comin and Gertler's (2006) 'medium-term cycle'.[10] *A priori*, the $2-15$ band pass filter might be regarded as the 'most relevant' to the underlying theoretical model because shocks in the model can cause low frequency events during the business cycle, such as a change in the permanent income level without a reversion to its previous level.[11]

The first estimation technique considered is OLS, as used by Taylor (1999) in the context of a contemporaneous interest rate rule. However, if expected future variables are correlated with the error term then a suitable set of instruments are required to proxy for these forward-looking terms.[12] Two instrumental variables (IV) techniques are considered and each differs by the instrument set employed. The first is a two stage least squares (2SLS) estimator under which the first lags of inflation, consumption growth, productive time growth and velocity growth and the second lag of the nominal interest rate are used as instruments. Adding a constant term to the instrument set provides a 'just identified' 2SLS estimator. In using lagged variables as instruments we exploit the fact that such terms are pre-determined and thus not susceptible to the simultaneity problem which motivates the use of IV techniques. The 2SLS procedure applies a Newey-West adjustment for heteroskedasticity and autocorrelation (HAC) to the coefficient covariance matrix.

The second IV procedure is a generalized method of moments (GMM) estimator under which three additional lags of inflation, consumption growth, productive time growth and velocity growth and two further lags of the nominal interest rate are added to the instrument set.[13] Expanding the instrument

of the Taylor condition almost exactly; we leave that exercise as an important part of future research that encompasses the entire alternative model; and then we could also compare it to the standard three equation central bank policy model.

[10] However, Comin and Gertler's 'medium-term cycle' is defined using a wider 2-200 quarter filter. However, the 2-15 filter will still retain periodicities that the HP and 3-8 filters consign to the 'trend'.

[11] In principle, the filtering procedure takes account of the Siklos and Wohar (2005) critique of empirical Taylor rule studies which do not address the non-stationarity of the data. However, standard ADF and KPSS tests suggest that the simulated data is stationary prior to filtering (results not reported). Accordingly, the filters do not implement a de-trending procedure.

[12] Empirical studies usually deal with expected future terms either by replacing them with realised future values and appealing to rational expectations for the resulting conditional forecast errors (e.g. Clarida et al., 1998, 2000) or by using private sector or central bank forecasts as empirical proxies (e.g. Orphanides, 2001; Siklos and Wohar, 2005).

[13] Carare and Tchaidze (2005, p.15) note that the four-lags-as-instruments specification is the standard approach in the interest rate rule literature (e.g. Orphanides, 2001). Although the GMM procedure in general corrects for autocorrelation and heteroskedasticity, in estimating with simulated data we use lags as 'valid' instruments for pre-determined variables. These instruments might prove to be 'relevant' because the data is serially correlated but no further

set in this manner reduces the sample size available for each of the 1000 simulated sample periods but the over-identifying restrictions can now be used to test the validity of the instrument set using the Hansen J-test. The GMM estimator used iterates on the weighting matrix in two steps and applies a HAC adjustment to the weighting matrix using a Bartlett kernel with a Newey-West fixed bandwidth.[14] A similar HAC adjustment is also applied to the covariance weighting matrix.

The results are presented in three sets of tables, one set for each estimating equation, and are further subdivided according to the statistical filter applied to the simulated data. Alongside the estimates obtained from an 'unrestricted' estimating equation, each table also reports estimates derived from a 'restricted' estimating equation which arbitrarily omits the forward interest rate term ($\beta_5 = 0$). This arbitrary restriction demonstrates the importance of the dynamic term in equation (22). Each table of results reports mean coefficient estimates along with the standard error of these estimates (as opposed to the mean standard error). The figures in square brackets report the number of coefficients estimated to be statistically different from zero at the 5% level of significance and this count is used as an indication of the 'precision' of the estimates. An 'adjusted mean' figure is also reported for each coefficient; this is obtained by setting non statistically significant coefficient estimates to zero when calculating the averages. The tables also report mean R-square and mean adjusted R-square statistics along with the mean P-value for the F-statistic for overall significance (these cannot be computed for the GMM estimator), the mean P-value for the Hansen J-statistic which tests the validity of the instrument set (these can only be calculated in the presence of over-identifying restrictions), and the mean Durbin-Watson (D-W) statistic which tests for autocorrelation. The number of estimations for which the null hypothesis of the J-statistic is *not* rejected - i.e. the instrument set is not found to be invalid - is reported alongside its mean P-value and the number of simulated series for which the D-W statistic exceeds its upper critical value - i.e. the null hypothesis that the residuals are serially uncorrelated cannot be rejected - is reported alongside the mean D-W statistic.[15]

lags are needed for the estimating equation itself. For actual data, Clarida et al. (QJE, 2000, p.153) use a GMM estimator "with an optimal weighting matrix that accounts for possible serial correlation in [the error term]" but they also add two lags of the dependent variable to their estimating equation on the basis that this "seemed to be sufficient to eliminate any serial correlation in the error term." (p.157), implying that the GMM correction was insufficient for this purpose.

[14] Jondeau et al. (2004, p.227) state that: "To our knowledge, all estimations of the forward-looking reaction function based on GMM have so far relied on the two-step estimator." They proceed to consider more sophisticated GMM estimators but nevertheless identify advantages to the "simple approach" (p.238) adopted in the literature.

[15] The D-W count excludes cases for which the test statistic falls in the inconclusive region of the test's critical values.

