QUALIFYING EXAMINATION IN GEOMETRY

1. Basic definitions
   (a) review of advanced calculus
   (b) definition of manifolds
   (c) examples: matrix groups, spheres, projective spaces, Stiefel and Grassman spaces
   (d) immersions, submersions, coverings
   (e) partitions of unity, sets of measure 0, Sard's theorem, elementary imbedding and immersion theorems
   (f) tangent bundles, frame bundles, general vector bundles (including Whitney sums, tensor products, and exterior products)
   (g) vector fields, flows, Lie bracket and Lie derivatives, Frobenius theorem
   (h) Lie groups, Lie algebras, 1–parameter subgroups
   (i) (optional) degree theory via Sard's theorem
2. Differential forms and integration
   (a) definition of p–forms, exterior products, and exterior derivatives
   (b) integration over singular p–chains, and Stokes' theorem
   (c) Poincaré lemma, polar coordinates, computation of $H^n(M)$ for $n = \dim M$.
   (d) degree theory, with application to curves
3. Riemannian geometry I
   (a) Riemannian structures, arc length, connections (especially, Riemannian connection), isometries
   (b) parallel translation, holonomy, geodesics, exponential map, normal coordinates
   (c) Gauss' lemma, convex neighborhood theorem
   (d) Hopf–Rinow theorems
4. Riemannian geometry II
   (a) curvature tensor, Bianchi identities, sectional and Ricci and scalar curvatures.
   (b) examples: surfaces, geodesic curvature, Gauss–Bonnet theorem
   (c) examples: Lie groups, symmetric spaces, and the classical space forms
   (d) Cartan–Hadamard theorem, and some of its consequences
5. Optional topics
   (a) second variation of arc length, Jacobi fields, comparison theorems
   (b) de Rham's theorem, Hodge star operator, de Rham Laplacian
   (c) Hodge theorem
   (d) Morse functions and handle-body decompositions