



The Graduate Center of CUNY Ph.D. Program
in Mathematics

Course Description -Spring 2021

Course Title: Computability Theory Throughout Mathematics

Course #: Math 713

Day & Time: Friday 9:30 - 11:30 a.m.

Instructor Name: Prof. Russell Miller

Contact Information: RUSSELL.MILLER@QC.CUNY.EDU

Pre-Requisites: None. (Math 711-712 are helpful but not necessary.)

Office Hours: Friday 11:30 - 12:30

Description:

Computability Theory Throughout Mathematics

This course will explore questions of decidability and computability in assorted areas of mathematics, drawn largely from the instructor's own research. From algebra, there will be a particular focus on fields and polynomials, including questions about classifying the isomorphism types within certain classes of countable fields, about computing isomorphisms between fields, and about identifying transcendence bases for non-algebraic field extensions. Related topics from number theory will also appear, centered around Hilbert's Tenth Problem. (The original version of this problem, for the integers, was resolved in 1970 by Matiyasevich, but its generalization to the rational numbers remains a major open question.) Some computable analysis will be discussed, and it is possible that differential equations, differential algebra, and/or some descriptive set theory could enter into the course.

The first-year logic sequence, and in particular the three or four weeks that cover computability theory, definitely provides adequate background for this course. Some of that background may be reviewed at the start of the course if it seems useful to do so. Moreover, it will be possible to take this course, and understand most of the concepts, without a rigorous focus on the details of Turing machines and theoretical computation. For most of our topics, simply replacing "Turing machine" by "computer" in the definitions will create reasonable (although not rigorous) and understandable concepts, making the rest of the course accessible to the non-logician.