Computations with Matrices and Polynomials:

Algebraic and Numerical Algorithms on

Serial and Parallel Computers

Fall 2015 **Code [92312], 3 credits**

Thursdays, 2.00 – 4.00 p.m.; also 4.15-6.15 pm
Room TBA

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Rationale

Algebraic and Numerical Algorithms, and in particular matrix and polynomial algorithms, are the backbone of the modern computations in Sciences, Engineering, and Signal and Image Processing. For example, they are routinely invoked when we turn on our computers, TVs or radios. Due to the demand, the subject has been enjoying high respect in the fields of Computer Science and Computational Mathematics. It is also the source of exciting research challenges in these relatively new dynamic fields. The course will introduce the students to some most fundamental methods and techniques of symbolic and numerical computations, including the techniques of concurrent and randomized computing. In spite of common prejudice, there is no conflict between formal study and practical application of these techniques. The course will provide insights into these fields and some experience in the design, analysis and implementation of modern algorithms and can lead students to research,

currently supported by the Instructor's NSF Grant, to publications in Journals and Proceedings of Conferences, and to the defenses of PhD Theses.

Description

The course will cover some fundamental topics in symbolic and numerical computations and the theory and practice of parallel and randomized computing. The instructor has decades-long experience of teaching and doing research on these subjects, covered in his 4 books and over 270 refereed publications. The course has no prerequisites. The entire area of this study is huge, but THE INSTRUCTOR PLANS TO ADJUST THE STUDY TO THE STUDENTS' INTERESTS. ACCORDINGLY, HE PLANS TO NARROW OR TO EXTEND THE LIST OF TOPICS BELOW. He will facilitate the students' study by supplying reading materials selectively. The course can bring the students to the research frontiers in both areas of Computer Science and Computational Mathematics, and can lead them to defending PhD degree. The students will have a chance to participate (under the instructors' NSF grant) in the design of new algorithms, their formal analysis and computer implementation. The instructor will encourage joint study and research of the students from Computer Science and Mathematics to the benefits of both groups. (In 2013 he has been designated a Fellow of the American Math. Society for his "Contributions to the Mathematical Theory of Computation".) He will be happy to arrange additional meetings with the advanced students to guide them towards their research publications and PhD degree and

would supply materials for this study, but the students can obtain 3 credits just for successful learning.

Topic List

- Fast Fourier transform
- Basic operations of computer algebra
- Structured matrices such as Toeplitz, Hankel, factor circulant, Vandermonde, Cauchy, Frobenius, Resultant, and HSS matrices. Efficient algorithms for them and their link to computations with polynomials
- Data compression by using matrix structures
- General matrices, their factorizations, norms and other basic concepts and techniques
- Parallel matrix and polynomial algorithms
- Randomization methods for matrix and polynomial computations

This high level list of the topics can be elaborated upon and adjusted to the students' interests and background and to the amount of time available for the course. For a particular example, parallel matrix computations can cover the basic techniques of parallel algorithm design, Flynn's taxonomy, Amdahl's law, problem scaling, message routing and MPI, vector and matrix products, the solution of triangular, tridiagonal, and banded linear systems of equations, the fan-in, fan-out, cyclic reduction, and nested dissection techniques,

Gaussian elimination with and without pivoting, and block matrix algorithms. The instructor will be happy to guide the independent study of such topics if for any reason they are not covered in the course.

Leaning Goals

Students are expected to

- Understand the basic principles, concepts and techniques of symbolic and numerical computing, both on serial and parallel computers.
- Learn some fundamentals of algorithm design and analysis, including parallelism and randomization techniques.
- Learn efficient algorithms for the most popular operations with polynomials, rational functions, and general and structured matrices, such as fast Fourier transform (FFT) and Fast Multipole Method (FMM), both listed among the ten most important algorithms of XX century.
- Learn the basic techniques of data compression for structured matrices
- Learn and possibly practice the basics of the implementation of symbolic and numerical algorithms
- Get a chance to advance in research, publications and preparation to the PhD defense based on their study of Algebraic and Numerical Algorithms

Assessment

- Class participations and discussions will be used to evaluate students' understanding of concepts of algebraic and numerical computations. The attendance and participation account for 10% of the final grade
- Homework assignments (40% of the final grade) will be designed to provide the opportunities for students to verify their understanding of the current subjects of the study and their ability to employ the relevant techniques and algorithms introduced for Algebraic and Numerical Computing
- Final and possibly midterm tests will represent 50% of the final grade. They will give students chances to show their overall understanding of the course subjects
- The students’ advances in research and implementation of recent and new algorithms can demonstrate their knowledge and understanding of the course materials. This can be counted as partial substitution for homework and exams towards the final grade