5.2 General Taylor Condition

Tables 3-5 present estimates obtained from the following 'correctly specified' estimating equation:

$$R_t = \beta_0+\beta_1 E_t\pi_{t+1}+\beta_2 E_t g_{c,t+1}+\beta_3 E_t g_{l,t+1}+\beta_4 E_t g_{V,t+1}+\beta_5 E_t R_{t+1}+\varepsilon_t. \quad (26)$$

Expected future variables on the right hand side are obtained directly from the model simulation procedure and are instrumented for as described above.

The key result is that Tables 3-5 consistently report an inflation coefficient which exceeds unity for the estimating equation which accurately reflects the underlying theoretical model. This result is found to be robust to the statistical filter applied to the data and to the estimator employed, subject to the estimator providing a 'precise' set of estimates. The forward interest rate term is also found to be important in terms of generating a coefficient on inflation consistent with the underlying Benk et al. (2010) model. Arbitrarily omitting this dynamic term yields much smaller estimates for the inflation coefficient to the extent that the mean estimate often falls below unity.

In terms of the general features of the results obtained from the unrestricted specification, the OLS and GMM procedures tend to generate a greater number of statistically significant estimates than the 2SLS estimator. Focusing on Table 5 for the $2-15$ filter, the 2SLS estimator provides a statistically significant estimate for the inflation coefficient for only 580 of the 1000 simulated histories while the OLS and GMM estimators both return 1000 statistically significant estimates. The OLS and GMM procedures generate reasonably large R-square and adjusted R-square statistics, whereas negative R-square statistics are obtained from the simple 2SLS estimator. Expanding the instrument set in order to implement the GMM procedure leads to 1000 rejections of the J-test for instrument validity across all three filters. One might also be wary of the high number of D-W null hypothesis rejections produced by the OLS estimator, although the mean D-W statistic remains 'reasonably large' in each case; 1.56 for the $2-15$ filter, for example. The results for the $3-8$ band pass filter (Table 4) are unusual in the sense that all three estimation procedures produce a high number of D-W test rejections. For the other two filters, this undesirable result is confined to the OLS estimator.

Table 5 reports that the mean estimate for the inflation coefficient is 2.179 using the OLS estimator and 2.306 using the GMM estimator.[16] These estimates compare favorably to the theoretical prediction of $\Omega = 2.125$. The right hand side of Table 5 shows that the mean estimate of the inflation coefficient falls below unity for the OLS and GMM estimators when the forward interest rate term is arbitrarily omitted from the estimating equation. A precise mean

[16] The discussion focuses on the OLS and GMM estimators because they produce more 'precise' estimates and also because the OLS estimator tends to reject the null hypothesis of the F-statistic more frequently than the 2SLS estimator (1000 vs. 907 rejections in Table 5, for example). The OLS regressions are possibly afflicted by autocorrelation however, as discussed above, thus one might favor the GMM estimates.

estimate of 0.614 is obtained from the OLS estimator and a similarly precise mean estimate of 0.964 is obtained from the GMM procedure. Similar OLS and GMM estimates are obtained for the inflation coefficient under the two alternative filters in Tables 3 and 4, both in terms of the mean coefficient estimates for the unrestricted specification and in terms of the decline in magnitude induced by the arbitrary restriction.

In contrast to the estimated inflation coefficients, the estimated coefficients for consumption growth and productive time growth diverge from their theoretical predictions for the 'unrestricted' estimating equation. Under log utility ($\theta = 1$), the former should take the same magnitude as the coefficient on inflation and the latter should take a value of zero. The coefficient estimates can be used to 'back-out' an estimate of the coefficient of relative risk aversion (θ). Firstly, using the mean GMM estimate for the coefficient on consumption growth of 0.302 (Table 5) and the corresponding estimate of Ω, an implied estimate of θ can be calculated as $\frac{\beta_2}{\beta_1} = \frac{0.302}{2.306} = 0.131$, which is substantially smaller than the baseline calibration of $\theta = 1$. Alternatively, the relationship $\beta_3 = \beta_1 \psi (1-\theta) l/(1-l)$, which is obtained from equation (22) with Ω replaced by its estimate β_1, can also be used to obtain an implied estimate of θ. Using the estimates presented in Table 5, the implied estimate would be $\theta = 1.103$, which is much closer to the calibrated value.

Table 5 also reports that both the OLS and GMM procedures generate 1000 statistically significant estimates for the coefficient on velocity growth under the unrestricted estimating equation and that the mean estimate is correctly signed for both estimators. The mean coefficient estimates are reported as -0.196 and -0.269 for OLS and GMM estimators respectively; these estimates are somewhat smaller than the theoretical prediction of -0.065. Similar estimates are obtained under the HP and $3-8$ filters. Finally, Table 5 reports mean estimates of -1.761 (OLS) and -1.729 (GMM) for the forward interest rate coefficient compared to a theoretical prediction of -1.125. The mean estimates are therefore correctly signed but, again, smaller than the theoretical prediction.

In a standard interest rate rule an inflation coefficient in excess of unity is interpreted to reflect policy-maker's dislike of inflation deviations from target. However, this interpretation is not applicable to the Taylor condition. The result that the coefficient on inflation exceeds unity is a consequence of a money growth rule not an interest rate rule. Similarly, the break-down of the Taylor principle under the 'restricted' estimating equation ($\beta_5 = 0$) cannot be interpreted as a softening of policy-makers' attitude towards inflation; this result simply emanates from model misspecification.

