The Graduate Center of CUNY

Ph.D. Program in Mathematics

Course Syllabus

Course Title: Dynamical Systems

Course #: MATH 81400

Time and Location: Fri. 11:00AM - 1:00PM

Instructor Name: Saeed Zakeri

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Pre-Requisites: TBA

Office Hours: TBA

Description:

This course will study in some depth two distinct but related topics: The dynamics of (smooth) homeomorphisms of the circle, and the structure of rotation sets under the multiplication by d map t 7! d t (mod Z) for some integer d • 2. Both theories arise in a variety of problems in onedimensional complex dynamics, and occasionally merge. An illustrative example is the dynamics on the boundary of a Siegel disk of a polynomial of degree d as seen from inside (irrational rotation) versus outside (multiplication by d). However, the results and tools that we discuss are general and go far beyond such particular examples.

The course will consist of 14 two-hour lectures. For each topic we start from the basics and take a path that gets us to interesting results as quickly as possible. Some modest background in dynamics is helpful, but much of the material will be accessible to anyone motivated enough to learn it. Here is a rough outline of the topics:

Part I (about 9 lectures):

 Continued fraction algorithm, arithmetic of rigid rotations, combinatorial and statistical properties of their orbits.

 Rotation number, the Poincaré semiconjugacy, Denjoy's theorem on linearization of circle diffeomorphisms.

- Cross-ratio distortion, S´wia¸tek's cross-ratio inequality for critical circle maps.
- Herman-S´wia¸tek's real a priori bounds, linearization theorems of Yoccoz and Herman.

 Applications in constructing Siegel disks via surgery.

Part II (about 5 lectures):

 Rotation sets under t 7! d t (mod Z), minor and major gaps, gap dynamics, maximal and minimal rotation sets.

 The invariant and gap measures associated with a rotation set.

 New unified approach to the Goldberg-Milnor-Tresser deployment theorem.

 Low-degree computations: Rotation sets under the doubling and tripling maps.

 Connections with the dynamic planes and parameter spaces of families of complex polynomial maps.