TOPICS IN PROBABILITY: *GAUSSIAN PROCESSES*
MATH 83600
CUNY GRADUATE CENTER, SPRING 2015

Instructor: Prof. Louis-Pierre Arguin
Time and Location: Th 4:15pm-6:15pm
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Prerequisites: A good background in real analysis (e.g. MATH 70100) or probability (MATH 83100 and MATH83200) is suggested. Students with an interest in real and functional analysis and/or probability are encouraged to attend.

1. Course Description

A stochastic process is a collection of random variables $(X_v, v \in V)$ indexed on a space V. Of all stochastic processes, the Gaussian processes form a class with nice properties that facilitate their analysis. The goal of the course is to cover the basic theory of Gaussian processes (over a metric space V) and to introduce the students to contemporary research problems on the subject. In the first part of the class, we will study the questions of existence and regularity of these processes. Important features such as the Markov property (on any V), comparison lemmas and concentration inequalities will be discussed [1, 2, 3, 4, 5]. In the second part, the focus will be on large values (extrema) of Gaussian processes and on general methods to find good bounds. These include generic chaining and majorizing measures [6]. The phenomenon of super-concentration about fluctuations of extrema will be studied [7]. In the third part (if time permits), contemporary research questions related to precise aymptotics of extrema of Gaussian fields and their relations to random matrix theory and the local maxima of the Riemann zeta function will be presented.

2. Evaluation

There will be a few homework assignments which will be collected throughout the semester. The course grade will be based on these and on participation.

References

[1] M. B. Marcus and J. Rosen, *Markov processes, Gaussian processes, and local times*, ser. Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge, 2006, vol. 100. [Online]. Available: http://dx.doi.org/10.1017/CBO9780511617997

[2] R. J. Adler and J. E. Taylor, *Random fields and geometry*, ser. Springer Monographs in Mathematics. Springer, New York, 2007.

[3] R. J. Adler, *An introduction to continuity, extrema, and related topics for general Gaussian processes*, ser. Institute of Mathematical Statistics Lecture Notes—Monograph Series, 12. Institute of Mathematical Statistics, Hayward, CA, 1990.

[4] M. Ledoux and M. Talagrand, *Probability in Banach spaces*, ser. Classics in Mathematics. Springer-Verlag, Berlin, 2011, isoperimetry and processes, Reprint of the 1991 edition.

[5] M. Ledoux, *The concentration of measure phenomenon*, ser. Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2001, vol. 89.
[6] M. Talagrand, *The generic chaining*, ser. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2005, upper and lower bounds of stochastic processes.
[7] S. Chatterjee, *Superconcentration and related topics*, ser. Springer Monographs in Mathematics. Springer, Cham, 2014. [Online]. Available: http://dx.doi.org/10.1007/978-3-319-03886-5