5.3 Taylor Condition with Output Growth

The same estimation procedure is now applied to an estimating equation which replaces consumption growth in equation (26) with output growth as follows:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (27)$$

HP filtered data, where HP $\lambda = 6.25$	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-9.68E-07 [0]	-2.04E-07 [0]	8.09E-07 [17]	-7.60E-07 [0]	-4.78E-07 [0]	-7.57E-08 [8]
Standard error	2.87E-05	2.15E-05	3.15E-05	2.39E-05	1.67E-05	4.29E-05
Adjusted mean	-	-	-4.27E-08	-	-	2.59E-07
$E_t\pi_{t+1}$	2.019 [1000]	2.309 [691]	2.299 [1000]	0.315 [830]	0.757 [397]	0.621 [925]
Standard error	0.248	1.488	0.268	0.126	0.856	0.265
Adjusted mean	2.019	1.800	2.299	0.293	0.475	0.614
$E_tg_{c,t+1}$	0.251 [1000]	0.336 [959]	0.293 [1000]	0.172 [1000]	0.313 [989]	0.231 [1000]
Standard error	0.024	0.096	0.020	0.020	0.048	0.025
Adjusted mean	0.251	0.324	0.293	0.172	0.311	0.231
$E_tg_{l,t+1}$	-0.243 [890]	-0.536 [774]	-0.374 [997]	-0.281 [864]	-0.530 [774]	-0.427 [996]
Standard error	0.094	0.321	0.079	0.100	0.231	0.111
Adjusted mean	-0.236	-0.448	-0.374	-0.265	-0.453	-0.427
$E_tg_{V,t+1}$	-0.137 [990]	-0.267 [800]	-0.212 [1000]	-0.098 [889]	-0.317 [888]	-0.190 [992]
Standard error	0.031	0.228	0.033	0.036	0.109	0.056
Adjusted mean	-0.137	-0.229	-0.212	-0.093	-0.293	-0.190
E_tR_{t+1}	-1.819 [1000]	-2.338 [646]	-2.005 [1000]	N/A	N/A	N/A
Standard error	0.221	2.282	0.277			
Adjusted mean	-1.819	-1.692	-2.005			
$Mean$;						
R-square	0.789	<0	0.796	0.544	<0	0.482
Adjusted R-square	0.778	<0	0.785	0.525	<0	0.459
Pr(F-statistic)	2.35E-15 (1000)	0.015 (974)	N/A	3.93E-09 (1000)	0.003 (992)	N/A
Pr(J-statistic)	N/A	N/A	0.258 {1000}	N/A	0.159 {482}	0.269 {1000}
Durbin-Watson	1.474 <151>	2.243 <1000>	2.194 <970>	1.732 <419>	2.145 <999>	2.047 <882>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 3: Taylor Condition Estimation, HP Filtered Data, Ravn and Uhlig (2002) Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-7.57E-07 [0]	6.54E-06 [0]	-7.10E-07 [3]	7.73E-07 [0]	-2.86E-06 [0]	-1.51E-06 [1]
Standard error	1.65E-05	3.24E-04	2.09E-05	1.45E-05	4.37E-05	2.68E-05
Adjusted mean	-	-	-1.83E-07	-	-	-9.81E-08
$E_t\pi_{t+1}$	2.166 [998]	2.484 [724]	2.423 [1000]	0.633 [969]	2.417 [965]	0.682 [974]
Standard error	0.391	32.906	0.298	0.195	1.125	0.222
Adjusted mean	2.166	2.141	2.423	0.628	2.291	0.679
$E_tg_{c,t+1}$	0.283 [1000]	0.304 [623]	0.314 [1000]	0.155 [1000]	0.175 [834]	0.168 [1000]
Standard error	0.043	7.324	0.027	0.029	0.074	0.030
Adjusted mean	0.283	0.231	0.314	0.155	0.160	0.168
$E_tg_{l,t+1}$	-0.237 [827]	-0.573 [430]	-0.312 [982]	-0.222 [685]	-0.595 [666]	-0.268 [870]
Standard error	0.131	10.199	0.099	0.133	0.361	0.135
Adjusted mean	-0.229	-0.193	-0.312	-0.195	-0.455	-0.259
$E_tg_{V,t+1}$	-0.152 [982]	-0.453 [351]	-0.174 [998]	-0.174 [976]	-0.604 [973]	-0.194 [984]
Standard error	0.043	15.068	0.039	0.052	0.252	0.057
Adjusted mean	-0.151	-0.128	-0.174	-0.173	-0.578	-0.193
E_tR_{t+1}	-2.026 [994]	-1.532 [424]	-2.289 [1000]			
Standard error	0.435	116.012	0.339			
Adjusted mean	-2.024	-1.211	-2.289			
$Mean;$						
R-square	0.789	<0	0.842	0.576	<0	0.590
Adjusted R-square	0.778	<0	0.833	0.558	<0	0.572
Pr(F-statistic)	4.75E-10 (1000)	0.077 (874)	N/A	2.08E-07 (1000)	0.005 (981)	N/A
Pr(J-statistic)	N/A	N/A	0.213 {1000}	N/A	0.344 {848}	0.249 {1000}
Durbin-Watson	1.568 <54>	1.653 <330>	1.517 <49>	1.728 <333>	1.728 <378>	1.715 <306>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 4: Taylor Condition Estimation, Band Pass Filtered Data (3-8 years), 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-2.26E-06 [0]	4.22E-05 [0]	2.54E-07 [22]	-1.82E-06 [0]	-2.46E-06 [0]	3.44E-06 [21]
Standard error	4.01E-05	0.001	4.82E-05	3.27E-05	4.83E-05	6.26E-05
Adjusted mean	-	-	4.25E-07	-	-	9.03E-07
$E_t\pi_{t+1}$	2.179 [1000]	3.816 [580]	2.306 [1000]	0.614 [999]	1.127 [763]	0.964 [999]
Standard error	0.195	51.040	0.272	0.108	0.640	0.169
Adjusted mean	2.179	1.402	2.306	0.614	0.936	0.963
$E_tg_{c,t+1}$	0.277 [1000]	0.570 [730]	0.302 [1000]	0.170 [1000]	0.262 [851]	0.207 [1000]
Standard error	0.016	5.546	0.025	0.017	0.103	0.026
Adjusted mean	0.277	0.265	0.302	0.170	0.230	0.207
$E_tg_{l,t+1}$	-0.295 [997]	-0.737 [526]	-0.359 [999]	-0.210 [732]	-0.263 [405]	-0.277 [935]
Standard error	0.067	8.208	0.085	0.088	0.203	0.111
Adjusted mean	-0.294	-0.242	-0.359	-0.182	-0.152	-0.272
$E_tg_{V,t+1}$	-0.196 [1000]	-0.347 [807]	-0.269 [1000]	-0.158 [998]	-0.307 [944]	-0.236 [1000]
Standard error	0.024	0.271	0.031	0.032	0.077	0.042
Adjusted mean	-0.196	-0.273	-0.269	-0.158	-0.292	-0.236
E_tR_{t+1}	-1.761 [1000]	-5.586 [335]	-1.729 [1000]			
Standard error	0.201	114.905	0.322			
Adjusted mean	-1.761	-0.712	-1.729			
$Mean;$						
R-square	0.830	<0	0.782	0.625	<0	0.522
Adjusted R-square	0.821	<0	0.770	0.609	<0	0.501
Pr(F-statistic)	2.50E-24 (1000)	0.051 (907)	N/A	5.04E-10 (1000)	0.003 (985)	N/A
Pr(J-statistic)	N/A	N/A	0.315 {1000}	N/A	0.298 {757}	0.298 {1000}
Durbin-Watson	1.558 <141>	2.059 <972>	2.040 <881>	1.954 <864>	2.052 <998>	2.205 <977>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 5: Taylor Condition Estimation, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

