SYLLABUS FOR FIRST EXAMINATION IN COMPLEX VARIABLES

The emphasis in the complex variables first examination may vary somewhat from year to year. To prepare for it, one should become familiar with the topics listed below, together with related examples and applications. The examination will not necessarily contain questions from all the listed topics, and the complex variables course may cover some more specialized additional topics.

TOPICS THAT MAY BE COVERED ON THE EXAM

Basic facts about complex numbers - geometric interpretation of complex numbers and arithmetic operations on them, completeness, conjugation, absolute value, inequalities, etc., conformality, the Cauchy-Riemann equations and the notion of complex differentiability, the Riemann sphere and stereographic projection, fractional linear transformations, the exponential and logarithmic functions.

Holomorphic and meromorphic functions - the Cauchy theorems, the significance of connectivity, Taylor and Laurent series, residues and the calculation of integrals by residues, the maximum-modulus theorem, Liouville's theorem, the fundamental theorem of algebra, Rouché's theorem and the argument principle, the open-mapping theorem, meromorphic functions, essential singularities, removable singularities, the Casorati-Weierstrass theorem, analytic continuation, essential boundaries, the monodromy theorem, uniform convergence of sequences of holomorphic functions, Runge's theorem, partial fractions, Mittag-Leffler's theorem on meromorphic functions with prescribed principal parts, infinite products, the Weierstrass product theorem for entire functions, Picard's theorem.

Conformal mapping - Schwarz's lemma, the mapping behavior of fractional linear transformations and other simple functions, the analytic automorphisms of the unit disk and upper half-plane, Blaschke products, explicit mappings of various simple domains onto the unit disk, the Schwarz reflection principle, normal families, the Riemann mapping theorem, boundary behavior in the Riemann mapping theorem for non-pathological domains.

Harmonic functions - connection with holomorphic functions, the Laplace operator, the Poisson integral and the mean value property, the maximum and minimum modulus theorems, Jensen's theorem.