Course Title: Topology II, Intermediate Algebraic Topology

Course #: MATH 82600

Time and Location: Mon. 10:45AM - 12:45PM

Instructor Name: Dennis Sullivan

Contact Information: Dsullivan@gc.cuny.edu

Pre-Requisites: TBA

Office Hours: TBA

Description:

In the fall we discussed cycles and homologies by analogy with
oriented closed manifolds and compact manifolds with boundary.
It was important that the boundaries of homologies have collar neighborhoods so that gluing was possible.
The new feature beyond manifolds per se was non manifold points formed a non empty sub complex. The
minimal gluing to obtain cycles resulted in the non manifold points having codimension three.
 Cycles mapping into a target space up to homology defined by homologies mapping into the space formed
a group graded by dimension. These groups were the usual homology groups of the space.

If the singularities were restricted in some way which was geometrically natural [preserved by transversal
intersection] then all but one of the axioms of usual homology theory held for the same construction using
these restricted singular cycles and homologies. These are called generalized homology theories when
some of the groups of a point are non zero in some nonzero degrees. A basic paper of George Whitehead
describes these theories by spatial objects [actually sequences of spaces indexed by all of the integers
with maps between the suspension of one to the next]. Examples are bott-atiyah complex K-theory and
Thom's cobordism.

These generalized theories are defined with coefficients and have dual versions called cohomology
theories which may have cup products.
 One has a canonical DeRham theorem isomorphism describing the dual cohomology theory with real
coefficients by differential forms labeled by the generalized cohomology groups of a point.

In 2004 Hopkins and Singer used this DeRham isomorphism to construct a differential cohomology theory
for anyone of these generalized cohomology theories which combines the latter with differential forms.

In 2008 axioms characterizing ordinary differential cohomology were described. In 2009 differential K
theory was described by vector bundles with connections. In 2015 a dual geometric description
to that one was described by numerical functions on odd dimensional geometric cycles called differential
characters for differential K theory. The numerical invariants lie in the complexified circle and are
computed by integrating certain differential forms defined by the curvature of the connection and the
domain if integration over even dimensional manifolds bounding the cycles and reducing module one. A