The simulated data remains unchanged, therefore equation (27) represents a misspecified version of the 'correct' estimating equation, which continues to be equation (26).[17] In particular, equation (27) can be seen to correspond to the misspecified Taylor condition, equation (24).

The results are similar across the HP, the $3-8$ band pass and the $2-15$ band pass filters but as the latter filter gives the most statistically significant coefficient estimates only estimates from the $2-15$ filter are presented (Table 6). Comparing the general features of the results to those presented in Tables 3-5, there has been a decline in the precision with which the coefficients are estimated, a decline in the magnitude of the R-square and adjusted R-square statistics and a decline in the number of rejections of the null hypothesis of the F-statistic for joint significance. This is not surprising given that an element of misspecification has been introduced into the estimating equation. The number of rejections of the null hypothesis of the D-W test statistic also tends to increase although the GMM procedure applied to $2-15$ filtered data still fails to reject the null for 94.5% of the simulated samples.

The estimated inflation coefficients are now found to be substantially greater than the coefficients obtained from the 'correctly specified' estimating equation (26) and hence substantially greater than the predicted value. For instance, the GMM estimate for the unrestricted estimating equation rises from 2.306 in Table 5 to 5.274 in Table 6 (or 5.235 according to the adjusted mean). Similarly, the OLS estimate increases from 2.179 to 4.219 (or 4.185 adjusted).[18]. The estimates clearly diverge further from the theoretical prediction of $\Omega = 2.125$ under this particular form of misspecification.

The incorrectly specified estimating equation also induces a substantial decrease in the estimated coefficients for the productive time growth rate and the forward nominal interest rate. The estimated coefficient on the productive time growth rate decreases from -0.294 to -2.073 (both adjusted means) between Table 5 and Table 6 according to the OLS estimator and from -0.359 to -2.790 for the GMM estimator, hence the estimates diverge further from their predicted value of $\beta_3 = 0$. The GMM estimates of the forward interest rate term also decrease from -1.729 in Table 5 to -4.372 in Table 6 (adjusted means where appropriate). Again, the estimates diverge further from theoretical prediction of -1.125.

The estimated coefficients for output growth in Table 6 are comparable to those for consumption growth presented in Table 5, despite the effect that the misspecification has on the other estimates. For example, the OLS estimate for β_3 is 0.300 (adjusted mean) in Table 6 compared to the corresponding estimate of 0.277 in Table 5. For the GMM estimator the coefficient on output growth is 0.402 (adjusted mean) in Table 6 compared to the corresponding estimate of

[17] The instrument sets used for the 2SLS and GMM estimators are modified by replacing consumption growth with output growth but remains unchanged in terms of the number of lags included.

[18] Corresponding upward shifts in the estimated inflation coefficient are found for the $3-8$ band pass filter results (results not reported) and even larger increases are found for the HP filtered data (results not reported).

0.302 reported in Table 5.

The velocity growth term is estimated precisely by the GMM estimator even after the modification to the estimating equation. Estimates of β_4 retain the correct sign and are of a similar magnitude as under the correctly specified estimating equation; for example, a GMM estimate of -0.190 in Table 6 compared to a corresponding estimate of -0.269 in Table 5.

For the restricted specification ($\beta_5 = 0$), the estimates undergo similar changes as those obtained from the restricted version of the 'correct' estimating equation (26). The OLS and GMM estimators generate inflation coefficients which often fall below unity in a manner incompatible with the theoretical model from which the Taylor condition is derived, although the GMM estimator provides a notable exception (Table 6).

