



The Graduate Center of CUNY
Ph.D. Program in Mathematics
Course Syllabus

Course Title: Logic I (Part II)

Course #: MATH 71200

Time and Location: Tues. & Thur. 6:30PM - 8:00PM

Instructor Name: Russell Miller

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Pre-Requisites: Math 711 or Permission of Instructor

Office Hours: Tues. & Thurs. 5:30PM - 6:30PM

Description:

The course will divide into three principle (related) segments. First we will consider Kurt Gödel's Incompleteness Theorems. Generally the subject matter is an analysis of exactly what mathematicians mean when they speak of a "proof." We give rigorous definitions for the concept of a mathematical statement, whether it is true or false, and what constitutes a proof of a statement. With these definitions, we can then consider questions such as "is every true statement provable?" The answers are often surprising.

The Incompleteness Theorems are based on the notion of an algorithm. In the second segment of the course, we will make this concept exact by defining Turing machines, a kind of ideal computer. A few weeks will be spent in considering the consequences of our definition. This area of logic is known as computability theory, or recursion theory; along with model theory, set theory, and proof theory, it is one of the four main branches of mathematical logic.

In the last few weeks, we will give some basic definitions and results from set theory as well, including the axiom set ZFC and the notions of ordinals and cardinals. These are less directly related to the Incompleteness Theorems, but they form the foundations of modern mathematics, so at some level they are very much relevant to the rest of the course. For example, until this segment, we will have taken the set ω of natural numbers for granted; now we will see how it arises from the axioms of ZFC.

In other years, Math 71200 may be given with a focus on set theory and just a brief overview of computability, or vice versa. Both set theory and computability are active areas of research in mathematics, and students have completed dissertations in each of these areas at the Grad Center during recent years.