# TYPICAL TOPICS FOR THE FIRST EXAMINATION IN REAL VARIABLES

Revised June 2011

The emphasis in the real variables first examination may vary somewhat from year to year. To prepare one should become familiar with the topics listed below, together with related examples and applications. The examination will not necessarily contain questions from all the listed topics, and the real variables *courses* may cover additional specialized topics.

TOPICS THAT MAY BE COVERED ON THE EXAM

**From Functions of a Real Variable - Math 70100**

The real number system, including the least upper bound property; elementary aspects of cardinal numbers (countable and uncountable sets, the uncountability of the reals); elementary point-set topology, including compactness, connectedness, metric spaces, complete metric spaces, metrizable spaces and the Baire Category Theorem; continuous functions between topological spaces; the topology of euclidean space $\mathbb{R}^n$, including the Heine-Borel and Bolzano-Weierstrass Theorems; function spaces, including sequences and series within such spaces, uniform convergence, the Stone-Weierstrass Theorem, and the Arzelà-Ascoli Theorem (a/k/a Ascoli's Theorem); differentiation in several variables, including the Inverse and Implicit Functions Theorems.

**References**

Each of the topics listed above can be found in one or more of the following references, although in some cases not under the given names. For example, the Arzela-Ascoli Theorem appears in Rudin's *Principles* as Theorems 7.23-7.25.


- J. Dieudonné, *Foundations of Modern Analysis*, Academic Press, 1960
- L. Loomis and S. Sternberg, *Advanced Calculus*, available on line.
- W. Rudin, *Principles of Mathematical Analysis*, Third Edition, McGraw-Hill, 1976.
- M. Spivak, *Calculus on Manifolds*, Perseus Books, 1965.


**From Functions of a Real Variable - Math 70200**

Measure theory and the Lebesgue integral, including outer measure, the monotone convergence theorem, Fatou's lemma and the dominated convergence theorem; $L^p$-spaces, including the Hölder and Minkowski inequalities, completeness, and the

$L^p - L^q$-duality; product measures and Fubini's theorem; measure theory and differentiation, including absolutely continuous functions, functions of bounded variation, and the Radon-Nikodym derivative; basic Hilbert and Banach space theory, including bounded linear operators, the Riesz representation theorems, orthonormal systems, Bessel's inequality, Parseval's equality, and applications to Fourier series.

**References**

Each of the topics listed above can be found in one or more of the following references.


- P.R. Halmos, *Measure Theory*, D. Van Nostrand, Princeton, 1950.
- E. Hewett & K. Stromberg, *Real and Abstract Analysis*, Springer-Verlag, 1968.
- J. Oxtoby, *Measure and Category*, Springer-Verlag, 1971.
- F. Riesz & B. Sz-Nagy, *Functional Analysis*, Dover, 1990.
- H. Royden, *Real Analysis*, Third Edition, Prentice-Hall, 1988.
- W. Rudin, *Real and Complex Analysis*, Third Edition, McGraw-Hill, 1987.