In short, the results obtained from applying equation (27) to the simulated data show that adapting the estimating equation in a seemingly minor way can have a substantial impact upon the coefficient estimates obtained. The erratic results produced by this misspecified estimating equation provide an illustration of the fundamental difference between the Taylor condition and a conventional interest rate rule. Unlike a Taylor rule, the Taylor condition cannot be modified in an *ad hoc* manner.[19] In order to make the progression from (26) to (27) in a legitimate manner, one would need to alter the underlying model by excluding physical capital, for example. A new set of artificial data would then need to be simulated from this alternative model prior to re-estimation.

5.4 A Conventional Interest Rate Rule

The estimation procedure is now re-applied to the following estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \qquad (28)$$

This estimating equation corresponds to the misspecified representation of the Taylor condition with output growth plus further restrictions on the terms in productive time and the velocity of money; see equation (25). Equation (28) can be interpreted as a 'dynamic forward-looking Taylor rule' for $\beta_5 \neq 0$ or a 'static forward-looking Taylor rule' under the restriction $\beta_5 = 0$. Notably, the term in velocity growth is absent from this expression. This omission might be expected to have a bearing on the estimates because equations (26) and (27) produced a large number of statistically significant estimates for the velocity growth coefficient.

The results are again similar across the HP and band pass filters so only the $2-15$ band pass results are presented (Table 7).[20] The estimates are generally

[19] In contrast, conventional interest rate rules are exogenously specified and thus amenable to arbitrary modifications. Clarida et al. (1998), for example, add the exchange rate to the standard Taylor rule and Cecchetti et al. (2000) and Bernanke and Gertler (2001) consider whether policymakers should react to asset prices.

[20] The instrument set now comprises of four lags of expected future inflation, four lags of expected future output growth, the second, third and fourth lags of the nominal interest rate and a constant term for the GMM estimator or just the shortest lag of each and a constant

BP Filter, 2-15 Window	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-3.66E-06 [0]	0.001 [0]	3.07E-06 [16]	-9.34E-07 [0]	-4.46E-06 [0]	2.80E-06 [5]
Standard error	6.19E-05	0.031	8.58E-05	2.43E-05	9.65E-05	6.42E-05
Adjusted mean	-	-	-9.38E-07	-	-	4.23E-07
$E_t\pi_{t+1}$	4.219 [971]	-25.003 [189]	5.274 [961]	0.541 [941]	2.338 [882]	1.101 [990]
Standard error	1.715	1290.303	2.582	0.170	1.151	0.264
Adjusted mean	4.185	2.522	5.235	0.532	2.010	1.100
$E_tg_{y,t+1}$	0.303 [967]	-2.019 [206]	0.406 [940]	0.038 [563]	0.211 [262]	0.082 [956]
Standard error	0.125	122.643	0.204	0.020	0.304	0.029
Adjusted mean	0.300	0.244	0.402	0.029	0.077	0.081
$E_tg_{l,t+1}$	-2.098 [959]	14.698 [211]	-2.815 [954]	-0.284 [424]	-1.462 [353]	-0.610 [941]
Standard error	0.892	825.197	1.417	0.189	1.740	0.232
Adjusted mean	-2.073	-1.672	-2.790	-0.189	-0.655	-0.598
$E_tg_{V,t+1}$	-0.118 [884]	0.069 [310]	-0.191 [970]	-0.095 [733]	-0.246 [587]	-0.158 [923]
Standard error	0.042	16.979	0.064	0.043	0.164	0.061
Adjusted mean	-0.113	-0.117	-0.190	-0.084	-0.161	-0.156
E_tR_{t+1}	-3.878 [907]	29.503 [128]	-4.498 [849]			
Standard error	1.812	1437.910	2.802			
Adjusted mean	-3.767	-1.757	-4.372			
$Mean;$						
R-square	0.361	<0	0.127	0.246	<0	<0
Adjusted R-square	0.327	<0	0.079	0.214	<0	<0
Pr(F-statistic)	0.001 (995)	0.379 (411)	N/A	0.020 (924)	0.055 (829)	N/A
Pr(J-statistic)	N/A	N/A	0.226 {1000}	N/A	0.260 {682}	0.264 {1000}
Durbin-Watson	1.882 <699>	1.982 <868>	2.342 <945>	2.295 <1000>	2.145 <997>	2.728 <999>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 6: Output Growth instead of Consumption Growth, Band Pass Filtered data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

found to be poor in terms of the number of statistically significant estimates produced and in terms of mean R-square and adjusted R-square statistics. This is not surprising given that yet another source of misspecification has been added to the estimating equation.

The mean coefficient on inflation does not exceed unity for any of the three estimators considered. The results are also comparatively weak in terms of the frequency with which the null hypothesis of the F-statistic is rejected and in terms of the number of non-rejections of the null hypothesis of the Hansen J-test. The latter finding calls into question the validity of the instrument set used for the GMM estimator for equation (28).

The inflation coefficients are estimated surprisingly precisely under the restriction on β_5. However, these estimates differ quite substantially between estimating procedures for the $2-15$ filter; 0.317 (adjusted mean) for OLS compared to 0.892 for GMM (adjusted mean).

In short, imposing a 'conventional Taylor rule' restricts the true estimating equation to such an extent that the theoretical prediction that the coefficient on expected inflation exceeds unity is not verified. An estimated inflation coefficient of this magnitude might erroneously be interpreted to signify that the Taylor principle is violated but this result is simply a product of a misspecified estimating equation in the present context. Only if the model excluded physical capital and set velocity to one, by exluding exchange credit for example, would such an estimating equation be appropriate.

6 Alternative Interpretations of the Taylor Condition

Consider two alternative representations of the Taylor condition; a backward-looking version and an alternative version which features credit.

6.1 Backward Looking Taylor Condition

The Taylor condition can be reformulated to feature a lagged dependent variable on the right hand side instead of the lead dependent variable which appears in equation (22). This yields a similar expression written in terms of R_{t+1} instead of R_t:

$$\begin{aligned}\overline{R}_{t+1}-\overline{R} &= \frac{\Omega}{(\Omega-1)}E_t\left(\overline{\pi}_{t+1}-\overline{\pi}\right)+\frac{\Omega\theta}{(\Omega-1)}E_t\left(\overline{g}_{c,t+1}-\overline{g}\right) \\ &\quad -\frac{\Omega\psi(\theta-1)\frac{l}{1-l}}{(\Omega-1)}E_t\overline{g}_{l,t+1}-\frac{\Omega_V}{(\Omega-1)}E_t\overline{g}_{V,t}-\frac{1}{(\Omega-1)}\left(\overline{R}_t-\overline{R}\right). \end{aligned} \tag{29}$$

term for the exactly identified 2SLS estimator.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-6.42E-07 [0]	7.62E-05 [0]	-2.48E-06 [8]	-7.00E-07 [0]	5.16E-06 [0]	-1.85E-07 [12]
Standard error	2.57E-05	0.006	1.15E-04	2.58E-05	2.12E-04	1.13E-04
Adjusted mean	-	-	-1.12E-06	-	-	4.92E-07
$E_t\pi_{t+1}$	0.310 [239]	0.671 [22]	0.132 [344]	0.326 [926]	1.472 [416]	0.894 [981]
Standard error	0.448	162.564	0.955	0.099	3.431	0.315
Adjusted mean	0.185	0.017	0.092	0.317	0.826	0.892
$E_tg_{y,t+1}$	0.020 [277]	-0.185 [40]	0.019 [237]	0.021 [408]	0.035 [163]	0.031 [460]
Standard error	0.017	16.569	0.027	0.012	0.582	0.027
Adjusted mean	0.011	0.012	0.012	0.013	0.041	0.024
E_tR_{t+1}	0.008 [147]	4.225 [27]	0.918 [524]	N/A	N/A	N/A
Standard error	0.475	88.304	1.002			
Adjusted mean	0.012	0.104	0.788			
$Mean$;						
R-square	0.169	<0	<0	0.153	<0	<0
Adjust R-square	0.142	<0	<0	0.136	<0	<0
Pr(F-statistic)	0.029 (887)	0.527 (162)	N/A	0.024 (891)	0.149 (599)	N/A
Pr(J-statistic)	N/A	N/A	0.050 {339}	N/A	0.352 {679}	0.058 {440}
Durbin-Watson	1.828 <850>	2.012 <974>	2.236 <991>	1.817 <783>	2.047 <996>	2.186 <990>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 7: Output Growth in a Standard Taylor Rule, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

While equation (29) compares better to interest rate rules which feature a lagged dependent variable on the right hand side as an 'interest rate smoothing' term, the lead nominal interest rate is now the dependent variable. Such an expression is more akin to a forecasting equation for the nominal interest rate than an interest rate rule. Such a transformation also raises the fundamental issue discussed by McCallum (2010). He argues that the equilibrium conditions of a structural model stipulate whether any given difference equation is forward-looking ("expectational") or backward-looking ("inertial") and that the researcher is not free to alter the direction of causality implied by the model as is convenient. The forward looking representation of the Taylor condition (22) is the long accepted rational expectations version; for example, Lucas (1980) suggests that the forward looking "filters" suit models which feature an optimizing consumer. In fact, we would argue that the timing of the cash-in-advance economy is such that our forward-looking rule in equation (22) is the correct model, while equation (29) is consistent with the alternative "cash-when-I'm-done" timing which we do not employ (see Carlstrom and Fuerst, 2001).

6.2 Credit Interpretation of the Taylor Condition

Christiano et al. (2010) have considered how the growth rate of credit might be included as part of a Taylor rule so that "allowing an independent role for credit growth (beyond its role in constructing the inflation forecast) would reduce the volatility of output and asset prices." The term in velocity growth can be rewritten as the growth rate of credit in the following way: Since $V_t = \frac{c_t}{m_t} = \frac{1}{1-\left(1-\frac{m_t}{c_t}\right)}$, then $V\widehat{V}_t = \left(1-\frac{m}{c}\right)\widehat{\left(1-\frac{m_t}{c_t}\right)}$ and $\overline{g}_{V,t} = \frac{m}{c}\left(1-\frac{m}{c}\right)\overline{g}_{\left(1-\frac{m}{c}\right),t}$ where $\overline{g}_{\left(1-\frac{m}{c}\right),t}$ is the growth rate of normalized credit. The Taylor condition is now rewritten as:

$$\begin{aligned}\overline{R}_t - \overline{R} \quad &= \quad \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\Omega_{\left(1-\frac{m}{c}\right)}E_t\overline{g}_{\left(1-\frac{m}{c}\right),t+1} - \Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right). \qquad (30)\end{aligned}$$

The credit coefficient can be derived as $\Omega_{\left(1-\frac{m}{c}\right)} \equiv \frac{(R-1)(1-\gamma)\left(\frac{m_t}{c_t}\right)^2}{(1-\gamma)^2}\frac{(1-\gamma)\left(1-\frac{m_t}{c_t}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m_t}{c_t}\right)\right]} \geq 0$. A positive expected credit growth rate decreases the current net nominal interest rate $\overline{R}_t$. With velocity set at one as in a standard cash-in-advance economy, neither credit nor velocity would enter the Taylor condition since the credit service does not exist and velocity does not vary over time.

7 Discussion

Expressing the monetary policy process in terms of the nominal interest rate carries the advantage of reconciling the language of economists, who have traditionally depicted the money supply as the instrument of monetary policy, with the language of central bankers, who are more accustomed to conducting policy deliberations in terms of a short-term interest rate (Mehrling, 2006). Alvarez et al. (2001) caution that modelling monetary policy solely in terms of a nominal interest rate rejects the quantity theory in spite of the strong empirical link between money growth, inflation and, interest rates. Schabert (2003), for example, uses the equilibrium conditions of a standard cash-in-advance model to derive the conditions under which a money supply rule and an interest rate rule are 'equivalent', while Fève and Auray (2002) generate simulated data from a similar model and demonstrate that an interest rate rule can be spuriously recovered from this data even though monetary policy is modelled in terms of a money growth rule.

This paper has derived an expression similar to a conventional interest rate rule as an equilibrium condition of an endogenous growth model with endogenous velocity in which monetary policy is characterized as a stochastic money supply rule. The theoretical model underpinning this expression implies that the coefficient on inflation exceeds unity in general, takes a value of unity as a special case at the Friedman (1969) optimum, but that it may not fall below unity. Simulation exercises support the theoretical restriction placed on this coefficient, so long as the estimating equation accurately reflects the equilibrium condition.

The Lucas critique perspective does apply strictly within this paper in that the variance distribution of the money supply process is invariant across the whole period of the simulated data. This means that the average long run money supply growth rate is a "structural" but "policy" parameter that the consumer takes as given and includes in the coefficient on the inflation rate in the equilibrium condition as expressed in 'Taylor rule form'. The inflation coefficient is invariant as all other parameters in this coefficient are also structurally given utility and technology parameters. Comparing this to reality and to results such as Leeper and Zha (2003) that suggest potential changes in the variance distribution of the money supply is a complex task. For example, Orlik and Veldkamp (2012) find that "uncertainty shocks" result from insufficiently complex models of the forecaster, which they reconcile by adding further degrees of state, model and parameter uncertainty within the forecasting process. Their results might be interpreted as allowing the unforecasted aggregate risk to be a result of rare crisis events that are hard to forecast and which require a fat tailed distribution rather than normal distributions, as assumed in our model above. Their unpredicted aggregate risk for U.S. data (Figure 2) appears correlated with unexpected inflation during periods when there were major wars or bank crises that demanded high fiscal expenditure. In other words, "abnormally" high inflation can follow after unexpectedly large government deficits due to wars or crises, after some of this debt gets "monetized" through the central bank buying some of the newly issued debt. If we could assume high kurtosis in order to include such rare events, then it may be that the inflation coefficient of a re-derived Taylor condition would in this environment be invariant over the entire sample period. In other words, rare events could seem to cause shifts in variance which are better represented by high kurtosis. This would mean for actual data that the Lucas critique might hold for one regime of money supply policy over the whole period rather than finding shifts in regime, a speculation that qualifies the broad interpretation of our results but remains a task for future research.

Our results can be interpreted in several other ways. First, the derivation could be said to represent an 'equivalence proposition' between the money supply process modelled and an 'interest rate rule', which actually represents an equilibrium condition of the model. This would be similar to the interpretation adopted in Alvarez et al. (2001), Végh (2002) and Schabert (2003).

Second, the Taylor condition can be interpreted as the interest rate rule which results from the money supply process in the context of the Benk et al. (2010) model. Woodford similarly derives the interest rate rule which "implements" strict inflation targeting in the New Keynesian model (Woodford, 2003, pp.290-295). However, the money supply does not enter that model. Changes in the velocity of money therefore play no role and thus cannot be used to help explain why traditional Taylor rules might be using misspecified estimating equations in finding an inflation coefficient of less than one in empirical applications. The fact that our framework assigns a central role to money potentially implies that the money growth rule can offer guidance to policy-makers at times when the conventional monetary policy instrument encounters the zero lower bound, as is the case at the present time.

Third the Taylor condition contrasts with the equilibrium condition for the nominal interest rate derived from a standard Euler equation: Canzoneri et al. (2007, p.1866), for example, derive an expression for the nominal interest rate from a conventional Euler equation in which the coefficient on the inflation term is one.[21] For post-1966 U.S. data, they show that the Euler-equation-implied nominal interest rate fits poorly to the observed nominal interest rate. On the other hand, a conventional Taylor rule with a coefficient on inflation in excess of unity has often been found to fit the observed nominal interest rate rather well (for example, Taylor, 1993). Clarida et al. (2000) estimate such an inflation coefficient for 'post-Volcker' subsamples of U.S. data but not for a pre-Volcker subsample. The Taylor condition (22) therefore represents an equilibrium condition which contains a coefficient on inflation consistent with empirical results which find evidence for a 'Taylor principle', while suggesting that results which fail to find support for the Taylor principle may omit potentially important variables such as the velocity of money. Our prelimary estimation results on actual US data that include velocity growth find an inflation coefficient above one for both subsamples, part of our future work.

The Taylor condition derivation here also resonates with Hetzel (2000) who warns that empirical correlations between a short-term interest rate and macroeconomic variables such as output and inflation cannot be interpreted to reveal the behavior of policy-makers (i.e. their policy rule) unless the relationship obtained can be declared as structural. It is also consistent with Cochrane (2011), who argues that the Taylor rule suffers from an identification problem in the

[21] Their expression is a log-normal approximation to a standard Euler equation and is written in terms of the inverse of the gross nominal interest rate. Therefore, it also contains second moments and the coefficient on inflation has a theoretical coefficient of minus one.

New Keynesian model. Our contribution has been to offer one very particular explanation based on a neoclassical monetary model extended to include endogenous growth and endogenous velocity in order to shed light on the structural relationships which might underpin the reduced form expressions to which Hetzel (2000) refers.

Finally, the Taylor condition here captures bank crises. A decrease in the productivity of banking causes a more costly supply of exchange credit and substitution towards a higher demand for real money balances. Velocity goes down. Velocity has in fact decreased significantly during the current period of bank-crisis-with-prolonged unemployment, and this is captured in the Taylor condition through a negative expected growth rate of money velocity. Following a narrow Taylor rule is not the point then in this environment. Behavior could still be described by the Taylor condition once the velocity and employment growth terms are included as in this model. The point is to rein in fiscal expenditure during crisis periods without printing so much money as to cause high inflation. Liquidity effects involved in driving the interest rate to zero are unlikely to be surprises in the current crisis due to the 'forward guidance' issued by the U.S. Federal Reserve. They have been printing money which fills the void created by the drop in bank's exchange credit, thus avoiding deflation in a way many have felt is appropriate. This means that equilibrium conditions like the Taylor condition of this model need not be departed from. The Taylor rule without velocity or employment terms only applies when these variables are not significantly varying, which has not been the case in the recent crisis or in the inflation run-up from the late 1960s to the early 1980s, nor during the Great Depression. Significant cycles in velocity from trend are the rule rather than the exception as shown in Benk et al. (2010) who document only two long cycles downward in U.S. monetary history since 1919: during the banking crisis of the Great Depression and during the first decade of the 21st century that has included the first major banking crisis since the Depression. Perhaps a more efficient comprehensive financial intermediary deposit insurance scheme, with risk-based premiums, would be more efficient government policy than the current amalgam, but this involves bank productivity which this model and its Taylor condition take as given exogenously.

8 Conclusion

The paper has derived a general equilibrium dynamic Taylor condition for a constant relative risk aversion economy with leisure, Lucas (1988) endogenous growth, and with endogenous velocity through production of exchange credit in a financial intermediary. The importance of a fluctuating velocity in replicating the 'Taylor principle' is consistent with the role for velocity reported by Reynard (2004, 2006) and with Benk et al. (2010).

While providing a theoretical means to overview the empirical literature relating to the Taylor rule, as reviewed by Siklos and Wohar (2005), here the focus is first to show that estimation of a Taylor rule may result in the spurious inference that the central bank is engaged in Taylor rule reaction behavior rather than simply supplying money. This is established here by generating artificial data as simulated from the model and then successfully estimating a theoretical Taylor condition. This condition is simply an equilibrium condition in the economy in which the central bank stochastically makes changes in the money supply growth rate to finance government spending. For example, such money supply changes tend to occur whenever the government needs to depart from its stationary money supply growth rate and resort to the 'fiscal inflation tax'. This typically can occur during banking crisis, recession, or war.

Money velocity growth itself enters as a variable and ends up playing a potentially significant role; in particular this occurs when velocity is changing, such as during the recent banking crisis and during the 1930s when U.S. velocity cycled downwards, as identified in Benk et al. (2010), and in the "pre-Volcker" U.S. high inflation of the 1970s. Velocity is endogenized in the model following the banking financial intermediation microeconomic literature, where financial services are produced according to a Cobb-Douglas production function that includes deposited funds as an input. This approach implies a bank service sector value-added that is consistent with the U.S. national income accounting treatment of the bank service sector, as emphasized by Inklaar and Wang (2013).

The paper shows how the banking production of exchange credit is surprisingly crucial to the derivation of the 'Taylor principle'. This result emanates from an endogenous velocity of money; a simple (cash-only) cash-in-advance constraint with a constant velocity of one is shown to provide an inflation coefficient of unity. Through endogenous growth, we can derive an output gap measure not inconsistent with Taylor and Wieland's (2010) emphasis on changes in output as a measure for the output gap. In our model, the output growth term does not enter directly unless we also include an investment growth term; otherwise the consumption growth is the 'output gap' term of the model's Taylor condition.

Estimation results are also provided for two misspecified models using the simulated data from the correct model. One includes output growth without including investment growth. The second is a standard Taylor rule which is appropriate for the model economy only if there is no physical capital and if exchange credit does not exist as an alternative to money ($A_F = 0$). Omitted variables cause significant misspecification bias in the reported results. The implication is that the results hold promise for explaining disparate estimated rules across different periods and countries, as well as during bank crises, sudden financial deregulation, or times of other significant shifts in money velocity. This could help organize and show greater robustness for this literature.

By simulating data of the model and estimating successfully a 'Taylor rule' from the data, the paper shows that such a rule can be identified econometrically from the economy's asset pricing behavior when the central bank simply prints money stochastically. In that case it would be spurious to claim that an estimated Taylor rule reveals how the central bank actually conducts policy through interest rate targeting. Rather, the central bank simply satisfies the government's fiscal needs via direct and indirect taxes, including the inflation tax. Put differently, if this economy were representative of the actual economy then estimating a standard Taylor (1993) model using actual data would be expected to produce an inflation coefficient above one in keeping with the Taylor principle only if velocity (or exchange credit), employment and investment did not change over the sample period. Reynard (2004, 2006) and Benk et al. (2010), for example, show the importance of time-varying velocity while significant business cycle fluctuations in employment and investment are a well-documented feature of business cycle research.

An important qualification is that the paper does not claim regime changes do not occur. Rather it analyzes the equilibrium within a regime of given first and second moments of the stochastic money supply process. Regime changes in our framework could cause different magnitudes of the Taylor condition inflation coefficient but still always above one for a positive nominal interest rate.

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